



## BILLANOOK COLLEGE

NAME:

Student Number:

### MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Exam 1 TECHNOLOGY FREE

Tuesday 17<sup>th</sup> July, 2018

Reading time: 15 minutes 2:30pm- 2:45pm

Writing time: 1 hour 2:45pm – 3:45pm

QUESTION AND ANSWER BOOKLET

#### Structure of Booklet

<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	44

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the test room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question and answer book with a detachable sheet of miscellaneous formulas.

#### Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.**

#### Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

**Question 1** (4 marks)

a. Let  $f: (-2, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = \frac{x}{x+2}$ .

Differentiate  $f$  with respect to  $x$ .

2 marks

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b. Let  $g(x) = (2 - x^3)^3$ .

Evaluate  $g'(1)$ .

2 marks

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**Question 2** (4 marks)

Let  $y = x \log_e(3x)$ .

- a. Find  $\frac{dy}{dx}$ . 2 marks

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- b. Hence, calculate  $\int_1^2 (\log_e(3x) + 1) dx$ . Express your answer in the form  $\log_e(a)$ , where  $a$  is a positive integer. 2 marks

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**Question 3** (4 marks)

a. Let  $y = e^{2x} \cos\left(\frac{x}{2}\right)$ .

Find  $\frac{dy}{dx}$ .

2 marks

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b. Let  $f: (0, \pi) \rightarrow \mathbb{R}$ , where  $f(x) = \log_e(\sin(x))$ .

Evaluate  $f'\left(\frac{\pi}{3}\right)$ .

2 marks

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**Question 4** (5 marks)

a. Find an antiderivative of  $\cos(1 - x)$  with respect to  $x$ .

1 mark

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b. Evaluate  $\int_1^2 \left( 3x^2 + \frac{4}{x^2} \right) dx$ .

2 marks

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c. Find  $f(x)$  given that  $f(4) = 25$  and  $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, x > 0$ .

2 marks

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**Question 5** (4 marks)

Let  $f: [-3, 0] \rightarrow \mathbb{R}$ ,  $f(x) = (x + 2)^2(x - 1)$ .

- a. Show that  $(x + 2)^2(x - 1) = x^3 + 3x^2 - 4$ .

1 mark

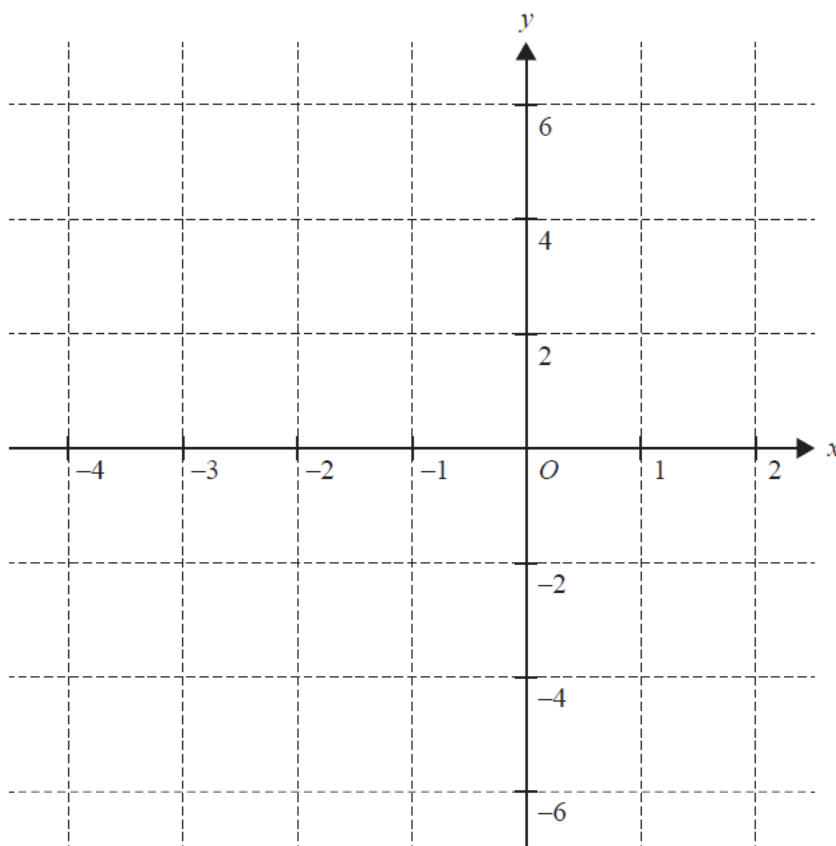
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- b. Sketch the graph of  $f$  on the axes below. Label the axis intercepts and any stationary points with their coordinates.

3 marks



**Question 6** (5 marks)

Let  $f: [0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = \sqrt{x+1}$ .

a. State the range of  $f$ .

1 mark

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b. Let  $g: (-\infty, c] \rightarrow \mathbb{R}$ ,  $g(x) = x^2 + 4x + 3$ , where  $c < 0$ .

i. Find the largest possible value of  $c$  such that the range of  $g$  is a subset of the domain of  $f$ . 2 marks

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ii. For the value of  $c$  found in **part b.i.**, state the range of  $f(g(x))$ .

1 mark

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c. Let  $h: \mathbb{R} \rightarrow \mathbb{R}$ ,  $h(x) = x^2 + 3$ .

State the range of  $f(h(x))$ .

1 mark

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**Question 7** (3 marks)

- a. State the smallest positive value of  $k$  such that  $x = \frac{3\pi}{4}$  is a solution of  $\tan(x) = \cos(kx)$ . 1 mark

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- b. Solve  $2\sin^2(x) + 3\sin(x) - 2 = 0$ , where  $0 \leq x \leq 2\pi$ . 2 marks

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**Question 8** (6 marks)

Let  $f: R \rightarrow R$ , where  $f(x) = 2x^3 + 1$ , and let  $g: R \rightarrow R$ , where  $g(x) = 4 - 2x$ .

- a. i. Find  $g(f(x))$ . 1 mark

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- ii. Find  $f(g(x))$  and express it in the form  $k - m(x - d)^3$ , where  $m$ ,  $k$  and  $d$  are integers. 2 marks

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- b. The transformation  $T: R^2 \rightarrow R^2$  with rule  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}$ , where  $a$ ,  $b$  and  $c$  are integers, maps the graph of  $y = g(f(x))$  onto the graph of  $y = f(g(x))$ .

Find the values of  $a$ ,  $b$  and  $c$ . 3 marks

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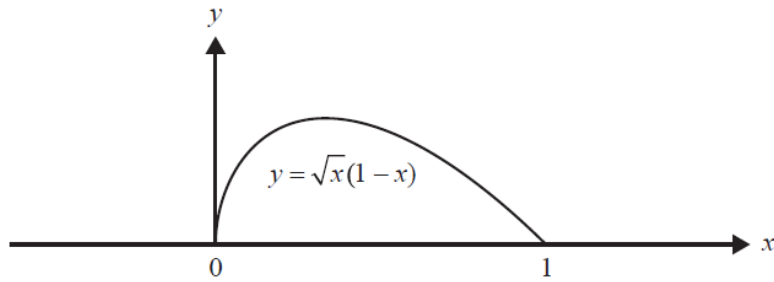
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## Question 9 (9 marks)

The graph of  $f : [0, 1] \rightarrow \mathbb{R}$ ,  $f(x) = \sqrt{x(1-x)}$  is shown below.



- a. Calculate the area between the graph of  $f$  and the  $x$ -axis.

2 marks

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- b. For  $x$  in the interval  $(0, 1)$ , show that the gradient of the tangent to the graph of  $f$  is  $\frac{1-3x}{2\sqrt{x}}$ .

1 mark

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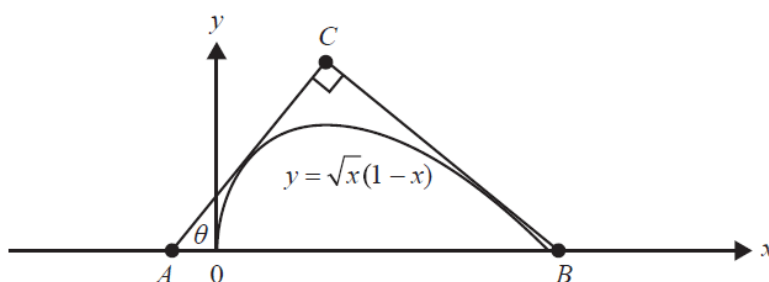
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The edges of the **right-angled** triangle  $ABC$  are the line segments  $AC$  and  $BC$ , which are tangent to the graph of  $f$ , and the line segment  $AB$ , which is part of the horizontal axis, as shown below.

Let  $\theta$  be the angle that  $AC$  makes with the positive direction of the horizontal axis, where  $45^\circ \leq \theta < 90^\circ$ .



c. Find the equation of the line through  $B$  and  $C$  in the form  $y = mx + c$ , for  $\theta = 45^\circ$ .

2 marks

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d. Find the coordinates of  $C$  when  $\theta = 45^\circ$ .

4 marks

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**END OF QUESTION BOOKLET**