BILLANOOK COLLEGE

NAME:	Student Number:

MATHEMATICAL METHODS (CAS) UNITS 3 & 4

Practice July Exam

Exam 1 TECHNOLOGY FREE

Friday 21st July, 2017
Reading time: 15 minutes 11:15am- 11:30am
Writing time: 1 hour 11:30am – 12:30pm

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of Questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the test room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book with a detachable sheet of miscellaneous formulas.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

Instructions

Answer **all** questions in the space provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.

Question 1 (5 marks)

a. If $y = x^2 \sin(x)$, find $\dfrac{dy}{dx}$. 2 marks

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b. If $f(x) = \sqrt{x^2 + 3}$, find $f'(1)$. 3 marks

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Question 2 (3 marks)

Sketch the graph of $f : R \setminus \{2\} \to R, \ f(x) = -1 + \dfrac{3}{x-2}$ on the set of axes below. Label axes intercepts with their coordinates. Label asymptotes with their equations
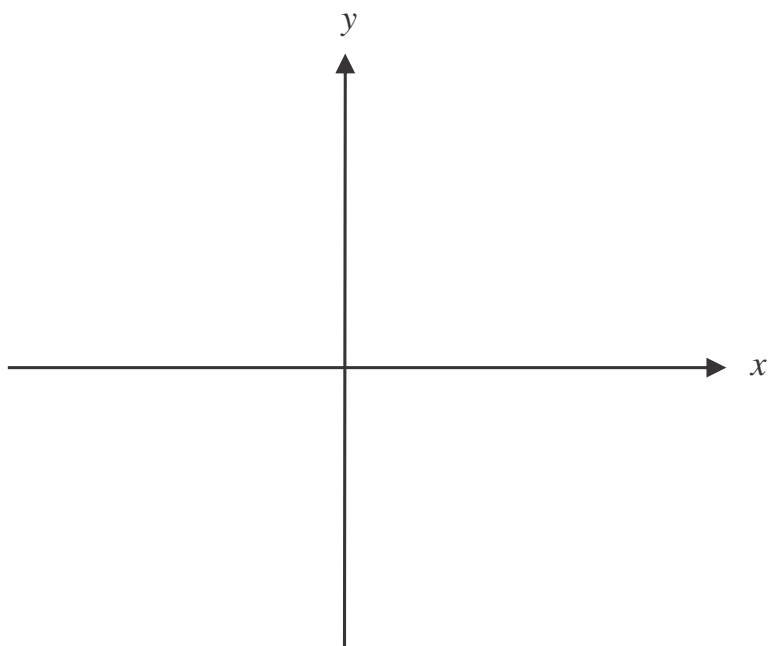
Question 3 (2 marks)

Find $\displaystyle\int_{1}^{3}\left(\frac{2}{x}+1\right)dx$.

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Question 4 (3 marks)

Let $f(x) = \dfrac{1}{\sqrt{3}}\cos(x)$ and $g(x) = \sin(x)$.

a. Solve the equation $f(x) = g(x)$ for $x \in [0, 2p]$. 2 marks

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b. Evaluate $f(g(0))$. 1 mark

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Question 5 (2 marks)

Solve the following equation:

$$\frac{4000}{2 + 7^{3x}} = 5$$

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Question 6 (2 marks)

The tangent to the curve $y = \frac{3}{x} - 2$ at the point $x = a$, where $a > 0$, has a gradient of -9.

Find the value of a.

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Question 7 (3 marks)

Solve the equation $\log_e(x) + \log_e(3x + 2) = 2\log_e(x + 1)$ for x, where $x > 0$.

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Question 8 (9 marks)

Consider the function with the rule $f(x) = \dfrac{x-2}{x+2}$

a.   Find the rule, f^{-1}, for the inverse of f.   3 marks

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b. Find the domain and range of the inverse of f. 2 marks

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c. Show that $f^{-1}(x)$ can be written in the form of $a + \frac{b}{1-x}$ and hence find $\int_0^{\frac{1}{2}} f^{-1}(x)\, dx$
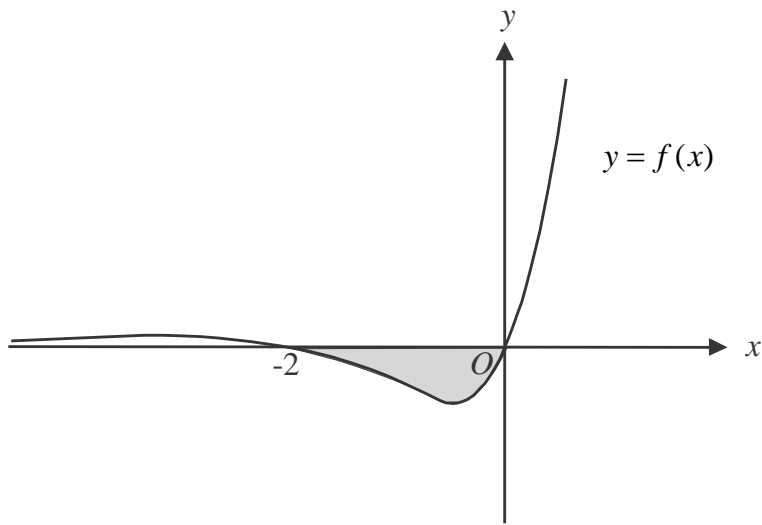
4 marks

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Question 9 (4 marks)

The graph of $f: R \rightarrow R, f(x) = (x^2 + 2x)e^x$ is shown below.

The region enclosed by the graph of f and the x-axis is shaded.

a. Find the derivative of $(3 - x^2)e^x$. Give your answer in the form $ae^x - f(x)$, where a is a positive constant.

1 mark

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b. Use your answer to part **a.** to find the area of the shaded region.

3 marks

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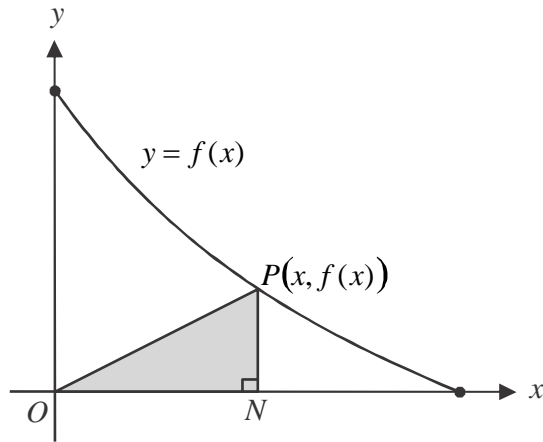**Question 10** (7 marks)

Let $f:[0,1]\to R,\ f(x)=1-x^{\frac{2}{3}}$. The graph of f is shown below.

The right-angled triangle *NOP* has vertex *N* on the *x*-axis, and vertex *O* at the origin. The vertex *P* lies on the graph of *f* and has coordinates $(x, f(x))$ as shown.

a.   Find the area *A*, of triangle *NOP* in terms of *x*.   1 mark

b.   Find

   i.   the value of *x* for which *A* is a maximum.   2 marks

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   ii.   the maximum area of triangle *NOP*. Give your answer in the form $\dfrac{a\sqrt{b}}{c}$

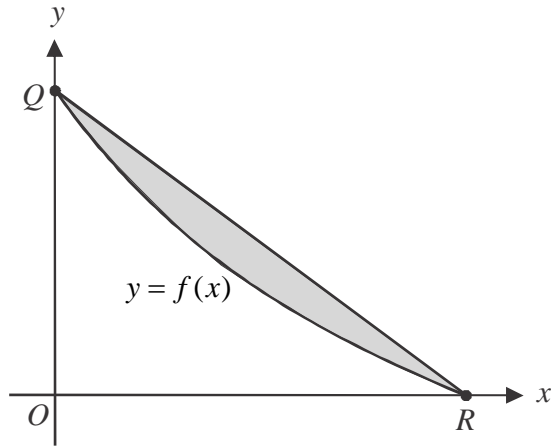   where *a, b* and *c* are positive integers.   1 mark

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c. The point Q lies on the graph of f and on the y-axis. The point R lies on the graph of f and on the x-axis.

Find the area enclosed by the line segment QR and the graph of f.   3 marks

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END OF QUESTION BOOKLET