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NAME: ______________________________

VCE® Mathematical Methods

Units 3 & 4 Practice Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	**9**	**40**

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

Let $f: R \rightarrow R, f(x) = 2x^2 e^{-x}$

a. Find $f'(x)$. Express $f'(x)$ in a form that contains no negative signs in the exponent(s). 2 marks

b. Hence, determine $f'(1)$. 1 mark

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Question 2 (3 marks)

Solve the equation:

$3 \sin sin\,(x) \; + \; 2(cos(x))^2 = 3, \;\; x \in [0, 2\pi]$

Question 3 (2 marks)

Consider the equation for $f(x)$ below.

$$f(x) = 4 \cos cos\,(x) \; - \; 1$$

What is the equation for $g(x)$, given $g(x)$ is obtained by dilation of $f(x)$ by a factor of 2 from the y - axis, followed by a translation of π units in the positive direction of the x-axis followed reflection about the x-axis.

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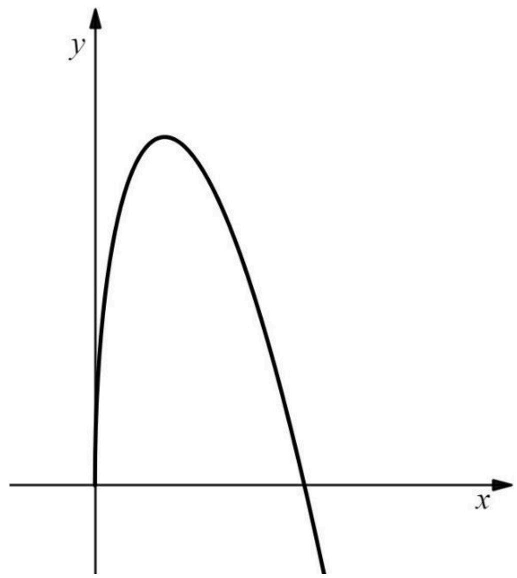
Question 4 (6 marks)

$Consider\ f(x) = \frac{ax-bx^2}{\sqrt{x}},\ \ a, b \in R\backslash\{0\}$

A sketch of $f(x)$ is shown below for a particular set of values for a and b.



a. Show that the x-intercept of $f(x)$ is located at $x = \frac{a}{b}$ 1 mark

b. Determine the *x*-coordinate of the turning point of *f*(*x*) in terms of *a* and *b*. 2 marks

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c. Let $a = 3$ and $b = 1$. Determine the value of x such that the gradient of $f(x)$ at this value is equal to the gradient of the line that passes through the turning point of $f(x)$ and the x-intercept located at $x = \frac{a}{b}$ 3 marks

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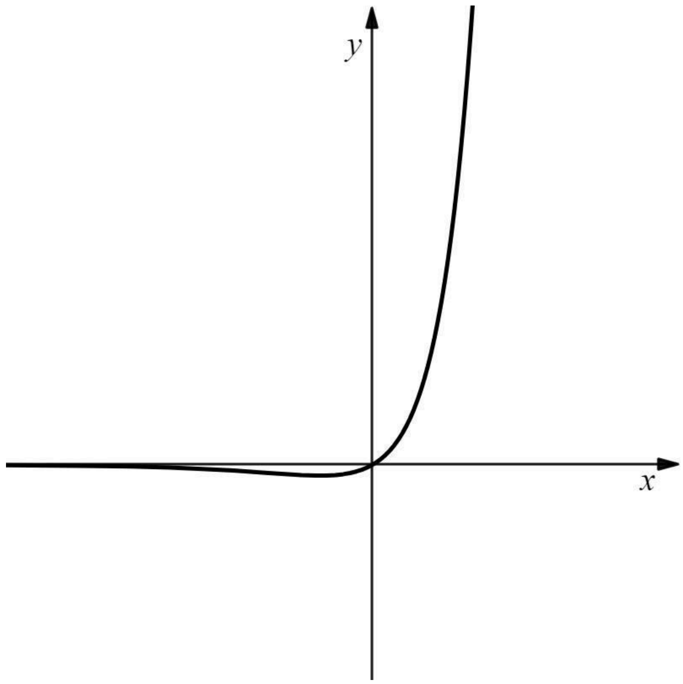
Question 5 (3 marks)

A function has the equation $g(x) = e^{2x} - e^{x},\ x \in R$. The graph of $g(x)$ is shown below.



a. Let $h(x) = g(x),\quad x \in [a, \infty)$. What is the minimum value of a for which $h^{-1}(x)$ the inverse of $h(x)$ is defined? 1 mark

b. Hence determine the equation of $h^{-1}(x)$ 2 marks

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Question 6 (8 marks)

a. The probability that Ralph arrives at class on time is 0.25. The teacher said that if he was not on time for at least two out of the next three mornings, he would have to make up the time at lunch.

(i) If Ralph decided to keep with his normal routine, what is the probability that he will be making up time at lunch? 1 mark

(ii) If Ralph was on time for at least one of the mornings, what is the probability that he will be making up time at lunch? 1 mark

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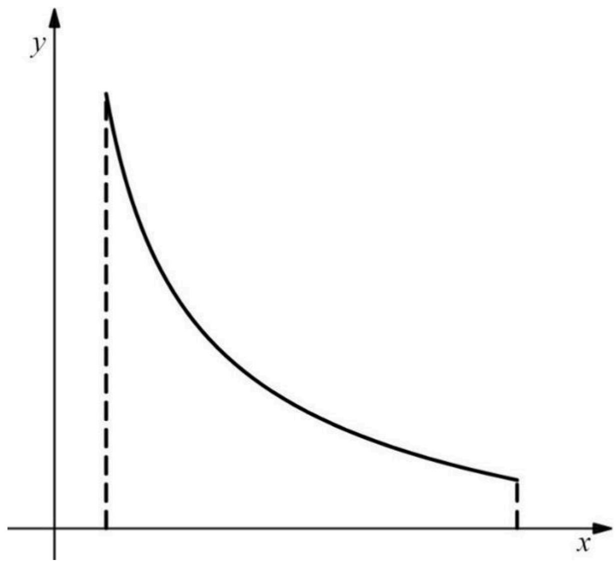
b. Ralph completes logic puzzles. The time required, in minutes, for him to complete a puzzle can be modelled by the random variable, X, with probability density function given by:

$$f(x) = \{\frac{1}{h\sqrt{t}} - k, \quad 1 \le x \le a \; 0, \qquad elsewhere$$

The probability that Ralph takes between 4 and 9 minutes is $\frac{3}{8}$ while the probability that it takes between 1 and 4 minutes is $\frac{5}{8}$. A graph of $f(x)$ is shown below.



(i) What is the shortest possible time that Ralph can complete a puzzle? 1 mark

(ii) Determine the values of the parameters h and k 3 marks

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(iii) Show that $a = 9$ in $f(x)$ 2 marks

Question 7 (5 marks)

Gertrude, a Year 12 student, wanted to improve the menu of the school's canteen. She planned to survey the students about their preference for hot snacks. A presurvey indicated that 14 out of the 20 students surveyed prefer the hot snacks currently offered.

Z is a standard normal random variable such that:

$$\Pr Pr\ (Z \leq 1.2816) = 0.90\ and\ \Pr Pr\ (Z \leq 1.6449) = 0.95$$

a. What is the value of $\hat{p}$? Express your answer as simplified fraction. 1 mark

b. Gertrude decided to sample 100 students from across the school for her survey. Using the information provided, write an expression for the 90% confidence interval. 2 marks

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c. If instead a sample of 200 students was used in **Part b**, by what scale factor would the width of the confidence interval change? Would its width increase or decrease? 2 marks

Question 8 (4 marks)

The average daily maximum temperature, T, throughout the year for a particular town can be modelled by the equation:

$$T(t) = a\cos(bt) + c, \quad a, b, c \in R$$

Where t is time in months and $t = 0$ corresponds to 1st January and $t = 1$ corresponds to the first of February and so on.

a. Determine the value of b in the model. 1 mark

b. Given that on the 1st of March the average daily maximum temperature is 26 °C, while on the 1st of September the average daily maximum temperature is 14 °C, determine the values of the other parameters in the model. 2 marks

c. Hence, what is the average daily maximum temperature on the 1st of May? 1 mark

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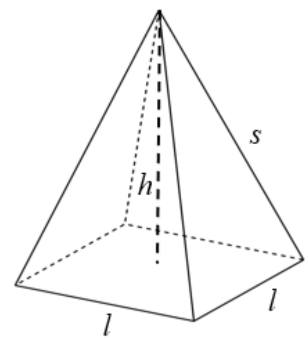
Question 9 (6 marks)

The frame (base and sloped edges) for a square-based pyramid is to be constructed from 4 m of wire. A diagram of the pyramid is shown below.



As can be seen in the diagram, the base of the pyramid has dimensions of $l \times l$, and the sloping edges have length s. The height of the apex of the pyramid above the centre of the base is h.

a. Express s in terms of l. 1 mark

b. The volume of a pyramid is equal to one third of the area of the base multiplied by the height. The volume contained within the wire-framed pyramid is to be maximised. Show that the volume, V, can be determined using the formula:

$$V = \frac{1}{3}\sqrt{\frac{1}{2}l^6 - 2l^5 + l^4}$$

2 marks

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c. Determine the dimensions of the frame (values of l and s) that will result in the maximum volume. 3 marks

END OF QUESTION AND ANSWER BOOK

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VCE® Mathematical Methods

Practice Written Examinations 1 and 2

FORMULA SHEET

Instructions

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Mensuration

area of a trapezium	$\frac{1}{2}(a + b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin sin\,(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax + b)^n\right) = an(ax + b)^{n-1}$	$\int (ax + b)^n dx = \frac{1}{a(n+1)}(ax + b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}((x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = (x) + c$
$\frac{d}{dx}(\sin sin\,(ax)) = a \cos cos\,(ax)$	$\int \sin sin\,(ax)dx = -\cos \frac{1}{a} cos\,(ax) + c$
$\frac{d}{dx}(\cos cos\,(ax)) = -a \sin sin\,(ax)$	$\int \cos cos\,(ax)dx = \sin \frac{1}{a} sin\,(ax) + c$
$\frac{d}{dx}(\tan tan\,(ax)) = \frac{a}{cos^2(ax)} = asec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

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Probability

$Pr(A) = 1 - Pr(A')$		$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$	
$Pr(A\|B) = \frac{Pr(A \cap B)}{Pr(B)}$			
mean	$\mu = E(X)$	Variance	$var(X) = \sigma^2 = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$Pr(X = x) = p(x)$	$u = \Sigma xp(x)$	$\sigma^2 = E((X - \mu)^2) = E(X)^2 - \mu^2$
continuous	$Pr(a < X < b) = \int_a^b f(x)$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \frac{X}{n}$		Mean	$E(\hat{P}) = p$
Standard deviation	$sd(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

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