Year 11 Mathematical Methods Test 2

VCE Unit 1 Mathematical Methods

SAC 1 - UNIT 1 – OUTCOMES 1, 2, 3

Thursday 23rd March 2017

You will have 75 minutes to complete this SAC.

The SAC is a Test

The total SAC contributes to 10/100 of SAC marks allocated for Unit 1.

Name _____

Note: *The grade or score for this task is only part of the internal assessment for this Unit. Your **total** School-assessed Coursework score may change as a result of statistical moderation by VCAA.*

Grade/Score: ___/55 Satisfactory Completion? S/N: _____
(Provisional)

Assessment Criteria

Students should be able to:

Define and explain key concepts and apply a range of related mathematical routines and procedures.
Apply mathematical processes in non-routine contexts including situations requiring problem-solving, modelling or investigative techniques or approaches and analyse and discuss these applications of mathematics.
Use numerical, graphical and symbolic functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

In particular students should draw on the following knowledge and skills:

- To understand set an interval notation
- To understand the concepts of function and relation
- To find the domain and range of a given relation
- To decide whether or not a function if one to one
- To work with restrictions of a function and piecewise defined functions
- To add, subtract, multiply and divide polynomials
- To use the remainder and factor theorem to identify the linear factors of cubic polynomials
- To find the rules for given cubic functions
- To apply cubic and quadratic functions to solving problems
- To sketch cubic and quadratic functions

Year 11 Mathematical Methods Test 2

FUNCTIONS AND RELATIONS AND POLYNOMIALS

Section A – consists of 7 short answer questions	28 marks
Section B – consists of 10 multiple choice questions.	10 marks
Section C – consists of 3 extended response questions.	17 marks

Section A Short answer questions

- NO technology or reference material permitted. Total: /28
- Show ALL working to get full marks.
- Time allowed: 40 mins

Question 1: (5 marks)

Given $f(x) = x^2 + 5$, find:

a) $f(-3)$ 1 mark

b) $f(a-3)$. (Give your answer in the expanded form.) 2 marks

c) $\{x : f(x) = 54\}$ 2 marks

Question 2: (5 marks)

For the graph of $f: [-1, \infty) \to R, f(x) = 2(x-1)^2 - 4$

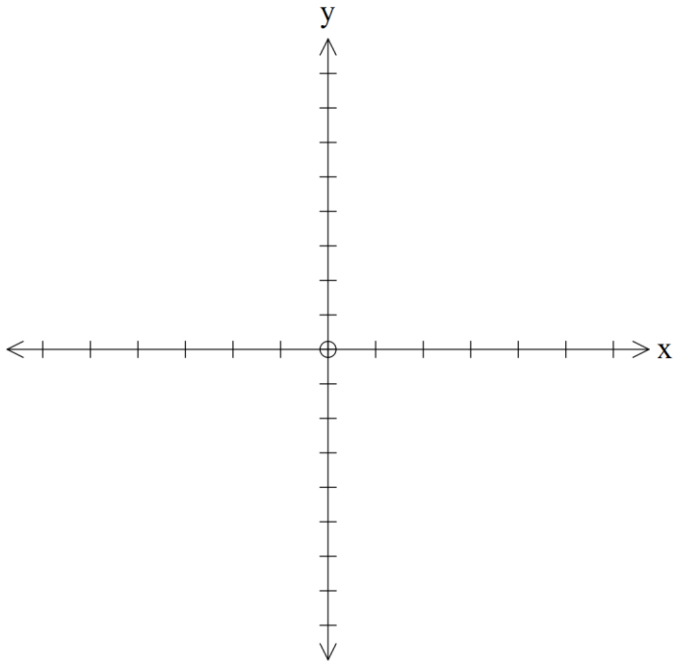
a) State the coordinates of the turning point. 1 mark

_____

b) Determine the coordinates of the *y* intercept 1 mark

_____

c) Sketch the graph of *y=f(x)* on the axes below. 2 marks



d) State the range of *f* 1 mark

_____

_____

Question 3: (4 marks)

a) Find the value of p such that $x+1$ is a factor of $P(x) = 9x^3 + 5x^2 - px - 5$

_____

_____

_____

_____

b) Find the other linear factors of $P(x)$

_____

_____

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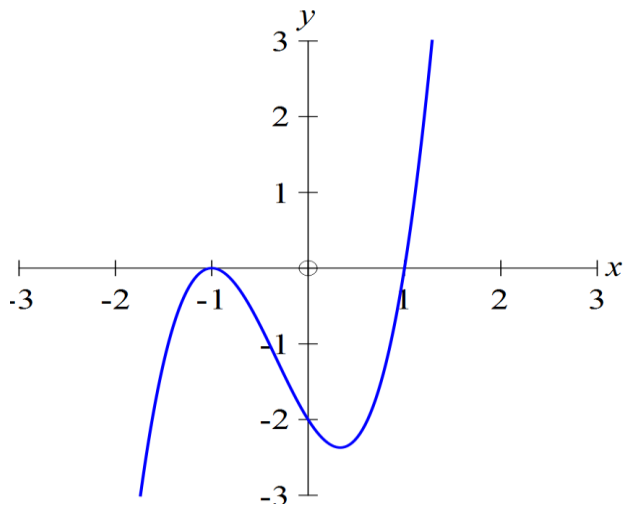
_____

Question 4: (3 marks)

Determine the equation of the cubic function $y = f(x)$ shown below, expressing your answer in factorised form.



Question 5: (4 marks)

Consider the following functions. State the implied domain of each of the following:

a $y = \sqrt{x-4}+5$ (2 marks)

_____

_____

b $f(x) = \frac{3}{x-1} + 2$ (2 marks)

_____

_____

Question 6: (7 marks)

a) Factorise the expression $x^3 - 5x^2 + 2x + 8$

_____

_____

_____

_____

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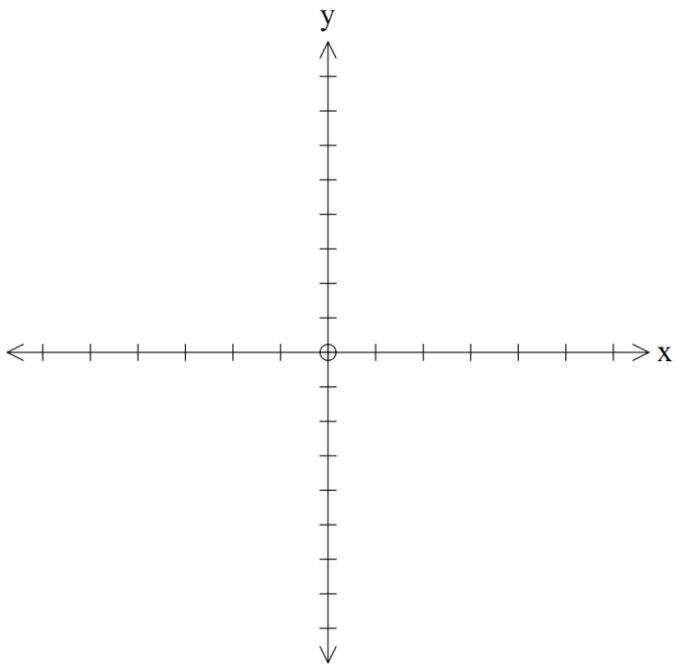
_____

b) Sketch the graph of $y = x^3 - 5x^2 + 2x + 8$ on the axes below



c) Hence calculate the values of for which $x^3 - 5x^2 < -2x - 8$

_____

_____

_____

Year 11 Mathematical Methods Test 2

Name: _____

Teacher: _____

Sections B and C

- Technology and reference material are permitted. Total: /27
- Show ALL working to get full marks.
- Time Allowed : 35 minutes

Section B: Multiple Choice Questions

Circle the correct response

1. If $f(x) = x^2 - x$, then $f(2-a)$ is equal to:

A $a^2 - a + 2$

B $-a^2 - a + 2$

C $a^2 + a + 2$

D $a^2 - 5a + 2$

E $a^2 - 3a + 2$

2. Which one of the following is **not** a one-one function?

A $f : R \to R,\ \text{where } f(x) = x^3 - 2$

B $f : R \to R,\ \text{where } f(x) = 4 - 2x$

C $f : R \to R,\ \text{where } f(x) = x^2 + 5$

D $f : [0, \infty) \to R,\ \text{where } f(x) = \sqrt{x}$

E $f : (-\infty, 2] \to R,\ \text{where } f(x) = 3 - (x-2)^2$

3. The implied domain of the function with rule $y = \dfrac{1}{\sqrt{2-x}}$ is:

A $(-2, 2)$

B $(-\infty, 2)$

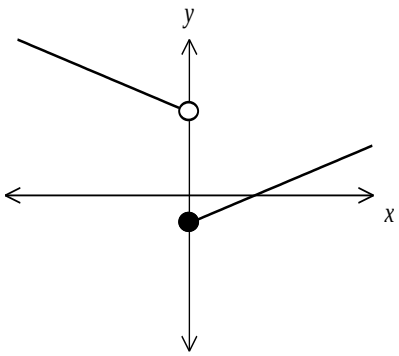
C $(-2, \infty)$

D $(-\infty, 2]$

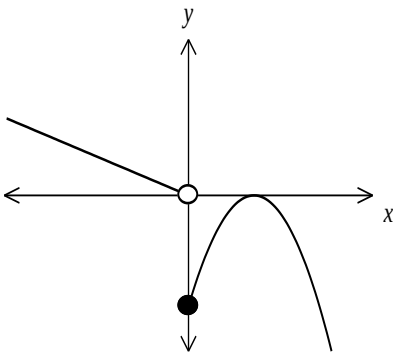
E $[2, \infty)$

4. Which of the following graphs is **not** the graph of a function?

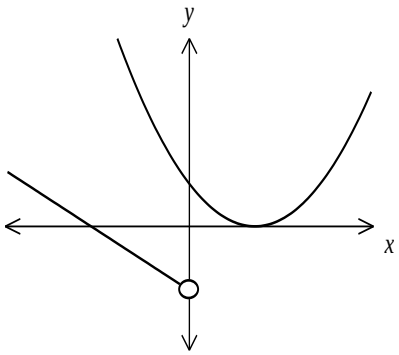
A.



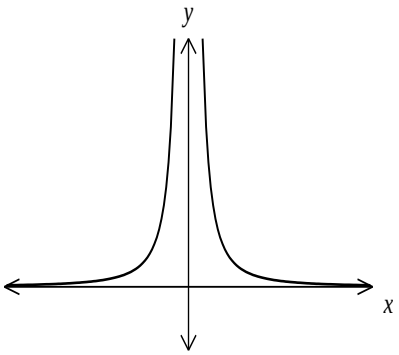
B.



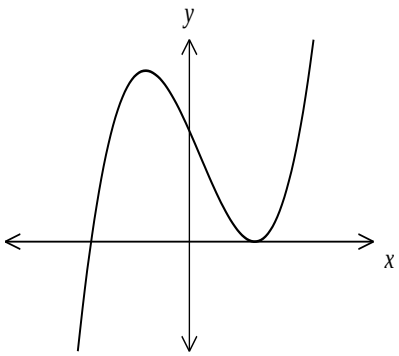
C.



D.



E.



5. The range of the function $f:(0,5)\to R,\ \text{where } f(x)=4+6x-x^2$ is:

A (4, 9)

B [4, 9]

C (4, 13)

D (4, 13]

E (3, 13)

6. The interval $(-5,7)\cap[2,9)$ can be described as

A $(-5,9)$

B $[2,7]$

C $(2,7]$

D $(2,7)$

E $[2,7)$

7. Given the graph with equation $f(x)=\frac{1}{2}(x+1)(x-2)^2$, what restriction to the domain would make this a one to one function?

A $[-1,2]$

B $[0,2)$

C $[0,\infty)$

D $[0,2]$

E $[-1,2)$

8. The maximum value of the graph of $y=-3\sqrt{x-9}+36$ is

A -3

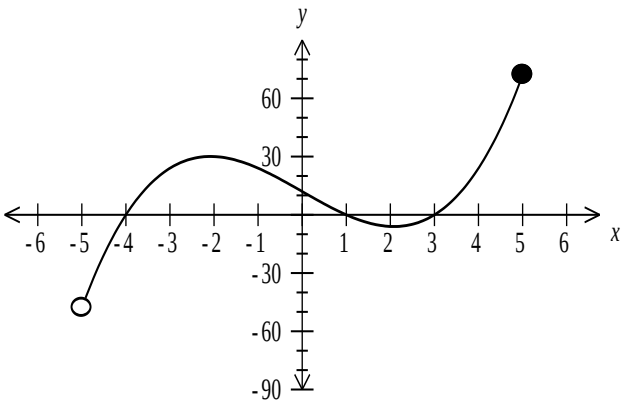
B 9

C -9

D 36

E -36

9. The domain of the function whose graph is shown on the right is



A. R

B. $[-5,5)$

C. $(-5,5]$

D. $(-5,5)$

E. $(-50,70]$

10. If x^3+3px^2-2x+7 has a remainder of 2 when divided by $x+1$, then p equals

A. 4

B. 1

C. 2

D. -2

E. -5

Part C: Extended Response

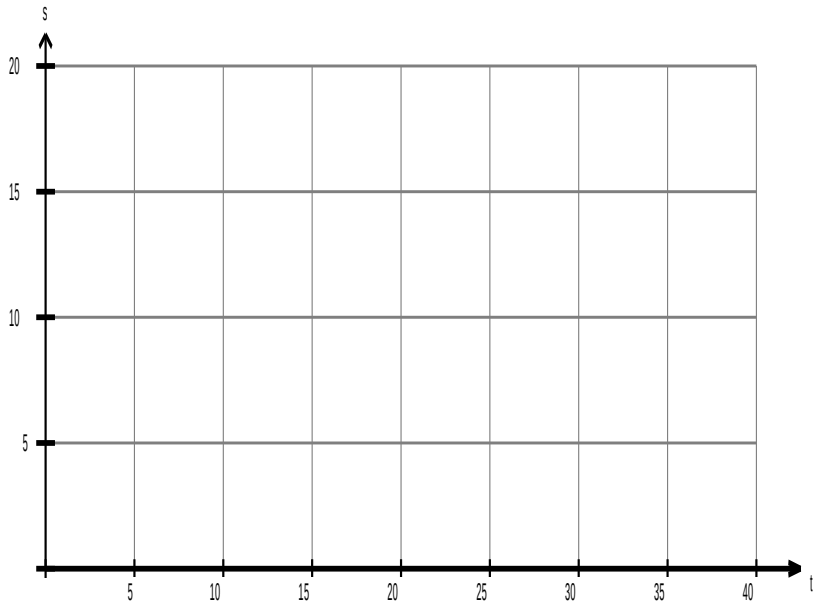
Question 1. (8 marks)

The speed, *s* metres per second, of a car as it travels between two sets of traffic lights can be modelled by the function:

$$s(t)=\begin{cases}1.5t & 0\le t<10\\ 15 & 10\le t<20\\ \frac{1}{15}(t-35)^2 & 20\le t\le 35\end{cases}$$

Where *t* is the time in seconds from when the first set of lights turns green.

a) Sketch the graph of *s* against *t* on the set of axes below. 3 marks



b) What is the speed of the car at: 3 marks

i) $t=6$ seconds?

_____

_____

ii) $t=13$ seconds?

_____

_____

iii) $t = 32$ seconds?

_____

_____

c) At what time(s) is the speed of the car 12 metres per second? 2 marks

_____

_____

_____

_____

_____

_____

Question 2 (9 marks)

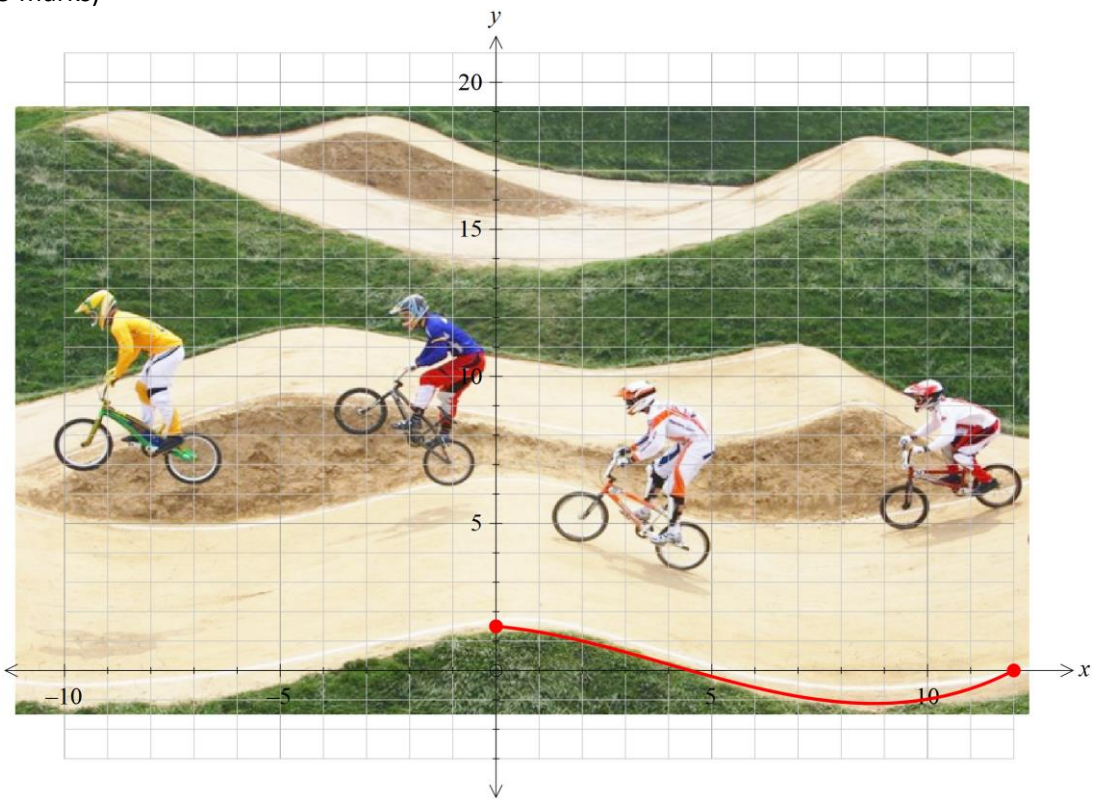


Figure 3: BMX Track

It is possible to model part of a BMX track with transformations of a cubic function of the form

$$y = (x - b)(x - c)(ax - d)$$

a) State the domain of the cubic function drawn in red in Figure 3.

_____

1 mark

b) Given that the cubic function cuts the *x* axis at $x = 4.5$ and $x = 12$ state the two linear factors $(x - b)(x - c)$

_____

_____

_____

2 marks

c) Given that the cubic function $y = (x - b)(x - c)(ax - d)$ also passes through the point $(2,1)$ and has a y intercept of $(0,1.5)$ construct two equations involving *a* and *d*

_____

_____

_____

3 marks

d) Hence find the values of *a* and *d* and write the equation which models the BMX track

_____

_____

_____

_____

_____

3 marks