Student Name: SOLUTIONS

Home Group: ____________

Teacher's name: (please circle): Mrs O'Rielly Ms Webb

Mathematical Methods

Unit 2

November 2015

Part I **Total 54 marks**

Topics covered:
- Probability
- Circular Functions
- Rates of Change
- Differential Calculus
- Integral Calculus
- Matrices
- Combinatorics

Complete working must be shown and simplified wherever possible in order to gain full marks.

Reading Time: 15 minutes

Writing Time: 60 minutes

Students are NOT permitted to use any calculators or reference books for this section.

No paper or electronic dictionaries may be used.

Useful formulae:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cap B) = \Pr(A).\Pr(B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$^{n}C_{r} = \frac{n!}{(n-r)!r!} \qquad ^{n}P_{r} = \frac{n!}{(n-r)!}$$

Any question worth more than 1 mark <u>must have</u> the appropriate working shown to justify the extra marks.

1) A menu offers a choice of five entrees, four mains and three desserts.

   Find the number of meal choices possible

   a) if one of each must be chosen for a 3 course meal.

   $5 \times 4 \times 3 = 60$

   b) if you can choose to omit the dessert.

   $5 \times 4 \times 3 + 5 \times 4 = 80$

(2 marks)

2) The digits 0, 1, 2, 3 and 4 are used to make a 3-digit number. No digit is repeated.

   a) How many different 3-digit numbers are possible, if 0 cannot be the first digit?

   $4 \times 4 \times 3 = 48$

   b) If any of the 3-digit numbers in part a is equally likely to have been made, find the probability that number made is greater than or equal to 230.

   no. ways $> 230 = 2\ 3\text{ or }4\ .\ . + 3\ .\ . + 4\ .\ .$

   $= 1 \times 2 \times 3 + 1 \times 4 \times 3 + 1 \times 4 \times 3$

   $= 6 + 12 + 12 = 30$

   $\therefore Pr(> 230) = \frac{30}{48} = \boxed{\frac{5}{8}}$

(3 marks)

3) Evaluate ${}^{100}C_2$

   $= \frac{100!}{98!\,2!} = \frac{100 \times 99}{2} = \frac{9900}{2} = 4950$

(1 mark)

4) In how many ways can four girls be selected for a table tennis team, if five girls try out?

   ${}^5C_4 = \frac{5!}{4!\,1!} = 5$

(1 mark)

5) If A = $\begin{bmatrix} 3 & ^{-}4 \\ 2 & ^{-}1 \end{bmatrix}$ and B = $\begin{bmatrix} ^{-}2 & ^{-}5 \\ 6 & 4 \end{bmatrix}$, find the following:

a) A - B

$$= \begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} -2 & -5 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -4 & -5 \end{bmatrix}$$

b) 5B

$$= 5 \begin{bmatrix} -2 & -5 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} -10 & -25 \\ 30 & 20 \end{bmatrix}$$

c) BA

$$= \begin{bmatrix} -2 & -5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -2\times 3 + -5\times 2 & -2\times -4 + -5\times -1 \\ 6\times 3 + 4\times 2 & 6\times -4 + 4\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 13 \\ 26 & -28 \end{bmatrix}$$

d) det (A)

$$= ad - bc = 3\times -1 - -4\times 2 = -3+8 = 5$$

e) A^{-1}

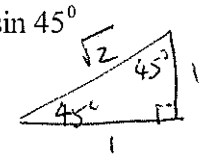
$$= \frac{1}{5} \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{5} & \frac{4}{5} \\ \frac{-2}{5} & \frac{3}{5} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -0.2 & 0.8 \\ -0.4 & 0.6 \end{bmatrix}$$

(1 + 1 + 2 + 1 + 1 = 6 marks)

6) Find the exact values of:

a) $\sin 45^0$

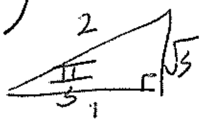


$= \frac{\sqrt{2}}{2}$

b) $\cos\frac{2\pi}{3}$ $= \cos\left(\pi - \frac{2\pi}{3}\right)$

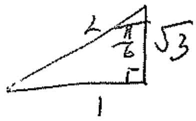
$= -\cos\frac{\pi}{3}$

$= -\frac{1}{2}$



c) $\tan\left(-\frac{\pi}{6}\right)$ $= \tan\frac{\pi}{6}$

$= \frac{\sqrt{3}}{3}$



(3 marks)

7) a) What is the period and the amplitude of the graph of

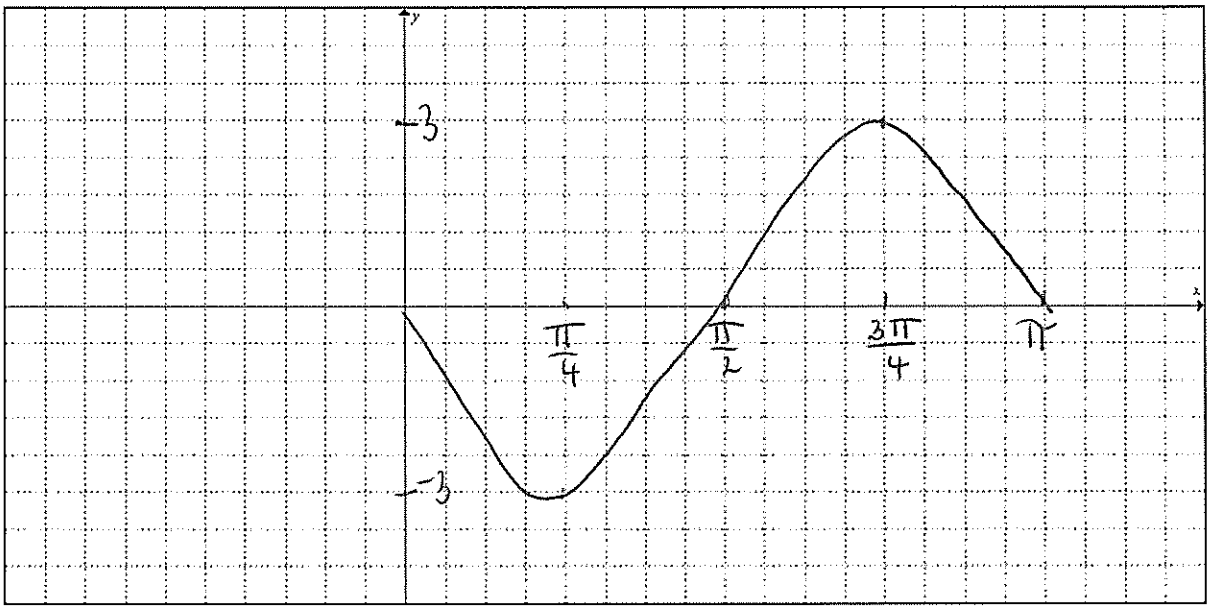
$y = -3\sin 2x$?

amp $= 3$ period $= \frac{2\pi}{6} = \frac{2\pi}{2}$

$= \pi$

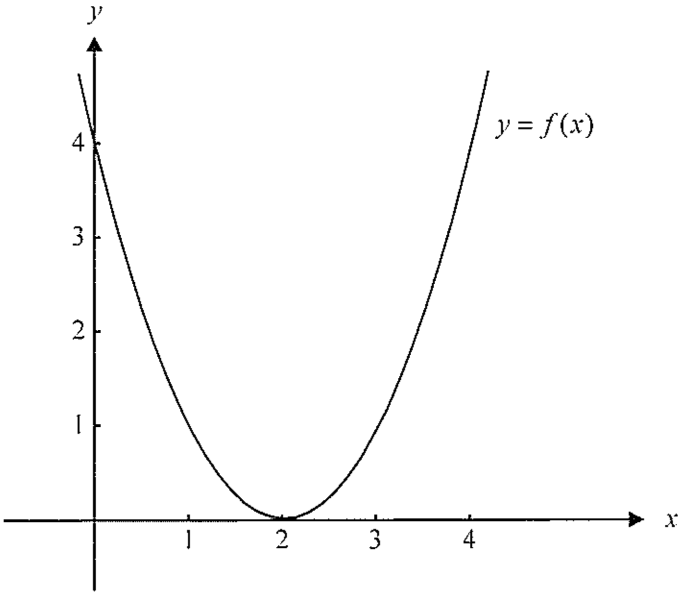
b) Sketch the graph, showing <u>one complete cycle</u>. Clearly label key points.

* reflected sin graph



(1 + 3 = 4 marks)

8) The graph of the function $f: R \to R,\ f(x) = (x-2)^2$ is shown below.



a) Find the average rate of change of $y = f(x)$ with respect to x, between x = 1 and x = 4.

av. rate of change $= \dfrac{f(4) - f(1)}{4-1}$

$= \dfrac{(4-2)^2 - (1-2)^2}{4-1}$

$= \dfrac{4-1}{3} = 1$

b) Find the instantaneous rate of change of $y = f(x)$ with respect to x at the point where x = 2.

Inst. rate of change $= f'(2)$

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$

$f'(2) = 2(2) - 4$

$= 0$

* can also read from graph as tp is at $x = 2$

(2 marks)

9) If Adam drives at 80km/h for 4 hours and 110km/h for 2 hours, what is his average speed for the entire journey?

distance after 4 hrs $= 4 \times 80 = 320$ km

" 2 more hrs $= 2 \times 110 = 220$

total $= 540$ km

av. speed $= \dfrac{540}{6} = 90$ km/hr.

(1 mark)

10) Find, using first principles, the derivative of

$$y = 2x^2 - 3x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} \quad \Rightarrow f'(x) = 4x - 3$$

$$= \lim_{h \to 0} 4x + 2h - 3, \ h \neq 0$$

(3 marks)

11) If $f(x) = (x-1)(x+4)$, find $f'(2)$.

$$= x^2 + 4x - x \cancel{4} - 4 \qquad \underline{f'(x) = 2x - 3}$$

$$= x^2 - 3x - 4$$

(2 marks)

12) Find $\lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 9}$

$$= \lim_{x \to 3} \frac{(x-4)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$$

$$= \lim_{x \to 3} \frac{x-4}{x+3}, \ x \neq -3, 3$$

(2 marks)

$$= \frac{3-4}{3+3}$$

$$= \frac{-1}{6}$$

13) Find the derivatives of:

a) $y = \dfrac{5x^4 - 2x^3}{x^2}$

$= \dfrac{5x^4}{x^2} - \dfrac{2x^3}{x^2}$

$= 5x^2 - 2x$

$\dfrac{dy}{dy} = 10x - 2$

b) $f(x) = \dfrac{3}{\sqrt{x}}$

$= 3x^{-\frac{1}{2}}$

$f'(x) = 3x - \frac{1}{2}\ x^{-\frac{3}{2}}$

$= \dfrac{-3}{2x^{\frac{3}{2}}}$

(4 marks)

14) Evaluate:

a) $\int (4x^3 - x^2 + 9)dx$

$= x^4 - \frac{1}{3}x^3 + 9x + c$

b) $\int \sqrt[3]{x}dx$

$= \int (x^{\frac{1}{3}})dx$

$= \dfrac{1}{\frac{4}{3}} x^{\frac{4}{3}} = \dfrac{3}{4} x^{\frac{4}{3}}$

(2 marks)

15) Find an expression for $f(x)$ if $f'(x) = 9x - 3$ and $f(2) = 4$.

$f(x) = \int f'(x)$

$= \int (9x - 3)\,dx$

$= \frac{9}{2}x^2 - 3x + c$

$f(2) = 4$

$\therefore 4 = \frac{9}{2} \times (2)^2 - 3(2) + c$

$4 = \dfrac{9 \times 4}{2} - 6 + c$

(3 marks)

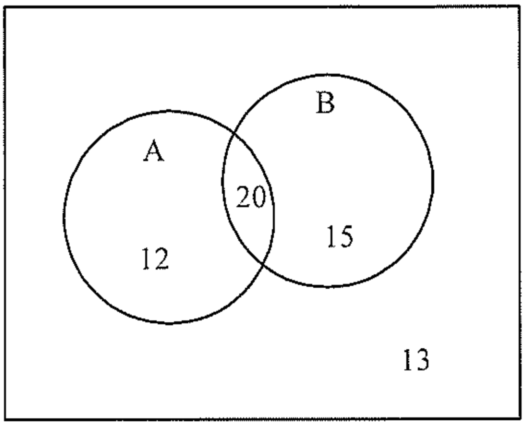
$4 = 18 - 6 + c$

$4 = 12 + c$

$c = -8$

$f(x) = \dfrac{9x^2}{2} - 3x - 8$

16)



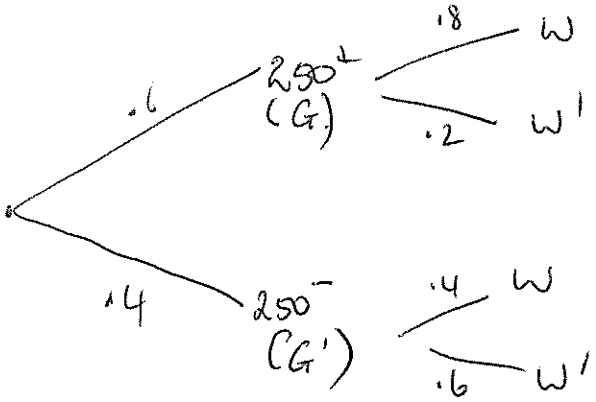
From the Venn Diagram find the $\Pr(A' \cup B)$.

$$= \frac{20+15+13}{60} = \frac{48}{60} = \frac{4}{5}$$

(1 mark)

17) If a cricket team makes more than 250 runs in a one-day game then on past performances they have an 80% chance of winning. If they make 250 runs or less then they have a 40% chance of winning. This team makes more than 250 runs on 60% of occasions.

a) Construct a tree diagram to illustrate this situation.



b) Calculate the overall chance of this team winning a one-day game.

$$\Pr(W) = .6 \times .8 + .4 \times .4$$

$$= .48 + .16$$

$$= .64$$

(1 + 2 = 3 marks)

18) A spinner has four equal parts; one is white and the other three are red.
A second spinner has six equal parts, two are red and the other four are white.
These spinners are spun simultaneously.

What is the probability that

* independent events

$\therefore Pr(A \cap B) = Pr(A) \times Pr(B)$

a. both spinners land on red.

$= \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}} = \frac{1}{4}$

(1 mark)

b. only one spinner lands on red:

$= Pr(1^{st} \text{ spinner red}) \text{ or } Pr(2^{nd} \text{ spinner red})$

$= \frac{\cancel{3}}{\cancel{4}_2} \times \frac{\cancel{2}}{\cancel{3}} + \frac{1}{4} \times \frac{1}{3} = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$

(2 marks)

c. at least one spinner lands on red.

$= 1 - Pr(\text{no red})$

$= 1 - \frac{1}{\cancel{4}_2} \times \frac{\cancel{2}^1}{3}$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

* or $= Pr(\text{both red}) + Pr(1 \text{ red})$

$= \frac{1}{4} + \frac{7}{12}$

$= \frac{3+7}{12}$

$= \frac{10}{12}$

$= \frac{5}{6}.$

(1 mark)

19) In Year 11 at River Valley College 27% of students failed the English exam and 62% of Year 11 students are male. Also, 20% of this Year 11 group are female **and** passed the English exam.

a) Use this information to complete the Karnaugh map below.

	Passed	Failed	
Male	·53	·09	·62
Female	·2	·18	·38
	·73	·27	1

(2 marks)

b) Find the probability that a student selected at random from the Year 11 group

i. is male and passed the exam.

0·53

(1 mark)

ii. is male and failed the exam.

0·09

(1 mark)

iii. is female and failed the exam.

0·18

(1 mark)

20) If $\Pr(A) = 0.1, \Pr(B) = 0.4$ and $\Pr(A|B) = 0.2$ find $\Pr(B|A)$.

$$Pr(A \backslash B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$0 \cdot 2 \times \cdot 4 = Pr(A \cap B)$$

$$\Rightarrow Pr(A \cap B) = \cdot 08$$

$$Pr(B \backslash A) = \frac{Pr(A \cap B)}{Pr(A)}$$

$$= \frac{\cdot 08}{\cdot 1}$$

$$= 0 \cdot 8$$

(2 marks)

END OF EXAMINATION