St Leonard's College

Year 10 10A Mathematics

EXAMINATION 2

Semester 1 2017

Question and Answer Booklet

STUDENT NAME: SOLUTIONS

TEACHER(S):

TIME ALLOWED: **Reading time 15 minutes**

Writing time 60 minutes

INSTRUCTIONS

A CAS calculator is permitted.
One bound book of notes is permitted. Not a textbook.

Section A: Multiple Choice Questions
Circle the letter corresponding to the correct alternative on the question paper.(This booklet)

Section B: Extended Response Questions.
Answer in the spaces provided on the exam paper.
Give answers in simplified form where applicable.

STRUCTURE OF BOOKLET / MARKINGSCHEME

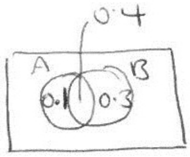
Exam Section	Number of questions to be answered	Total marks
A	10	10
B	4	30

Section A

Circle the letter corresponding to the correct alternative for the questions below.

1. The probability of rain on Monday is 0.3 and the probability of rain on Tuesday is 0.2. The probability of rain on Monday and Tuesday is:
   A. 0.06
   B. 0.25 0.3×0.2
   C. 0.3
   D. 0.5
   E. 0.6

2. For two events A and B, $\Pr(A \cap B) = 0.4$, $\Pr(A) = 0.5$ and $\Pr(A' \cap B) = 0.3$.
   $\Pr(A \cup B) =$



   A. 0.5
   B. 0.6
   C. 0.7
   D. 0.8
   E. 0.9

3. The gradient of the line containing the points $(0,-2)$ and $(2,6)$ is:

   $m = \frac{8}{2}$

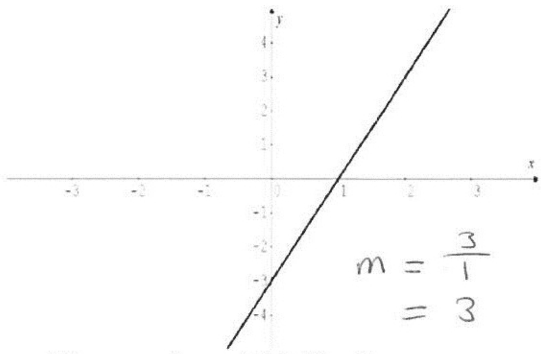
   A. $\frac{1}{4}$
   B. -2
   C. 2
   D. -4
   E. 4

4. The equation of a line which is parallel to the line with equation $y = 2x - 7$ is:

   need $m = 2$

   A. $y = -\frac{1}{2}x - 5$
   B. $2x + y = 3$
   C. $x + 2y = 3$
   D. $2x - y = 11$
   E. $y = 7x - 2$

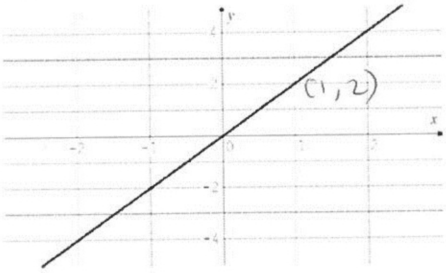
5. The line shown has intercepts $(0,-3)$ and $(1,0)$:



   $m = \frac{3}{1}$
   $= 3$

   The equation of this line is:
   $y = 3x - 3$
   A. $y + 3 = x$
   B. $y + x = 3$
   C. $3x - y - 3 = 0$
   D. $y + 3x + 3 = 0$
   E. $y - 3x = 1$

6. The equation of the graph shown is:



$y = 2x$

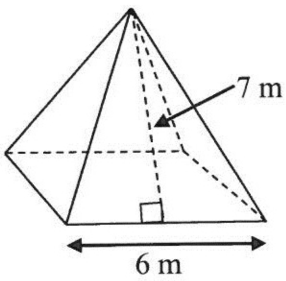
A. $x = 1$
B. $y = 2$
C. $y + 2x = 0$
D. $2y - x = 0$
E. $y - 2x = 0$

7. If $(x+1)(x+a) = x^2 + bx - 6$ then the values of a and b respectively are:

$a = -6$

A. 6 and 5
B. 6 and −5
C. −6 and 5
D. −6 and −5
E. 5 and 6

8. The surface area of the square-based pyramid shown below is



$6 \times 6 + 4 \times \frac{6 \times 7}{2}$

$= 36 + 84$

A. 78 m^2
B. 204 m^2
C. 84 m^2
D. 108 m^2
E. 120 m^2

9. The height of a cylinder, with radius 5 cm and volume 1250 cm^3, correct to one decimal place is

$\pi(5)^2 h = 1250$

$h = \frac{1250}{25\pi}$

$= 15.9\ldots$

A. 22.6 cm
B. 157.1 cm
C. 46.9 cm
D. 79.6 cm
E. 15.9 cm

10. $4\sqrt{5}$ is equivalent to

$\sqrt{16} \times \sqrt{5}$

$= \sqrt{80}$

A. $2\sqrt{10}$
B. $\sqrt{20}$
C. $\sqrt{100}$
D. $\sqrt{80}$
E. $\sqrt{10}$

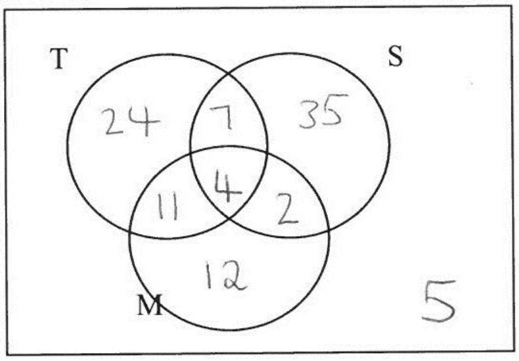
Section B:

Answer in the spaces provided.

1. A resort offers many activities as part of a holiday package. Three of these activities that can be booked are tennis, sailing and massage. The resort currently has 100 guests and it is found that;

- 5 guests have not booked any of these activities
- 4 guests have booked all 3 activities
- 2 have booked sailing and massage only
- 11 have booked tennis and sailing
- 15 have booked massage and tennis
- 24 have booked tennis only
- 35 have booked sailing only

a) Represent all of the information above in the Venn Diagram below.



b) How many guests booked a massage only? 12

c) Use the Venn Diagram to calculate the probability that a guest booked;

i) Tennis

$$\frac{46}{100} = \frac{23}{50} \text{ or } 0.46$$

ii) A massage given that they booked tennis

$$\frac{15}{46}$$

iii) Are booking a massage and booking tennis independent events based on the given data? Justify with a mathematical reason.

No, $Pr(M|T) \neq Pr(M)$

[2 + 1 + 1 + 2 + 2 = 8 marks]

2. Jack and Jill are keen gardeners. They both buy a baby beanstalk at the St. Leonard's Fair. They decide to have a competition to see which of them can grow the tallest beanstalk. The height, h cm, of each beanstalk after time, t weeks can be modelled by linear functions.

The rule for the height of Jack's beanstalk is $h = 18 + 30t$

After 1 week, Jill's, beanstalk has a height of 58 cm and its height after 5 weeks is 170 cm,

a) Write down the rate of change of Jack's beanstalk including units.

30 cm / week

b) Write the rule for the height, h cm, of Jill's beanstalk after time, t weeks.

$h = mt + c$

$h = 28t + c$

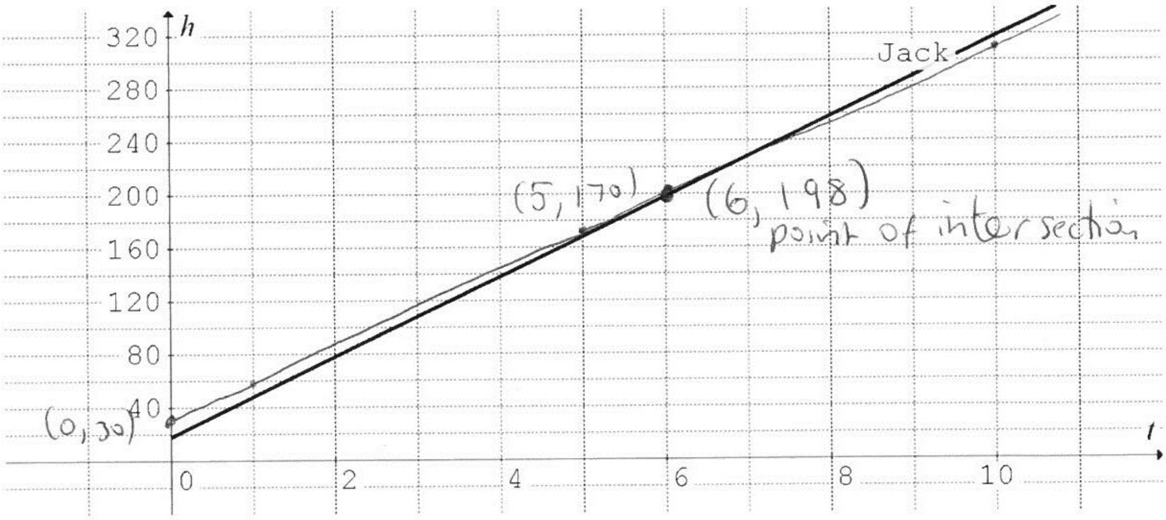
$58 = 28(1) + c$

$c = 30$

$\therefore \underline{h = 28t + 30}$

$m = \frac{170 - 58}{5 - 1} = \frac{112}{4} = 28$

c) The graph of h against t for Jack's beanstalk is shown on the axes below. Sketch the graph modelling Jill's beanstalk on the same set of axes labelling two points.



d) Label the point of intersection of the two graphs explaining what it represents in terms of the height of the beanstalks.

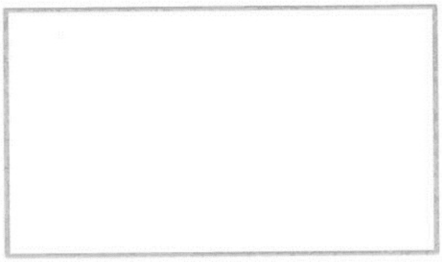
It shows after how many weeks the two beanstalks have the same height

e) Given that Jack and Jill decided that the winner was the one with the tallest beanstalk after 4 weeks, who won the competition?

Jill

[1 + 2 + 2 + 2 + 1 = 8 marks]

3. A rectangle has the length and width as shown in the diagram below.



$\frac{x}{2}$

$\frac{2x+1}{3}$

a) Given that the perimeter of this rectangle is 24 cm, find the length and width of this rectangle.

$$2\left(\frac{2x+1}{3} + \frac{x}{2}\right) = 24$$

* Solve on CAS on actual exam

$$\frac{2(2x+1) + 3x}{6} = 12$$

$$4x + 2 + 3x = 72$$

$$7x = 70$$

$$x = 10$$

$\therefore$ length 7 cm and width 5 cm.

b) A different rectangle has length $\frac{3x+1}{2}$ cm and width $\frac{2x+3}{3}$ cm. The length is 12 cm longer than the width. Calculate the perimeter of the rectangle in cm.

$$\frac{3x+1}{2} = \frac{2x+3}{3} + 12$$

$\times 2$

$$3x+1 = \frac{2(2x+3)}{3} + 24$$

$\times 3$

$$3(3x+1) = 2(2x+3) + 72$$

$$9x + 3 = 4x + 6 + 72$$

$$5x + 3 = 78$$

$$5x = 75$$

$$x = 15$$

* Solve on CAS on exam

$\frac{3 \times 15 + 1}{2}$ $\qquad$ $\frac{2 \times 15 + 3}{3}$

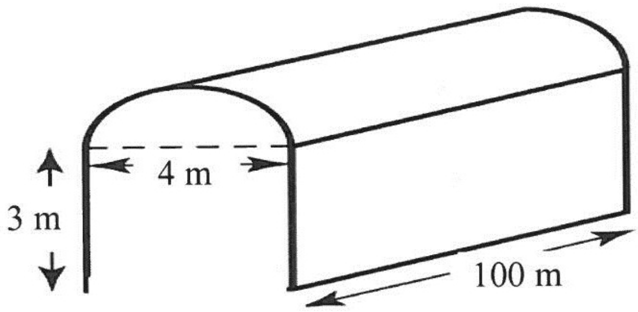
$\therefore$ width 23 cm, length 11 cm

perimeter $= 2 \times (23 + 11)$

$= 68$ cm

[3 + 3 = 6 marks]

4. An underground train tunnel 100 m long is to be constructed with a half cylindrical top of diameter 4 m as shown.



a. The first step in the construction process is to remove the earth. Calculate, to 1 decimal place, the volume of earth that needs to be removed for the tunnel.

$$\text{Volume} = 4 \times 3 \times 100 + \frac{1}{2}\pi(2)^2(100)$$
$$= 1200 + 200\pi$$
$$= 1828.3\ m^3 \text{ (to 1 dec. place)}$$

b. i. The inside surface of the tunnel requires strengthening. Calculate the internal surface area of the tunnel, not including the entrances or base, to 1 decimal place.

$$\text{Surface area} = 2 \times (3 \times 100) + \frac{1}{2} \times 2\pi(2)(100)$$
$$= 600 + 200\pi$$
$$= 1228.3\ m^2 \text{ (to 1 dec place)}$$

ii. Convert your answer from **i.** to cm^2 and calculate, to the nearest dollar, the total cost of strengthening the tunnel, if strengthening costs \$0.50 per 100 cm^2.

$$1228.3 \times 100^2 = 12283000\ cm^2$$
$$12283000 \div 100 \times 0.5 = \$61415$$

[3 + (3 + 2) = 8 marks]

END OF EXAM