James Hancock Methods Exam 1 –

Brackets denote mark distribution.

1.

$2x^3e^{2x} + 3x^2e^{2x}$ (1)

$f'(x) = 2\sec^2(2x)/\tan(2x)$ (1)

$f'\left(\frac{\pi}{8}\right) = \frac{2\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} = 2 * \frac{2}{1} = 4$ (1)

2.

$2e^{3x^2} + C$ (+C is not necessary as it asked for AN antiderivative) (1)

$2\log_e\sqrt{(x-2)} + \log_e(x-4) = 1$

$\log_e(x-2)(x-4) = \log_e(e)$ $(1) \textit{for correct} \log \textit{law usage}$

$(x-2)(x-4) = e$

$x^2 - 6x + 8 - e = 0$

Using the quadratic formula:

$$x = \frac{6 \pm \sqrt{36 - 4(8-e)}}{2} = \frac{6 \pm 2\sqrt{9 - (8-e)}}{2} = 3 \pm \sqrt{1+e} \qquad (1)mark$$

Providing an argument as to why $3 - \sqrt{1+e}$ is an unacceptable solution (since ln(x-4) will have x-4 <0) yields the third mark. (1mark)

3.

$\tan\left(3x + \frac{\pi}{4}\right) = 0$

$$3x + \frac{\pi}{4} = tan^{-1}(0)$$

$$3x + \frac{\pi}{4} = 0, \pi, 2\pi \qquad (1\ mark)$$

$$3x = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12} \qquad (1\ mark)$$

Asymptote at x = pi/12 and 5pi/12 (1 mark)
General tan shape (1 mark)

4.

f(1/x) = 1/(2(1/x)^3) -2 = x^3/2 – 2 (2 marks, 1 for substitution, 1 for correct answer)

x^3/2 – 2 = 0
x^3 = 4

x = (4^(1/3)) = 2^(2/3) (1 mark either is fine)

5.

a .

Given E(X) = 2.05

$$m * 0 + 0.2 + 0.8 + 0.45 + 4k = 2.05$$

$$4k = 0.6$$

k = 0.15 (1 mark)

Since the sum of all probabilities = 1, m = 0.1 (1 mark)

b.

4 combinations (writing them down gives one mark, or having them in the form Pr(0 and 3) etc gives one mark)

Pr(sum to 3) = Pr(0 and 3) + Pr(1 and 2) + Pr(2 and 1) + Pr(3 and 0) (1 mark)

= (0.1)(0.15) + (0.15)(0.1) + (0.2)(0.4) + (0.4)(0.2) = 0.19 (19/100) (if arrived at answer, award all 2 marks)

6.

$$\frac{dV}{dt} = \frac{-2T}{5}$$

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dr} = 4\pi r^2 \ (1\ mark)$$

$$\frac{dr}{dt} = \frac{dr}{dV} * \frac{dV}{dt} = \left(\frac{1}{4\pi r^2}\right) * \left(\frac{-2T}{5}\right) = \frac{-T}{10\pi r^2} \quad (1\ mark)$$

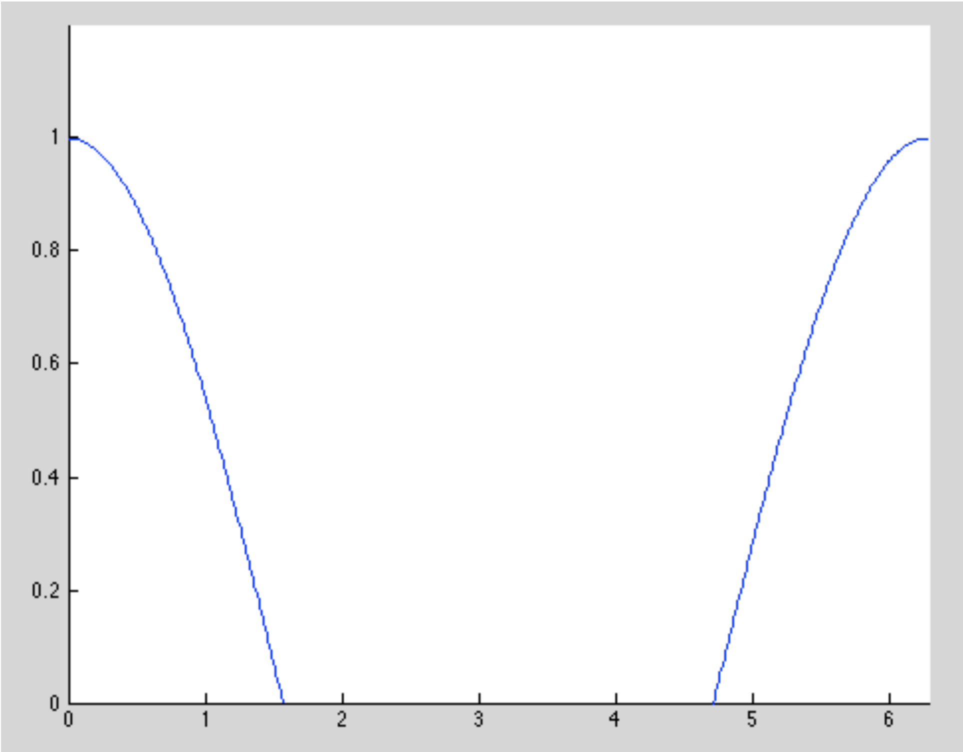
b.

When V = 32pi/3, r = 2 (1 mark)

$$\frac{dr}{dt}(r = 2) = -\frac{25}{10\pi * 4} = -\frac{5}{8\pi}$$

7.

(1 mark, remember the y-axis is not 1, it is k).



b.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Since the total area under the graph is equal to 2 (1 mark for showing this using integrals), k has to equal ½ in order to have f(x) define a probability distribution function. (1 mark for stating why k has to equal 1/2, by referring to the total area under the graph equaling 1).

c. $\Pr(X<\frac{\pi}{2} \mid X<\frac{7\pi}{4}) = \frac{\Pr\left(X<\frac{\pi}{2} \cap X<\frac{7\pi}{4}\right)}{\Pr\left(X<\frac{3\pi}{2}\right)} = \frac{\Pr\left(X<\frac{\pi}{2}\right)}{\Pr\left(X<\frac{7\pi}{2}\right)}$ $(1\ mark)$

Remember that

$\Pr\left(X < \frac{\pi}{2}\right) = 0.5, and \Pr\left(X < \frac{7\pi}{2}\right) =$

$0.5 + \int_{\frac{3\pi}{2}}^{\frac{7\pi}{4}} \cos(x)\, dx = 0.5 + \frac{1}{\sqrt{2}}$ $(1\ mark\ for\ correct\ probabilities\ )$

Therefore $\frac{\Pr\left(X<\frac{\pi}{2}\right)}{\Pr\left(X<\frac{3\pi}{2}\right)} = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{\sqrt{2}}} = \frac{1}{1+\sqrt{2}}\ or\ \frac{1}{1+\frac{2}{\sqrt{2}}}$ (1 mark, for equivalent answer, award 1 mark)

8.

$$f'(x) = \frac{1}{2\sqrt{x}} \quad (1\ mark)$$

Need to find where f'(x) = 1/8 (getting this from the normal equation).

This occurs when x = 16. Therefore c = 16 (1 mark)

Therefore, f(16) = 9, so therefore b = 5. (1 mark)

When now know that the point x = 16, y = 9 lies on the normal equation.

Therefore, a = y + 8x = 9 + 16*8 = 137 (1 mark)

9.

Pr(3T) = (1-p)^3 (1 mark)

b.

Pr(X = 2) = nCr(3,2)*p^2(1-p) = $3p^2(1-p)$ (1 mark)

c.

Therefore let, P = $3p^2(1-p)$

$$P = 3p^2 - 3p^3$$

$$\frac{dP}{dt} = 6p - 9p^2 = 0 \quad (1\ mark)$$

This occurs at p = 2/3. (1 mark)

10.

Since the cubic passes through the point (0,0) d = 0. (1 mark)

Integrating from -1 to 0 will yield an area of 19/12. Therefore a = 12. (1 mark for correct a and 1 mark for correct integration evaluation)

We know have the cubic $12(x^3 + 5x^2 + 6x)$. The x-intercepts are $x = -2$ and $x = -3$. Therefore $b = -2$ $c = -3$. (these values can be swapped). (1 mark).