BILLANOOK COLLEGE

NAME: **Student Number:**

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Test B TECHNOLOGY ACTIVE

Thursday 19th July, 2018
Reading time: 15 minutes 8:30am – 8:45am
Writing time: 1.5 hours 8:45am – 10:15am

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of Questions*	*Number of questions to be answered*	*Number of marks*
1	15	15	15
2	4	4	39

Students are permitted to bring into the SAC room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality maybe used.

Students are NOT permitted to bring into the SAC blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book, Multiple Choice answer sheet, Formula sheet

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

SECTION A – Multiple-choice questions

Question 1

Let $f: R \to R, f(x) = 3 - 2\cos\left(\frac{\pi x}{4}\right)$.

The period and range of this function are respectively

A. 4 and $[-2, 2]$

B. 8 and $[1, 5]$

C. 8π and $[1, 5]$

D. 8π and $[-2, 2]$

E. $\frac{1}{2}$ and $[-1, 5]$

Question 2

Let $f: R \to R, f(x) = 5\sin(2x) - 1$.

The period and range of this function are respectively

A. π and $[-1, 4]$

B. 2π and $[-1, 5]$

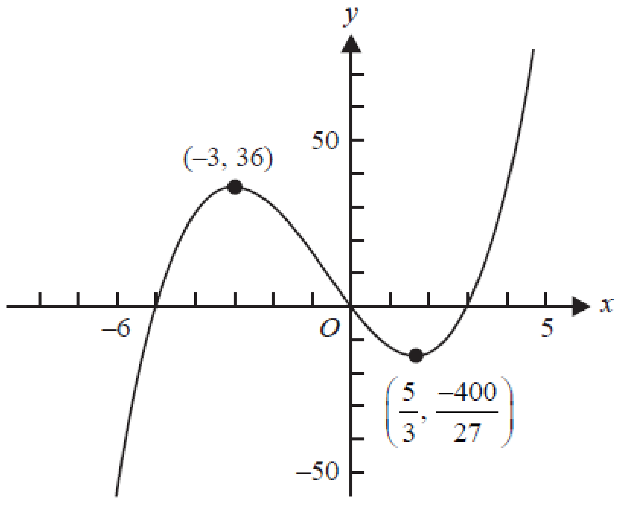
C. π and $[-6, 4]$

D. 2π and $[-6, 4]$

E. 4π and $[-6, 4]$

Question 3

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval

A. $(0, 3)$

B. $(-\infty, -5) \cup (0, 3)$

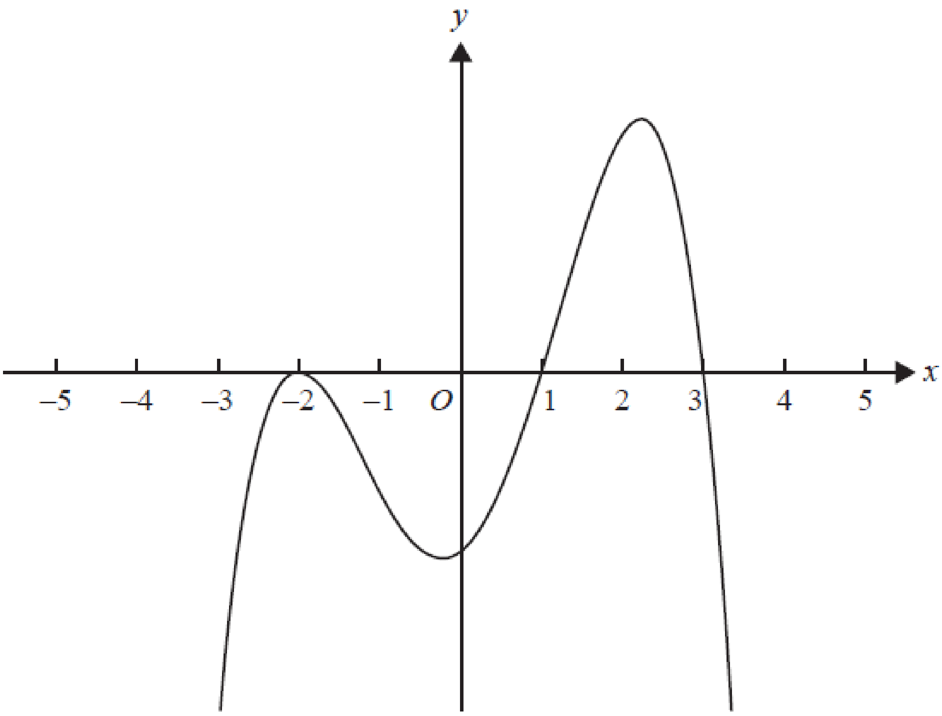
C. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left(-3, \frac{5}{3}\right)$

E. $\left(\frac{-400}{27}, 36\right)$

Question 4

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is

A. $y = (x + 2)(x - 1)(x - 3)$

B. $y = (x + 2)^2(x - 1)(x - 3)$

C. $y = (x + 2)^2(x - 1)(3 - x)$

D. $y = -(x - 2)^2(x - 1)(3 - x)$

E. $y = -(x + 2)(x - 1)(x - 3)$

Question 5

A set of three numbers that could be the solutions of $x^3 + ax^2 + 16x + 84 = 0$ is

A. $\{3, 4, 7\}$

B. $\{-4, -3, 7\}$

C. $\{-2, -1, 21\}$

D. $\{-2, 6, 7\}$

E. $\{2, 6, 7\}$

Question 6

Let f and g be functions such that $f(2) = 5$, $f(3) = 4$, $g(2) = 5$, $g(3) = 2$ and $g(4) = 1$.
The value of $f(g(3))$ is

A. 1

B. 2

C. 3

D. 4

E. 5

Question 7

The average rate of change of the function with the rule $f(x) = x^2 - 2x$ over the interval $[1, a]$, where $a > 1$, is 8.
The value of a is

A. 9

B. 8

C. 7

D. 4

E. $1+\sqrt{2}$

Question 8

The sum of the solutions to the equation $\sqrt{3}\sin(2x) = -3\cos(2x)$ for $x \in [0, 2\pi]$ is equal to

A. $\frac{\pi}{3}$

B. $\frac{7\pi}{6}$

C. $\frac{11\pi}{3}$

D. $\frac{13\pi}{3}$

E. $\frac{14\pi}{3}$

Question 9

The range of the function $f:\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R,\ f(x) = 2x^3 - 3x + 4$ is

A. $\left(4-\sqrt{2}, 4+\sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

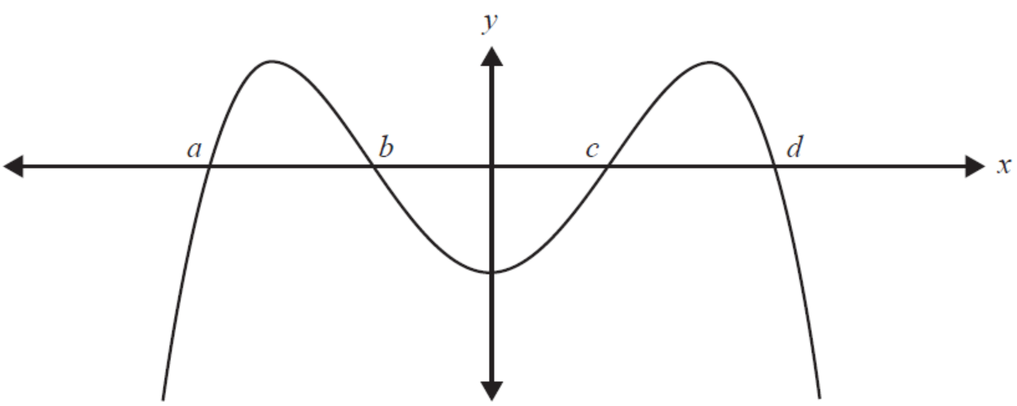
C. $\left(4-\sqrt{2}, 4+\sqrt{2}\right]$

D. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right]$

E. $\left[4-\sqrt{2},\ 4+\sqrt{2}\right]$

Question 10

The graph of a function f, where $f(-x) = f(x)$, is shown below.



The graph has x-intercepts at $(a, 0)$, $(b, 0)$, $(c, 0)$ and $(d, 0)$ only.
The area bound by the curve and the x-axis on the interval $[a, d]$ is

A. $\int_a^d f(x)\,dx$

B. $\int_a^b f(x)\,dx - \int_c^b f(x)\,dx + \int_c^d f(x)\,dx$

C. $2\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

D. $2\int_a^b f(x)\,dx - 2\int_b^{b+c} f(x)\,dx$

E. $\int_a^b f(x)\,dx + \int_c^b f(x)\,dx + \int_d^c f(x)\,dx$

Question 11

The graph of the function f is obtained from the graph of the function g with rule $g(x)=3\cos\left(x-\frac{\pi}{6}\right)$ by a dilation of a factor of $\frac{1}{2}$ from the x-axis, a reflection in the y-axis, a translation of $\frac{\pi}{6}$ units in the negative x direction and a translation of 4 units in the negative y direction, in that order.
The rule of f is

A. $f(x)=\frac{3}{2}\cos\left(-x-\frac{\pi}{3}\right)-4$

B. $f(x)=\frac{3}{2}\cos(-x)-4$

C. $f(x)=-\frac{3}{2}\cos(x)-4$

D. $f(x)=-3\cos\left(\frac{x}{2}-\frac{\pi}{3}\right)-4$

E. $f(x)=\frac{3}{2}\cos\left(-x+\frac{\pi}{3}\right)-4$

Question 12

Let $h:(-1, 1)\rightarrow R$, $h(x)=\dfrac{1}{x-1}$.
Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

Question 13

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for

A. $m = 7$ or $m = -7$

B. $m = 12$ or $m = 3$

C. $m \in R\backslash\{-7, 7\}$

D. $m = 4$ or $m = 3$

E. $m \in R\backslash\{12, 1\}$

Question 14

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

Question 15

Question 10

A transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ maps the graph of $y = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right)$ onto the graph of

A. $y = \sin(x + \pi)$

B. $y = \sin\left(x - \frac{\pi}{2}\right)$

C. $y = \cos(x + \pi)$

D. $y = \cos(x)$

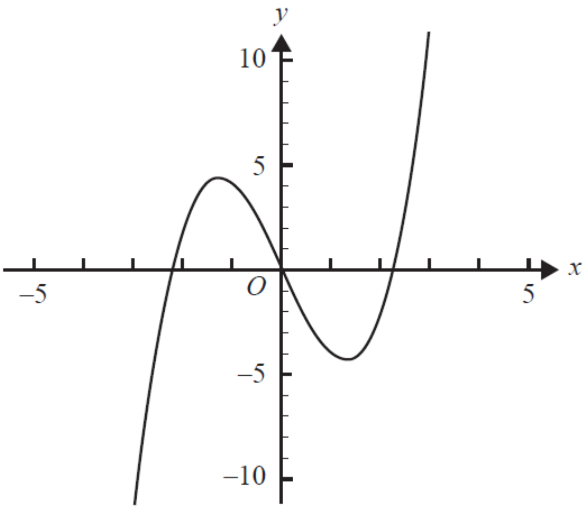
E. $y = \cos\left(x - \frac{\pi}{2}\right)$

SECTION B

Answer all questions in this section.

Question 1 (11 marks)

Let $f: R \rightarrow R,\ f(x) = x^3 - 5x$. Part of the graph of f is shown below.



a. Find the coordinates of the turning points. 2 marks

b. $A(-1, f(-1))$ and $B(1, f(1))$ are two points on the graph of f.

i. Find the equation of the straight line through A and B. 2 marks

ii. Find the distance AB. 1 mark

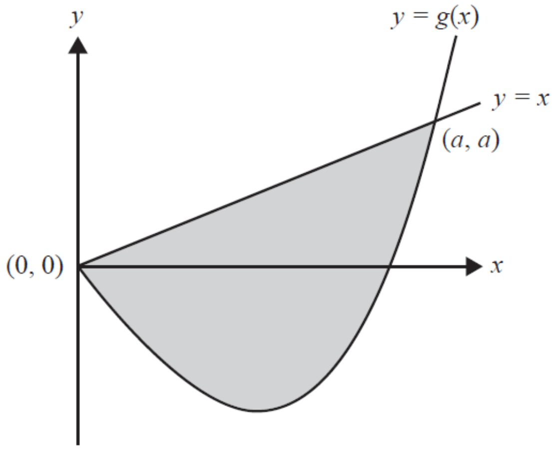
Let $g : R \to R, g(x) = x^3 - kx, k \in R^+$.

c. Let $C(-1, g(-1))$ and $D(1, g(1))$ be two points on the graph of g.

i. Find the distance CD in terms of k. 2 marks

ii. Find the values of k such that the distance CD is equal to $k + 1$. 1 mark

d. The diagram below shows part of the graphs of g and $y = x$. These graphs intersect at the points with the coordinates $(0, 0)$ and (a, a).



i. Find the value of a in terms of k. 1 mark

ii. Find the area of the shaded region in terms of k. 2 marks

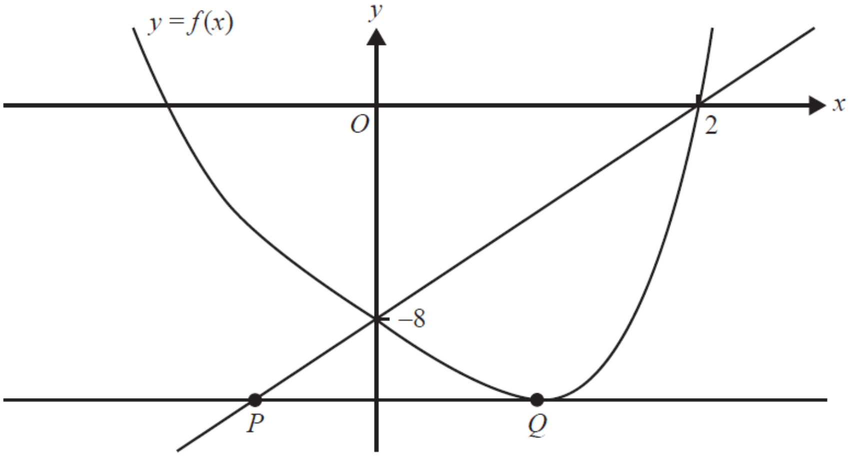
Question 2 (9 marks)

Let $f : R \to R, f(x) = x^4 - 4x - 8$.

a. Given $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find a, b and c. 1 mark

b. Find two consecutive integers m and n such that a solution to $f(x) = 0$ is in the interval (m, n), where $m < n < 0$. 2 marks

The diagram below shows part of the graph of f and a straight line drawn through the points $(0, -8)$ and $(2, 0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph of f at the point Q. The two straight lines intersect at the point P.



c. **i.** Find the equation of the line through $(0, -8)$ and $(2, 0)$. 1 mark

ii. State the equation of the line through the points P and Q. 1 mark

iii. State the coordinates of the points P and Q. 2 marks

d. A transformation $T: R^2 \to R^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d \\ 0 \end{bmatrix}$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. 1 mark

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(n, 0)$ respectively, under the transformation T, where m and n are defined in **part b.**

Find the values of m' and n'. 1 mark

Question 3 (11 marks)

The temperature, T °C, in an office is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function

$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right), 0 \le t \le 24.$

a. What are the maximum and minimum temperatures in the office? 2 marks

b. What is the temperature in the office at 6.00 am? 1 mark

c. Most of the people working in the office arrive at 8.00 am.

What is the temperature in the office when they arrive? 1 mark

d. For how many hours of the day is the temperature greater than or equal to 19 °C? 2 marks

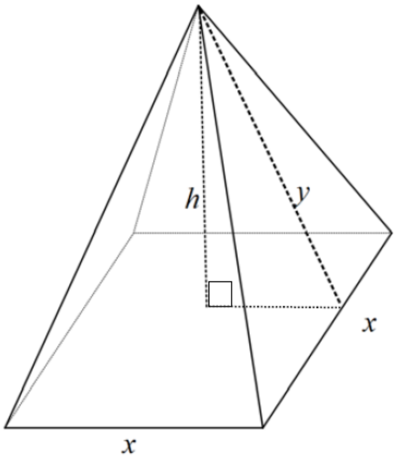
e. What is the average rate of change of the temperature in the office between 8.00 am and noon? 2 marks

f. **i.** Find $T'(t)$. 1 mark

ii. At what time of the day is the temperature in the office decreasing most rapidly? 2 marks

Question 4 (8 marks)

A group of architects, the *Pyramid Group*, are planning a cast iron installation in Federation Square in Melbourne. The first design is a simple right square based pyramid with vertical height, h metres, and length of the sides of the base, x metres.



The *Pyramid Group* know that the total cast iron required for the four sloping sides as well as the square base is 60 m^2.

a. Show that the slant height, y m, of the pyramid can be expressed as $y = \sqrt{h^2 + \frac{x^2}{4}}$. 1 mark

b. Find an expression for the Total Surface Area (TSA) of the pyramid in terms of both x and h. 2 marks

c. Hence show that the volume, V m^3, of the square based pyramid can be expressed in terms of x only as

$$V = \frac{1}{3}x\sqrt{900 - 30x^2}\ .$$

2 marks

d. State the implied domain for V. 1 mark

e. Find the maximum volume, in m^3, that the *Pyramid Group* can have for this square based pyramid. State the value of x, in m, that gives this maximum volume. 2 marks

END OF EXAMINATION

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ \ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

MATHEMATICAL METHODS

TRIAL EXAMINATION 2

MULTIPLE - CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ▬ C D E

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	11.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E