Billanook College

NAME: Answers Student Number:

MATHEMATICAL METHODS UNIT 3

Thursday 8th March, 2018

Topic Test: Transition, Functions and Transformations and Exponentials and Logarithmic Functions

Technology and Reference book permitted.

Expected Writing time for this section: 60 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of Questions*	*Number of questions to be answered*	*Number of marks*
1	10	10	10
2	7	7	26

- Students are permitted to bring into the TEST room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality maybe used.
- Students are NOT permitted to bring into the SAC blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorized electronic devices into the test room.

Multiple Choice Answer Sheet

Use a **PENCIL** for **ALL** entries. For each question, shade the box which indicates your answer.
All answers must be completed like **THIS** example: A ▮ C D E
Marks will **NOT** be deducted for incorrect answers.
NO MARK will be given if more than **ONE** answer is completed for any question.
If you make a mistake, **ERASE** the incorrect answer – **DO NOT** cross it out.

ONE ANSWER PER LINE

1	A		C	D	E
2	A	B	C	D	
3		B	C	D	E
4	A	B	C	D	
5	A	B		D	E
6		B	C	D	E
7	A		C	D	E
8	A		C	D	E
9		B	C	D	E
10	A		C	D	E

Section B: Multiple Choice questions:(10 Marks)

Choose the correct answer and shade the relevant box provided on the multiple choice answer page.

1 The maximal domain D of the function $f: D \to R$ with the rule $f(x) = log_e(2x+1)$ is

A $R\backslash\left\{-\frac{1}{2}\right\}$

B $\left(-\frac{1}{2}, \infty\right)$

C R

D $(0, \infty)$

E $\left(-\infty, -\frac{1}{2}\right)$

$2x+1>0$

$x > -\frac{1}{2}$

2 The function f has rule $f(x) = 3log_e(2x)$. If $f(5x) = log_e(y)$ then y is equal to

A $30x$

B $6x$

C $125x^3$

D $50x$

E $1000x^3$

$f(5x) = 3\log_e(10x)$

$\log_e y = \log_e(10x)^3$

$y = 1000x^3$

3 The transformation that maps the graph of $y = \sqrt{8x^3+1}$ onto the graph of $y = \sqrt{x^3+1}$ is a

A Dilation by a factor of 2 from the y axis

B Dilation by a factor of 2 from the x axis ✗

C Dilation by a factor of $\frac{1}{2}$ from the x axis ✗

D Dilation by a factor of 8 from the y axis ✗

E Dilation by a factor of $\frac{1}{2}$ from the y axis

$y = \sqrt{8x^3+1} \Rightarrow y = \sqrt{x^3+1}$

$y = \sqrt{(2x)^3+1} \Rightarrow y = \sqrt{(2(\tfrac{1}{2}))^3+1}$

4 If $log_m p = 3^2$ then p is equal to

A m^6

B 9

C $9m$

D $\sqrt{m^3}$

E m^9

$m^9 = p$.

5 $log_3 7$ could be evaluated as

A $\frac{log 3}{log 7}$

B $\frac{log_e 7}{log_3 e}$

C $\frac{log_e 7}{log_e 3}$

D $log_{10}7 - log_{10}3$

E $log_{10}3^7$

Change of base rule.

6 $\frac{1}{2}log_{10}49 - log_{10}\sqrt{7} + log_{10}\frac{1}{7}$ simplifies to

A $-\frac{1}{2}log_{10}7$

B $log_{10}\sqrt{7}$

C $-log_{10}\sqrt{7}$

D $\frac{3}{2}log_{10}7$

E $\frac{50}{7} - \sqrt{7}$

$\log_{10}\sqrt{49} - \log_{10}\sqrt{7} + \log_{10}\frac{1}{7}$

$\log_{10}7 - \log_{10}\sqrt{7} + \log_{10}\frac{1}{7}$

$\log_{10}\left(\frac{7}{\sqrt{7}} \times \frac{1}{7}\right)$

$= \log_{10}\left(\frac{7}{7\sqrt{7}}\right)$

$= \log_{10}7^{-1/2} \Rightarrow -\frac{1}{2}\log_{10}7$.

t.

7 The maximum value of $y = 2.6e^{0.3t}$ for $\cancel{x}\epsilon[-1,4]$ is approximately:

A 4

B 8.63

C 1.93

D 2.6

E 0.78

Increasing function.

Max occurs when $\overset{t}{\cancel{x}} = 4$.

$y_{max} = 2.6e^{0.3(4)}$

$= 8.63$.

8 The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

A $k = 6$

B $k > 6 \text{ or } k < -2$

C $-2 \le k \le 6$

D $6 - 2\sqrt{3} \le k \le 6 + 2\sqrt{3}$

E $k = -2$

$x^2 + 2x = kx - 4$

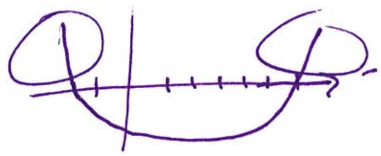
$x^2 + (2-k)x + 4 = 0$

$\Delta = b^2 - 4ac$

$= (2-k)^2 - 16$

$= k^2 - 4k - 12$

$= (k-6)(k+2)$



$\Delta > 0$

9 If $5^{x-1} = 10$ then x can be found by evaluating

A $\frac{1}{log_{10}5} + 1$

B $\frac{1}{log_e 5} + 1$

C $log_5 10 - 1$

D $1 + log_{10}5$

E $log_e 5 + log_e 10$

$\log_{10} 5^{x-1} = \log_{10} 10$

$x - 1\,(\log_{10}5) = \log_{10}10\,(1)$

$x - 1 = \frac{1}{\log_{10}5}$

$x = \frac{1}{\log_{10}5} + 1$

10 Let $g(x) = log_2(x), x > 0$. Which one of the following equations is true for all positive real values of x?

A $2\,g(8x) = g(x^2) + 8$

B $2\,g(8x) = g(x^2) + 6$

C $2\,g(8x) = [g(x) + 8]^2$

D $2\,g(8x) = g(2x) + 6$

E $2\,g(8x) = g(2x) + 64$

$2g(8x) = 2\log_2(8x)$

$= \cancel{2}\log_2(2^6x^2)$

$= 6 + \log_2 x^2$

$= g(x^2) + 6$.

Short Answer Section:

Any question worth more than 1 mark must have the appropriate working shown to justify the extra marks.

1. Solve $2log_e(x-2) - log_e(x) = 0, where\ x > 2$

$\log_e (x-2)^2 - \log_e x = 0$

$\log_e \left(\frac{(x-2)^2}{x}\right) = 0$

$\boxed{x = 4}$

1 marks

2. If $log_{10}(y) = -2 + 4log_{10}(x) - log_{10}(y^2)$, **show** that $y = \sqrt[3]{\frac{x^4}{100}}$.

$\log_{10}(y) = \cancel{\log} -2 + 4\log_{10}(x) - 2\log_{10}(y)$

$3\log_{10}(y) = -\log_{10}(100\cancel{0}) + 4\log_{10}(x)$

$\log_{10} y^3 = \log_{10}\left(\frac{x^4}{100\cancel{0}}\right)$

$y^3 = \frac{x^4}{100\cancel{0}} \Rightarrow y = \sqrt[3]{\frac{x^4}{100\cancel{0}}}$

3 marks

3. State the gradient of a line perpendicular to the line that passes through the points (3,0) and (0,-6).

$m = \frac{-6-0}{0-3} = \frac{-6}{-3} = 2.$

$\boxed{m_\perp = -\frac{1}{2}.}$

2 marks

4. The transformation $T: R^2 \to R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

The image of the curve $y = 2x^2 + 1$ under the transformation T has equation $y = ax^2 + bx + c$. Find the values of a, b and c.

$x' = 3x - 1 \Rightarrow x = \frac{x'+1}{3}$

$y' = 2y + 4 \Rightarrow y = \frac{y'-4}{2}$

$y = 2x^2 + 1$ becomes $\frac{y'-4}{2} = 2\left(\frac{x'+1}{3}\right)^2 + 1$

$y' = \frac{4}{9}(x')^2 + \frac{8}{9}(x') + 6\frac{4}{9}$

$a = \frac{4}{9}$ $b = \frac{8}{9}$ $c = \frac{58}{9}$

3 marks

5 If $log_e(y) = log_e(x) - log_e(p)$, write an equation relating x, y and p that **does not** involve logarithms.

$\log_e(y) = \log_e(x) - \log_e(p)$

$\log_e(y) = \log_e\left(\frac{x}{p}\right)$

$\boxed{y = \frac{x}{p}}$

2 marks

6. A species of small fish is being farmed in the sea to keep up supplies for the exporting of seafood to other countries. The number of fish, F, is increasing according to the rule: $F = 5000 \times 1.5^t$ where t is the number of years since the farm started

a) How many fish were there initially?

When $t = 0$ $F = 5000 \times 1.5^0$

$= 5000$ fish.

1 mark

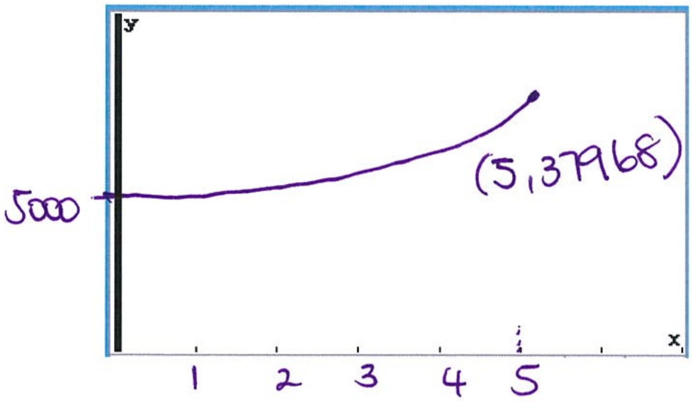
b) How many fish were there after 2 years?

$t = 2$ $F = 5000 \times 1.5^2$

$= 11250.$

1 mark

c) Sketch the graph of F against t for the first 5 years.



3 marks

d) How many years did it take for the number of fish to double? (to 1 decimal place)

When $F = 10{,}000$

$10000 = 5000 \times 1.5^t$

$2 = 1.5^t$

$t = 1.7$ years.

1 marks

e) In which month between $t = 5$ and $t = 6$ did the number of fish reach 50000?

$50{,}000 = 5000 \times 1.5^t$

$t = 5.6789$

$\therefore$ in September.

2 marks

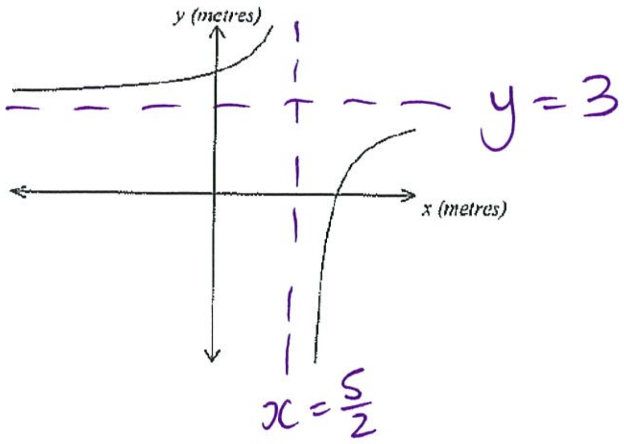
7. A go-kart racing track has 2 different tracks that can be modelled by the function $f: D \to R, f(x) = \frac{-5}{2x-5} + 3$ where x and $f(x)$ are in meters.

a) State the largest possible domain of D

$\text{Dom} f = R \setminus \{\frac{5}{2}\}$.

1 marks

The tracks are sketched below:



b) i) Give the equations of the asymptotes and sketch these above.

$x = \frac{5}{2} \qquad y = 3.$

ii) Give the coordinates of axis intercepts of $f(x)$.

When $y = 0$.

$0 = \frac{-5}{2x-5} + 3$

$-3 = \frac{-5}{2x-5}$

$2x - 5 = \frac{5}{3}$

$2x = \frac{5}{3} + \frac{15}{3}$

$2x = \frac{20}{3}$

$x = \frac{20}{6}$

$x = \frac{10}{3}$

$\underline{x = 0}$

$y = \frac{-5}{-5} + 3$

$= 1 + 3$

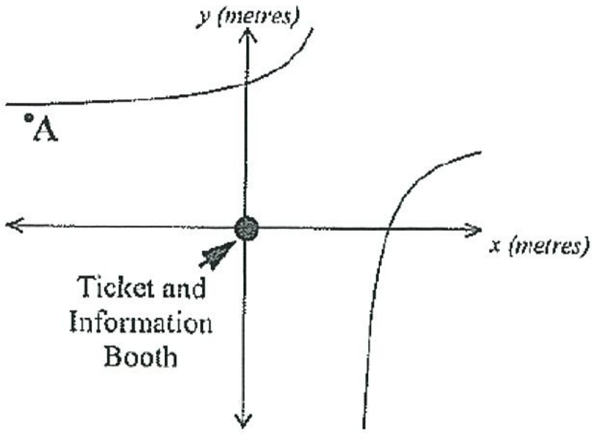
$= 4 \quad (0, 4).$

$\therefore$ x int

$(10/3, 0).$

2 mark

Safety regulations require spectators to be at least one meter away from the track. A ticket and information booth is located at the origin of the graph. A spectator is standing at point A with co-ordinates (-5,3) on the diagram shown.



c) Is the spectator breaking safety regulations? Justify your answer. yes

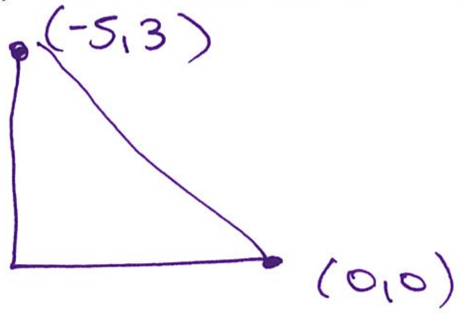
When $x = -5$ $\quad y = \frac{-5}{-15} + 3$ $\quad \therefore$ Breaking regulations.

$= 3\frac{1}{3}$

$\therefore$ spectator is $\frac{1}{3}$ metre south of track

2 marks

d) What is the exact distance from the spectator to the Ticket and Information Booth?



2 marks

distance $= \sqrt{(0-3)^2 + (0+5)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$ metres.

<u>**End of question booklet.**</u>