MATHEMATICAL METHODS (CAS) UNITS 3 & 4

2014 Trial **EXAMINATION 1**

July 2014

Section B: No CAS or reference book allowed

There is a total of 40 marks available for this section.

Writing time: 1 hour

Instructions to students

This section consists of 12 questions.
All questions should be answered in the spaces provided.
The marks allocated to each of the questions are indicated throughout.
An exact answer is required for all questions unless specified otherwise.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.

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Question 1

a. Let $y = \sqrt{2x^2 - 1}$. Find $\frac{dy}{dx}$.

2 marks

b. Find the derivative of $\log_e(\sin(x))$.

1 mark

c. Let $f(x) = \frac{x}{e^{3x}}$. Find $f'(1)$.

3 marks

Question 2

Solve $\log_e(3) + 2\log_e(x) = \log_e(4x)$ for x.

3 marks

Question 3

Let $g:(2,\infty) \rightarrow R$, $g(x) = 3\log_e(x-2)$. Find g^{-1}, the inverse function of g.

Inverse functions are not on the July exam

3 marks

Question 4

Let $g: R\setminus\{0\} \to R,\ g(x) = 1 + \frac{1}{x}$.

Show that $4g(2u) - g(-u) = 3g(u)$.

2 marks

Question 5

Solve the equation $\sin\left(\frac{x}{2}\right) + \frac{1}{\sqrt{3}}\cos\left(\frac{x}{2}\right) = 0$ for $x \in R$.

3 marks

Question 6

Find the exact area enclosed by the graph of $y = e^{\frac{x}{2}}$, the line $x = 1$ and the positive x and y axes.

3 marks

Question 7

A spherical balloon is being inflated. Its volume is increasing at the rate of 2cm^3 per second. Find the rate in cm/sec, at which the radius of the balloon is increasing when the radius is 4cm.

3 marks

Question 8

A transformation is described by the equation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right).$$

Find the image of the curve with equation $y = \dfrac{2}{x+1} - 1$ under this transformation. Give your answer in the form $y = \dfrac{a}{x} + b$ where a and b are real constants.

3 marks

Question 9

Given that $f(x+h) \approx f(x) + hf'(x)$, where h is small, find an approximate value of $\sqrt{9.03}$.

3 marks

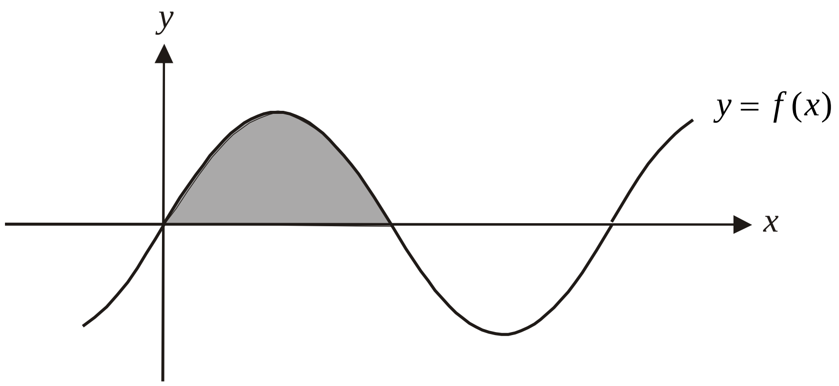
Question 10

The graph of $y = 3x^2 + a$; where a is a real constant, has a normal with equation $y = \frac{x}{3} + 1$. Find the value of a.

4 marks

Question 11

Part of the graph of the function $f : R \to R,\ f(x) = a\sin(2x)$ where a is a positive constant is shown below.



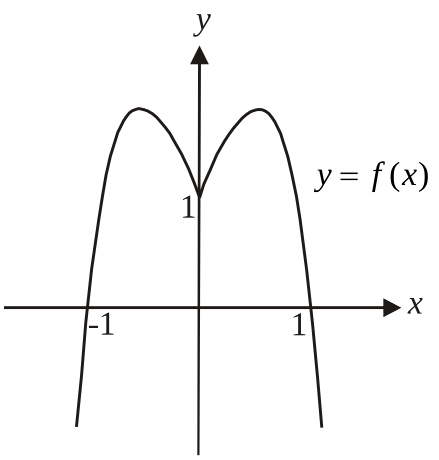
The shaded region represents an area of 4 square units.
Find the value of *a*.

4 marks

Question 12

Let $f : R \to R,\ f(x) = 2|x| - 3x^4 + 1$.

The graph of $y = f(x)$ is shown below.



a. Write down the domain of the derivative function $f'(x)$.

1 mark

b. Find the rule for $f'(x)$.

2 marks

End of Section B

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x\, p(x)$	$\sigma^2 = \Sigma(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$