Name:
Teacher:

Unit 2 Maths Methods (CAS) Exam 2012

Tuesday November 6 - 1.50 pm

Reading time: 10 Minutes
Writing time: 80 Minutes

Instruction to candidates:
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a single bound exercise book containing notes and class-work, CAS calculator.

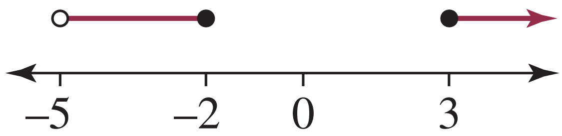
Materials Supplied:
Question and answer booklet, detachable multiple choice answer sheet at end of booklet.

Instructions:
- Write your name and that of your teacher in the spaces provided.
- Answer all short answer questions in this booklet where indicated.
- Always show your full working where spaces are provided.
- Answer the multiple choice questions on the detachable answer sheet.

Section A	Section B	Total exam
/20	/30	/50

Section A – Multiple choice questions (20 marks)

Question 1
Which of the sets correctly describes the interval shown on the number-line below?



a) $(-5,-2) \cup (3,\infty)$

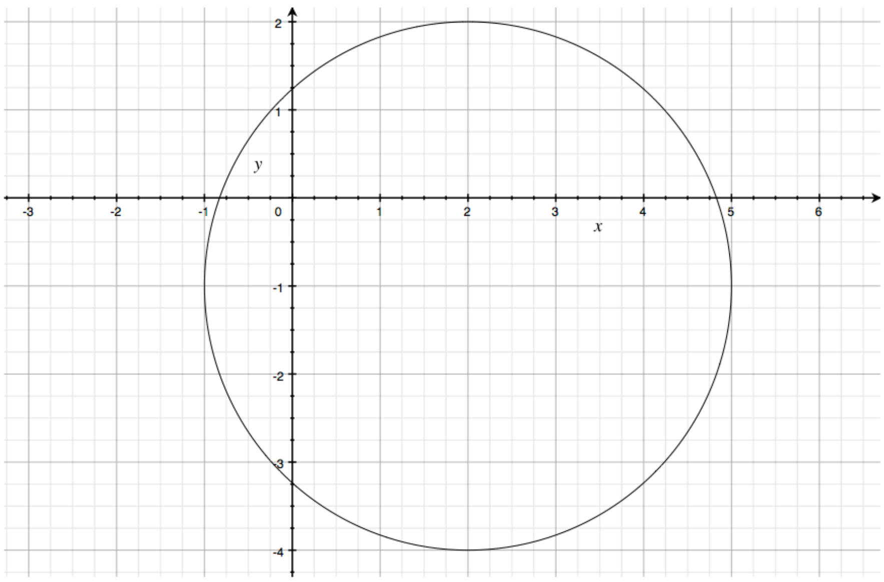
b) $[-5,-2] \cup [3,\infty]$

c) $[-5,-2) \cup [3,\infty)$

d) $(-5,-2] \cup (3,\infty]$

e) $(-5,-2] \cup [3,\infty)$

Question 2
What is the equation of the circle shown here?



a) $(x-2)^2 + (y+1)^2 = 9$

b) $(x-2)^2 + (y+1)^2 = 3$

c) $(x+2)^2 + (y-1)^2 = 9$

d) $(x+2)^2 + (y-1)^2 = 3$

e) $(x-2)^2 - (y+1)^2 = 9$

Question 3

The angle $140°$ in radians is equal to:

a) 140π

b) $\frac{140}{\pi}$

c) $\frac{\pi}{140}$

d) $\frac{7\pi}{9}$

e) $\frac{9\pi}{7}$

Question 4

For an angle (θ) in the first quadrant $(0 < \theta < \frac{\pi}{2})$, which of the following statements is **incorrect**?

a) $\sin(\theta) > 0$

b) $\cos(\theta) > 0$

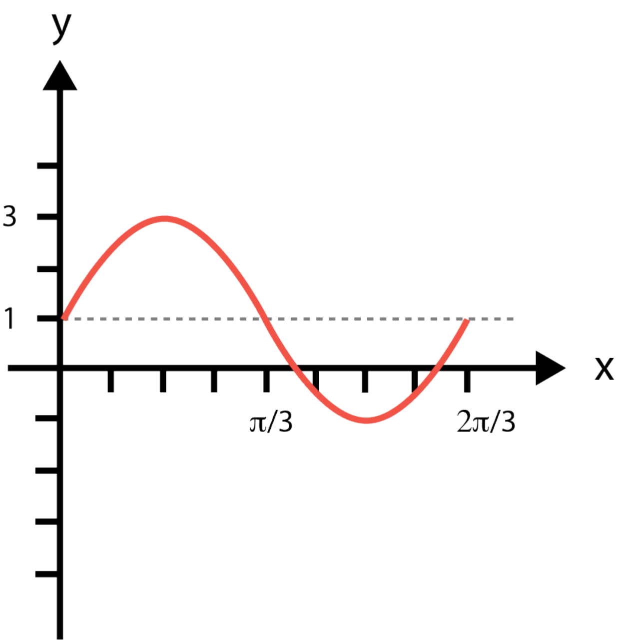
c) $\tan(\theta) > 0$

d) $\sin(\pi + \theta) = \sin(\theta)$

e) $\sin(2\pi - \theta) = -\sin(\theta)$

Question 5

The graph shown here could be described by the equation:



a) $y = \frac{2\pi}{3} sin2x$

b) $y = 2sin3x + 1$

c) $y = 2sin\frac{2\pi}{3}x - 1$

d) $y = 2cos\frac{2\pi}{3}x + 1$

e) $y = 2sin\frac{2\pi}{3}x + 1$

Question 6

The solutions to the equation $\sqrt{3} = 2cos\left(\frac{\theta}{3}\right)$ where $0 < \theta < 12\pi$ are:

a) $\frac{\pi}{2}, \frac{11\pi}{2}$

b) $\frac{\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{23\pi}{2}$

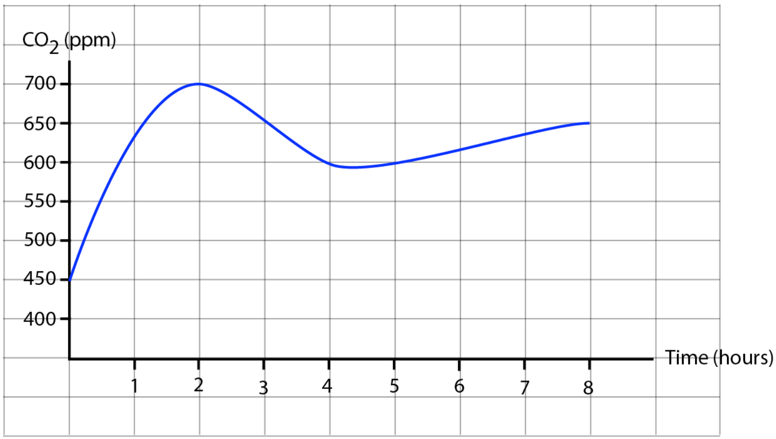
c) $\frac{\pi}{3}, \frac{11\pi}{3}, \frac{21\pi}{3}, \frac{31\pi}{3}$

d) $\frac{\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{23\pi}{3}$

e) undefined

Question 7

The graph here shows the concentration of CO_2 in the air inside a gymnasium over the 8 hours that the gym is in use.



The average increase in concentration during the first four hours was

a) 0 ppm / hour
b) 0.027 ppm / hour
c) 38 ppm / hour
d) 150 ppm / hour
e) 600 ppm / hour

Question 8

The cubic function $y = \frac{x^3}{3} - \frac{7x^2}{2} + 10x + 2$ has stationary points at:

a) $\left(-2, \frac{32}{3}\right)$ and $\left(-5, \frac{37}{6}\right)$

b) $\left(2, \frac{32}{3}\right)$ and $\left(5, \frac{37}{6}\right)$

c) $\left(-2, -\frac{32}{3}\right)$ and $\left(5, \frac{37}{6}\right)$

d) $\left(-2, -\frac{32}{3}\right)$ and $\left(-5, -\frac{37}{6}\right)$

e) $\left(2, \frac{37}{6}\right)$ and $\left(5, \frac{32}{3}\right)$

Question 9

At the point $x = 2$, the gradient of the curve $y = 4x^2 - 16x$ is equal to:

a) -16
b) -12
c) 0
d) 16
e) 20

Question 10

The function $f(x) = ax^3 + bx^2 + cx + d$ has the derivative function:

a) $f'(x) = -3ax^2 - 2bx^2 - cx - d$

b) $f'(x) = ax^2 + bx + c$

c) $f'(x) = 3ax^2 + 2bx^2 + cx + d$

d) $f'(x) = 3ax^2 + 2bx + c$

e) $f'(x) = 0$

Question 11

The tangent to the curve $y = 2x^2 + 3$ at the point $(1,5)$ is given by the equation:

a) $y = 4x + 1$

b) $y = -4x + 1$

c) $y = 4x - 9$

d) $y = 4x - 1$

e) $y = 2x + 1$

Question 12

The derivative of the function $y = x^2 + \frac{1}{x}$ is:

a) $2x - \frac{1}{x^2}$

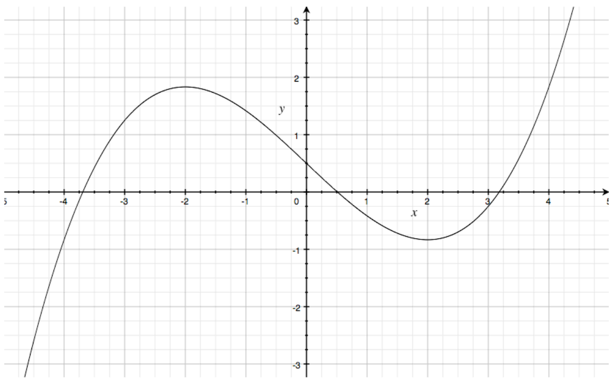
b) $2x + \frac{1}{x^2}$

c) $2x$

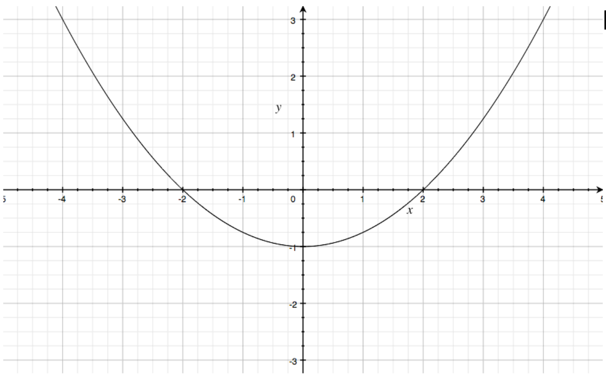
d) $2x + 1$

e) $2x - 1$

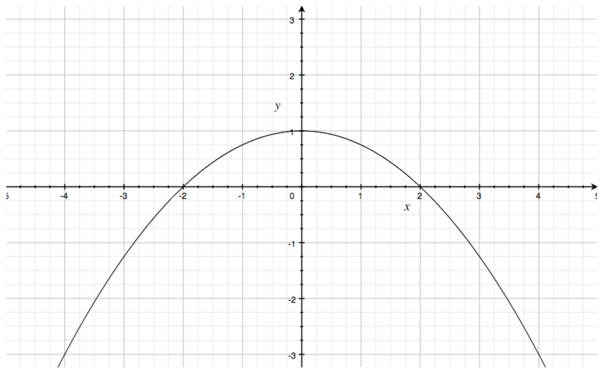
Question 13
Which of the following graphs correctly shows the derivative of the function shown here?



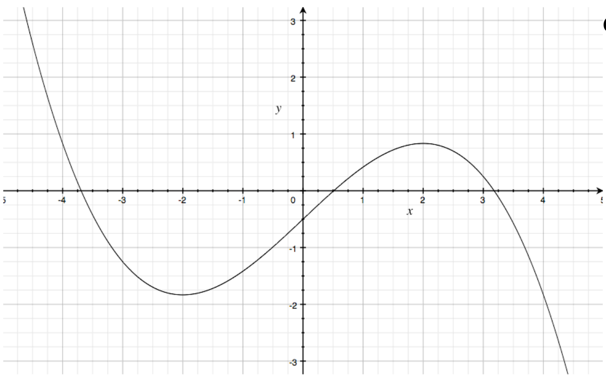
a)



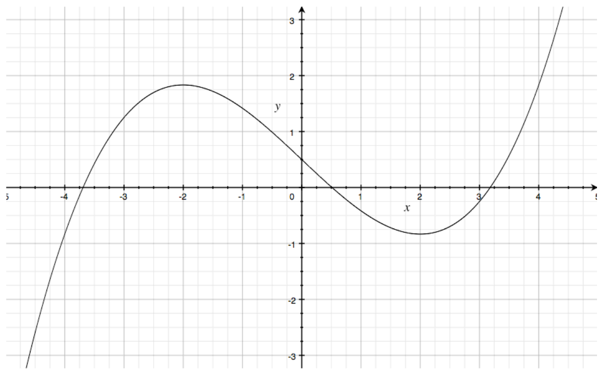
b)



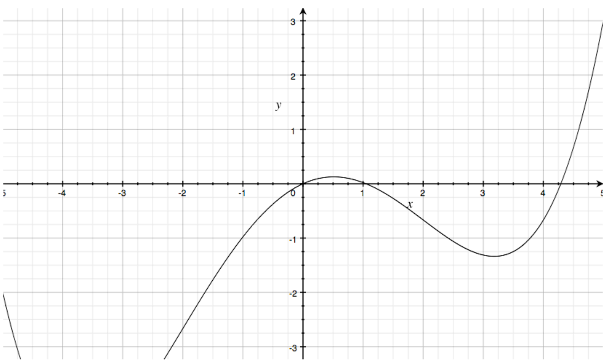
c)



d)



e)



Question 14

A possible **anti-derivative** of the function $f(x) = 5x^2 + 2x$ could be:

a) $F(x) = 10x^3 + 2x^2$

b) $F(x) = \frac{5x^3}{3} + x^2 + 2$

c) $F(x) = 5x^3 + x^2 + c$

d) $F(x) = -\frac{5x^3}{3} - x^2 + 6$

e) $F(x) = 2x^5 + x^2 - 4$

Question 15

For the matrix multiplication shown below, what is the value of x?

$$\begin{bmatrix} 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 41 \end{bmatrix}$$

a) 1
b) 2
c) 3
d) 4
e) 5

Question 16

Which the following multiplications of the matrices shown below **cannot** be performed?

$$A = \begin{bmatrix} 4 & 3 \\ -4 & 7 \end{bmatrix} B = \begin{bmatrix} -4 \\ 3 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 4 & -4 \\ 9 & -8 \\ 6 & 0 \end{bmatrix}$$

a) AB
b) DA
c) DB
d) CD
e) DC

Question 17

In a game involving rolling a standard die, a player has rolled a 6 two times in a row. The chance that the next roll is a 6 is:

a) 1 in 6
b) 1 in 2
c) 1 in 36
d) 1 in 216
e) 1 in 1296

Question 18

A particular industrial machine produces a percentage of defective products. It was found that if a defective product is made, then there is a 20% chance of the next one also being defective. There is only a 10% of a product being defective after a non-defective one is made. If the first product of the day is defective, what is the chance that the third one is also defective?

a) 11%
b) 12%
c) 20%
d) 81%
e) 82%

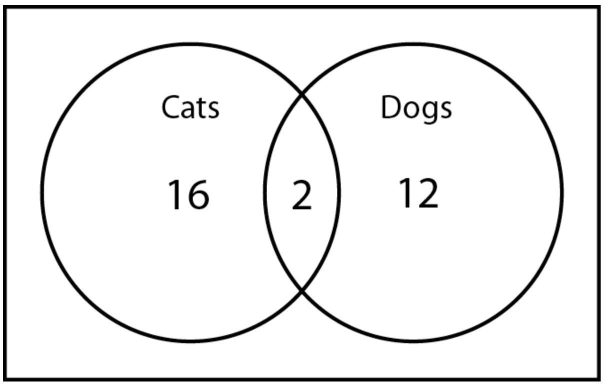
Question 19

Two independent events have the probabilities Pr(A) = 0.7 and Pr(B) = 1.Which of the following statements is **incorrect**?

a) The outcomes in event A are a subset of the outcomes in event B.
b) Pr(A∪B) = 1
c) Pr(A∩B) = 0.7
d) Pr(A'∩B) = 0.3
e) Pr(A'∩B') = 0.3

Question 20

The Venn diagram shown here gives the results of a survey of 100 Warrnambool residents, asking whether they own cats, dogs or both.



Which of the following statements about the results is **incorrect**?

a) There are 14 dog owners.
b) 2 of the dog owners also own cats.
c) 70% of respondents don't own a cat or dog.
d) There are 16 cat owners
e) A person who owns a dog is most likely not to own a cat.

Section B – Short answer questions (30 marks)

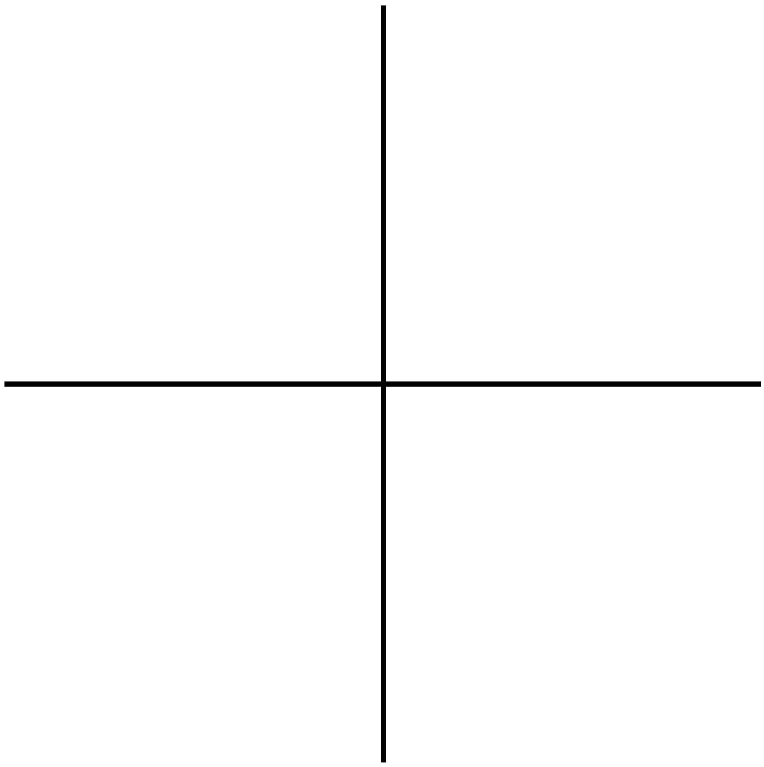
Question 1

For the function $f(x) = \sqrt{x} + 3$:

a) State the implied domain of the function. (1 mark)

b) State the range of the function. (1 mark)

c) Sketch the graph of the function, including labels on any endpoints. (1 mark)



d) On the same axes, sketch the graph of the inverse function $f^{-1}(x)$, including labels on any endpoints. (2 marks)

Question 2

The height (in metres) of a bungee jumper over time is given by the equation:

$$h(t) = 15\cos\left(\frac{\pi t}{2}\right) + 18$$

(Time is measured in seconds from the start of the jump.)

a) Calculate the time that it takes the jumper to return to the starting height. (2 marks)

b) State the minimum and maximum height above the ground that the jumper reaches. (1 mark)

Minimum:	m	Maximum:	m

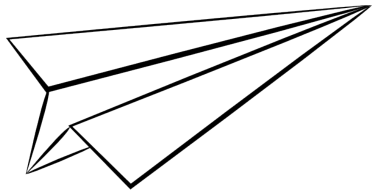
c) Calculate the times (to the nearest tenth of a second) in the first 5 seconds at which the jumper is at a height of 8 m. (2 marks)

s	s

d) The bungee apparatus has been modified by using a stiffer bungee cord so that now the jump only takes 3 s to fall 12 m and return to the same starting height as before. State the equation that would be used to model this fall. (1 mark)

Question 3

In a paper aeroplane competition, participants can launch their planes from a maximum height of 2 metres.



During a flight, the height H(t) of one aeroplane is given by the rule:

$$H(t) = \frac{1}{64}t^3 - \frac{7}{32}t^2 + \frac{1}{2}t + 2, t > 0$$

where t is the time (in seconds) from launch, H the height is in metres, and the rule H(t) only applies until the aeroplane first touches the ground (H=0).

a) Show that the initial height complies with competition rules. (1 mark)

b) State the maximum height of the aeroplane. (1 mark)

c) State the time taken for the aeroplane to hit the ground. (1 mark).

d) Find a rule that gives the rate of change of height with respect to time. (2 mark)

e) Calculate the rate of change of height with respect to time at t=4. (2 marks)

Question 4

Two matrix transformations are applied to the point (3,6). $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

a) Explain what each matrix transformation does to the point. (2 marks)

b) Find the new co-ordinates of the point. (1 mark)

Question 5

For the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & -4 \end{bmatrix}$:

a) Find the determinant of A. (1 mark)

b) Find the inverse A. (1 mark)

c) Use this information to find the solution to the simultaneous equations $2x + 4y = 12$ and $x - 4y = -8$. (1 mark)

Question 6

A survey of 200 adults was conducted about the the relationship between vision problems and age. It was found that 70 of the people surveyed wore glasses. 15 of the 75 young adults (under 30 years of age) wore glasses.

a) Complete the table below, showing the number in each group in each group. (4 marks)

	G	**G'**	
Y			
Y'			
			200

b) What is the probability that a randomly selected young adult wears glasses? (1 mark)

c) What is the probability that a randomly selected person who does not wear glasses is over 30? (1 mark)

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Answer sheet for section A

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e
12.	a	b	c	d	e
13.	a	b	c	d	e
14.	a	b	c	d	e
15.	a	b	c	d	e
16.	a	b	c	d	e
17.	a	b	c	d	e
18.	a	b	c	d	e
19.	a	b	c	d	e
20.	a	b	c	d	e

Answer sheet for section A

1.	a	b	c	d	**e**
2.	**a**	b	c	d	e
3.	a	b	c	**d**	e
4.	a	b	c	**d**	e
5.	a	**b**	c	d	e
6.	a	**b**	c	d	e
7.	a	b	**c**	d	e
8.	a	**b**	c	d	e
9.	a	b	**c**	d	e
10.	a	b	c	**d**	e
11.	**a**	b	c	d	e
12.	**a**	b	c	d	e
13.	**a**	b	c	d	e
14.	a	**b**	c	d	e
15.	a	b	c	d	**e**
16.	a	b	c	d	**e**
17.	**a**	b	c	d	e
18.	a	**b**	c	d	e
19.	a	b	c	d	**e**
20.	a	b	c	**d**	e

Section B – Short answer questions (30 marks)

Question 1

For the function $f(x)=\sqrt{x}+3$:

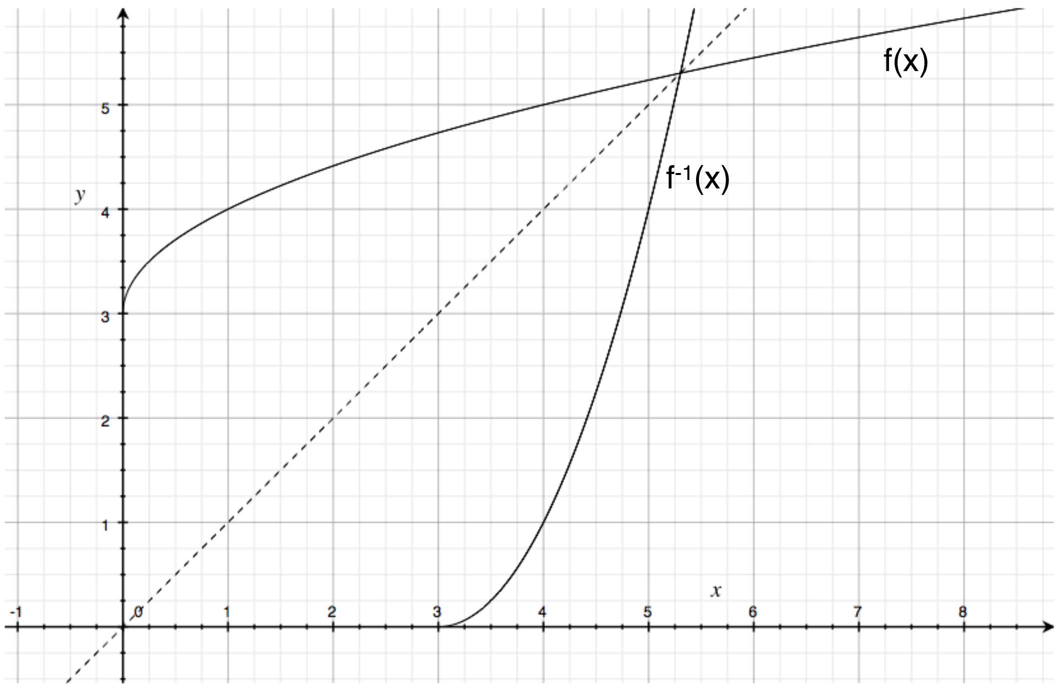
a) State the implied domain of the function. (1 mark)

$[0,\infty)$

b) State the range of the function. (1 mark)

$[3,\infty)$

c) Sketch the graph of the function, including labels on any endpoints. (1 mark)



d) On the same axes, sketch the graph of the inverse function $f^{-1}(x)$, including labels on any endpoints. (2 marks)

Question 2

The height (in metres) of a bungee jumper over time is given by the equation:

$$h(t) = 15\cos\left(\frac{\pi t}{2}\right) + 18$$

(Time is measured in seconds from the start of the jump.)

a) Calculate the time that it takes the jumper to return to the starting height. (2 marks)

Time period: $T = \frac{2\pi}{\pi/2} = 4$ s

b) State the minimum and maximum height above the ground that the jumper reaches. (1 mark)

Minimum:	3 m	Maximum:	33 m

c) Calculate the times (to the nearest tenth of a second) in the first 5 seconds at which the jumper is at a height of 8 m. (2 marks)

$8 = 15cos\left(\frac{\pi t}{2}\right) + 18$ $\qquad$ $\frac{-10}{15} = cos\left(\frac{\pi t}{2}\right)$

$\frac{2}{\pi}cos^{-1}\left(\frac{-10}{15}\right) = t$

t = 1.5 s	t = 2.5 s

d) The bungee apparatus has been modified by using a stiffer bungee cord so that now the jump only takes 3 s to fall 12 m and return to the same starting height as before. State the equation that would be used to model this fall. (1 mark)

$k = \frac{2\pi}{3}$ $\qquad$ amplitude = 6m $\qquad$ centre = 27 m

$h = 6cos\left(\frac{2\pi t}{3}\right) + 27$

Question 3

In a paper aeroplane competition, participants can launch their planes from a maximum height of 2 metres.

During a flight, the height H(t) of one aeroplane is given by the rule:

$$H(t) = \frac{1}{64}t^3 - \frac{7}{32}t^2 + \frac{1}{2}t + 2, t > 0$$

where t is the time (in seconds) from launch, H the height is in metres, and the rule H(t) only applies until the aeroplane first touches the ground (H=0).

a) Show that the initial height complies with competition rules. (1 mark)

$$H(0) = \frac{1}{64}0^3 - \frac{7}{32}0^2 + \frac{1}{2}0 + 2 = 0$$

b) State the maximum height of the aeroplane. (1 mark)

2.31 m

c) State the time taken for the aeroplane to hit the ground. (1 mark).

8 s

d) Find a rule that gives the rate of change of height with respect to time. (2 mark)

$$H'(t) = \frac{3}{64}t^2 - \frac{7}{16}t + \frac{1}{2}$$

e) Calculate the rate of change of height with respect to time at t=4. (2 marks)

$$H'(4) = \frac{3}{64}4^2 - \frac{7}{16}4 + \frac{1}{2}$$

$$H'(4) = \frac{48}{64} - \frac{28}{16} + \frac{1}{2}$$

$$H'(4) = -\frac{1}{2}$$

Question 4

Two matrix transformations are applied to the point (3,6). $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

a) Explain what each matrix transformation does to the point. (2 marks)

$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ Dilation of 3 from the x axis.

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$: Reflection around the line y = x

b) Find the new co-ordinates of the point. (1 mark)

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

Question 5

For the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & -4 \end{bmatrix}$:

a) Find the determinant of A. (1 mark)

$$\det A = (2 \times -4) - (4 \times 1) = -12$$

b) Find the inverse A. (1 mark)

$$\begin{bmatrix} 2 & 4 \\ 1 & -4 \end{bmatrix}^{-1} = -\frac{1}{12}\begin{bmatrix} -4 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{12} & -\frac{1}{6} \end{bmatrix}$$

c) Use this information to find the solution to the simultaneous equations $2x + 4y = 12$ and $x - 4y = -8$. (1 mark)

$$\begin{bmatrix} 2 & 4 \\ 1 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ -8 \end{bmatrix} == \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{12} & -\frac{1}{6} \end{bmatrix}\begin{bmatrix} 12 \\ -8 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ \frac{7}{3} \end{bmatrix} \qquad x = \frac{4}{3} \quad y = \frac{7}{3}$$

Question 6

A survey of 200 adults was conducted about the the relationship between vision problems and age. It was found that 70 of the people surveyed wore glasses. 15 of the 75 young adults (under 30 years of age) wore glasses.

a) Complete the table below, showing the number in each group in each group. (4 marks)

	G	G'	
Y	15	60	75
Y'	55	70	125
	70	130	200

b) What is the probability that a randomly selected young adult wears glasses? (1 mark)

$$Pr(G|Y) = \frac{Pr(G \cap Y)}{Pr(Y)} = \frac{15}{75}$$

$$\frac{1}{5} = 20\%$$

c) What is the probability that a randomly selected person who does not wear glasses is over 30? (1 mark)

$$Pr(Y'|G) = \frac{Pr(Y' \cap G')}{Pr(G')} = \frac{70}{130}$$

$$\frac{7}{13} = 54\%$$