# MATHEMATICAL METHODS

## Units 3 & 4 – Written examination 1

TSSM
Creating VCE Success

Test A
Tech Free .

***(TSSM's 2015 trial exam updated for the current study design)***

## ***SOLUTIONS***

**Question 1**

**a.** $3 - 5x \geq 0$

$x \leq \frac{3}{5}$

$Domain: \left(-\infty, \frac{3}{5}\right]$

A1

1 mark

**b.** $f'(x) = \frac{1}{2}(3-5x)^{-\frac{1}{2}} \times -5$

$f'(x) = -\frac{5}{2\sqrt{3-5x}}$

M1+A1

2 marks

**c.** $f'\left(\frac{1}{5}\right) = -\frac{5}{2\sqrt{3-1}} = -\frac{5}{2\sqrt{2}} = -\frac{5\sqrt{2}}{4}$

A1

1 mark

**Question 2**

**a.** $\int \sin(3x)\,dx = -\frac{\cos(3x)}{3} + c$

$0 = -\frac{1}{3} + c \ \ which\ gives\ \ c = \frac{1}{3}$

$F(x) = -\frac{\cos(3x)}{3} + \frac{1}{3}$

M2+A1

3 marks

**b.** $-\frac{\cos(3x)}{3}+\frac{1}{3}=\frac{1}{2}$

$\cos(3x)=-\frac{1}{2}$

$3x=\frac{2\pi}{3},\frac{4\pi}{3},\frac{8\pi}{3}$

$x=\frac{2\pi}{9},\frac{4\pi}{9},\frac{8\pi}{9}$

M2+A1

3 marks

**Question 3**

**a.** $y=\frac{x-2}{x+2}$

$x=\frac{y-2}{y+2}$

$yx+2x=y-2$

$y(x-1)=-2-2x$

$y=\frac{2+2x}{1-x}$

$f^{-1}(x)=\frac{2+2x}{1-x}$

M2+A1

3 marks

**b.** $Domain: R\backslash\{1\}$

$Range: R\backslash\{-2\}$

A2

2 marks

**c.** Using long division,

$f^{-1}(x)=-2+\frac{4}{1-x}$

$\int_0^{\frac{1}{2}}\left(-2+\frac{4}{1-x}\right)\,dx=[-2x-+4\log_e(1-x)]_0^{\frac{1}{2}}=-1-4\ln\left(\frac{1}{2}\right)=-1+4\ln 2$

M3+A1

4 marks

**Question 4**

**a.** $f'(x)=x^3-3x^2+2x$

$f'(x)=0\ gives\ \ x(x^2-3x+2)=0$

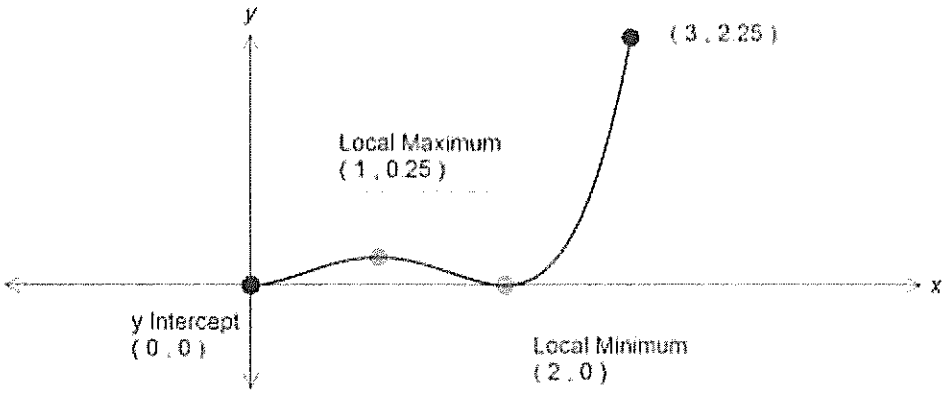
$x(x-2)(x-1)=0\ gives\ x=0,1,2$

$(0,0),\ \left(1,\frac{1}{4}\right),\ (2,0)$

M2+A1

3 marks

**b.**




1 for shape, 1 for stationary points, 1 for end points.

3 marks

**c.** $Area = \int_0^2 \left(\frac{1}{4}x^4 - x^3 + x^2\right)\,dx = \left(\frac{x^5}{20} - \frac{x^4}{4} + \frac{x^3}{3}\right)_0^2$

$Area = \frac{8}{5} - 4 + \frac{8}{3} = \frac{4}{15}\ \ square\ units$

M1+A1

2 marks

**Question 5**

**a.** $4000 = 5(2 + 7^{3x})$

$800 = 2 + 7^{3x}$

$798 = 7^{3x}$

$3x = log_7(798)$

$x = \frac{1}{3}log_7(798)$

M1+A1

2 marks

**b.** $2 \times 2^{2x} + 2^x - 1 = 0$

$2y^2 + y - 1 = 0,\ \ where\ y = 2^x$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2}, -1$

$2^x = \frac{1}{2},\ 2^x = -1$

$x = -1\ \ (2^x = -1\ has\ no\ solution)$

M2+A1

3 marks

**Question 6**

**a.** $\frac{1}{5}+\frac{1}{10}+\frac{1}{3}+k=1$

$k=\frac{11}{30}$

A1
1 mark

**b.** $\Pr(X<2)=\frac{1}{5}+\frac{1}{3}=\frac{8}{15}$

A1
1 mark

**c.** $\text{Mean}=\sum x\Pr(X=x)=0+\frac{1}{3}+\frac{1}{5}+\frac{11}{10}=\frac{49}{30}$

M1+A1
2 marks

**Question 7** 6

$\frac{dy}{dx}=-\frac{3}{x^2}$

$grad\ of\ tangent=-\frac{3}{a^2}$

$-\frac{3}{a^2}=-9$

$a=\pm\frac{1}{\sqrt{3}}$

$a=\frac{\sqrt{3}}{3}$

M1+A1
2 marks

**Question 8**

**a.** $\hat{p}=0.9$

A1
1 mark

**b.** $M=1.96\sqrt{\frac{0.9\times0.1}{r}}$

If you double r

$M=1.96\sqrt{\frac{0.9\times0.1}{2r}}$

Margin of error will decrease by a factor of $\sqrt{2}$

A1
1 mark

# MATHEMATICAL METHODS

## Units 3 & 4 – Written examination 2


Test B
Tech Active.

***(TSSM's 2015 trial exam updated for the current study design)***

## ***SOLUTIONS***

**SECTION 1: Multiple-choice questions (1 mark each)**

**Question 1**

*Answer:* A

*Explanation:*

Solve the two equations on CAS.

**Question 2**

*Answer:* C

*Explanation:*

It is negative cubic so either C or D. Check the x-intercept.

**Question 3**

*Answer:* E

*Explanation:*

Define the functions on CAS and find $f(g(x))$

**Question 4**

*Answer:* D

*Explanation:*

$f(x) = 2\left(\sqrt{x} + \frac{1}{2}\right)$
$g(x) = 2 \times \frac{1}{2}\left(\sqrt{x} + \frac{1}{2}\right)$

**Question 5**

*Answer:* C

*Explanation:*

$Domain: 4 - x \geq 0$ $gives\ x \leq 4$ and the graph is above the x-axis.

**Question 6**

*Answer:* A

*Explanation:*

$Av\ ROC = \frac{f(8)-f(2)}{8-2}$

**Question 7**

*Answer:* C

*Explanation:*

Note the shaded end-points.

**Question 8**

*Answer:* ~~C~~ B

*Explanation:*

$f(g(x)) = \frac{3}{x+5},\ x \neq -2$ ✓ $\therefore R \setminus \{-2\}$.

**Question 9**

*Answer:* E

*Explanation:*

Eliminate incorrect options

**Question 10**

*Answer:* D

*Explanation:*

$Amp = 2,\ Period = \frac{2\pi}{\frac{1}{5}}.$

**Question 11**

*Answer:* E

*Explanation:*

$\frac{dy}{dx}\ at\ x = 4$ on CAS.

**Question 12**

*Answer:* B

*Explanation:*

$A_1 = A_2$

**Question 13**

*Answer:* B

*Explanation:*

$normalline(f(x), x = 0)$ on CAS.

**Question 14**

*Answer:* C

*Explanation:*

$\left(f(x)\right)^2 \times \left(f(y)\right)^2 = e^{2x} \times e^{2y} = e^{2x+2y} = f(2x+2y)$

**Question 15**

*Answer:* A

*Explanation:*
$\frac{1}{k}\int_0^k x^3 dx = 9 \ \ gives \ \ k = 6^{\frac{2}{3}}$ on CAS.

**Question 16**

*Answer:* B

*Explanation:*

$binompdf\left(10, \frac{1}{5}, 6\right)$

**Question 17**

*Answer:* C

*Explanation:*

$normcdf(165,170,165,7.62)$.

**Question 18**

*Answer:* A

*Explanation:*

$binomcdf(6,0.2,5,6)$ on CAS.

**Question 19**

*Answer:* D

*Explanation:*

$50th$ percentile means she is on average, due to the symmetry of the normal distribution

**Question 20** 16

*Answer:* C

*Explanation:*

Sketch on CAS and read the maximum value.

**Question 21**

*Answer:* C

*Explanation:*

$k = 0.2,\ E(X) = 3.9$

**Question 22**

*Answer:* B

*Explanation:*

$\frac{\pi}{n} = 3$ gives $n = \frac{\pi}{3}$

**SECTION 2: Analysis Questions**

**Question 1**

**a.** $r = l\,sin\alpha,\ h = l\,cos\alpha$

A2
2 marks

**b.** $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(lsin\alpha)^2(lcos\alpha) = \frac{\pi}{3}l^3 \sin^2\alpha\, cos\alpha$

M1
1 mark

**c.** $V'(\alpha) = \frac{\pi}{3}l^3\,(\sin^2\alpha \times -sin\alpha + cos\alpha \times 2sin\alpha\, cos\alpha) = 0$

$sin\alpha\,(-\sin^2\alpha + 2\cos^2\alpha) = 0$

$sin\alpha = 0,\ \tan^2\alpha = 2$

$\alpha = 0,\ \alpha = \pm\tan^{-1}\sqrt{2}$

$\alpha = \tan^{-1}\sqrt{2},\quad V(\alpha) = \frac{2\sqrt{3}}{27}\pi l^3$

$\left(\tan^{-1}\sqrt{2},\ \frac{2\sqrt{3}}{27}\pi l^3\ \right)$

Alternate form: $\left(\cos^{-1}\frac{\sqrt{3}}{3},\ \frac{2\sqrt{3}}{27}\pi l^3\ \right)$ also correct

M3+A1
4 marks

**d.** ~~$\alpha = \tan^{-1}\sqrt{2}$ is a point of maximum volume.~~

~~$Max\ volume = \frac{2\sqrt{3}}{27}\pi \times 6^3 = 16\sqrt{3}\pi\ cm^3$.~~

~~M1+A1~~
~~2 marks~~

**Question 2**

**a.** Period $= \frac{2\pi}{\frac{\pi}{2.2}} = 4.4$ years and Amplitude $= 300$

A2
2 marks

**b.** Min $= 200$, Max $= 800$

A2
2 marks

**c.** Solve $P(t) = 800$ over $[0, 5]$
$t = 0.7$. After 8.4 months

M1+A1
2 marks

**d.** Sketch the graph on CAS and read the domain when $P < 300$
$2.3 < t < 3.5$ *and* $6.7 < t < 7.9$

M1+A2
3 marks

**e.** Strictly increasing for $t \in [0, 0.7] \cup [2.9, 5]$
Note that we include endpoints for strictly increasing intervals.

A3
3 marks

**Question 3**

**a.** Sketch on CAS and read the max: 0.45 μg/mL

A1
1 mark

**b.** 3.5 minutes

A1
1 mark

**c.** $C(10) = 0.32$ μg/mL

M1+A1
2 marks

**d.** $\frac{C(5)-C\left(\frac{3}{2}\right)}{5-\frac{3}{2}} = 0.0115 \frac{\mu g}{mL}$/minute

M1+A1
2 marks

**e.** Solve $\frac{dC}{dt} < 0$ on CAS
$t > 3.53$ minutes

M1+A1
2 marks