

The Mathematical Association of Victoria

Trial Examination 2024

**MATHEMATICAL METHODS**

**Trial Written Examination 2 - SOLUTIONS**

**SECTION A: Multiple Choice**

| Question | Answer | Question | Answer |
|----------|--------|----------|--------|
| 1        | C      | 11       | A      |
| 2        | C      | 12       | D      |
| 3        | A      | 13       | C      |
| 4        | B      | 14       | A      |
| 5        | B      | 15       | B      |
| 6        | D      | 16       | D      |
| 7        | B      | 17       | A      |
| 8        | D      | 18       | C      |
| 9        | C      | 19       | B      |
| 10       | D      | 20       | A      |

**Question 1**                      **Answer C**

$$f(x) = -\frac{3}{2}\sin(2x - \pi)$$

Amplitude:  $A = \frac{3}{2}$

Period:  $P = \frac{2\pi}{2} = \pi$

**Question 2**                      **Answer C**

$$f(x) = \sqrt{x+2} \text{ and } g(x) = e^{2x}$$

Test range of  $f \subseteq$  domain of  $g$

$$[0, \infty) \subset R$$

domain of  $g \circ f =$  domain of  $f = [-2, \infty)$

**Question 3**                      **Answer A**

$$0 = ax^2 + 4x + c$$

two unique solutions if  $\Delta > 0$

$$4^2 - 4ac > 0$$

$$4ac < 16$$

$$ac < 4$$

**Question 4**      **Answer B**

$$x + (m-1)y = 2 \Rightarrow y = \left(\frac{-1}{m-1}\right)x + \frac{2}{m-1}$$

$$(m+1)x + 3y = 8 - m \Rightarrow y = -\left(\frac{m+1}{3}\right)x + \frac{8-m}{3}$$

Equate gradients

$$-\frac{1}{m-1} = -\frac{m+1}{3}$$

Gives  $m = \pm 2$ 

Test for infinite number of solutions

$$m = 2 \quad \begin{array}{l} x + y = 2 \\ 3x + 3y = 6 \end{array}$$

$$m = -2 \quad \begin{array}{l} x - 3y = 2 \\ -x + 3y = 10 \end{array}$$

Answer:  $m = 2$ 

The screenshot shows a CAS calculator window titled "Edit Action Interactive". The input field contains the following commands and results:

```

solve(x+(m-1)*y=2, y)
      {y=-x/(m-1)+2/(m-1)}
(solve((m+1)*x+3*y=8-m, y))
      {y=-(m*x+x+m-8)/3}
solve(-1/(m-1)=-((m+1)/3), m)
      {m=-2, m=2}
  
```

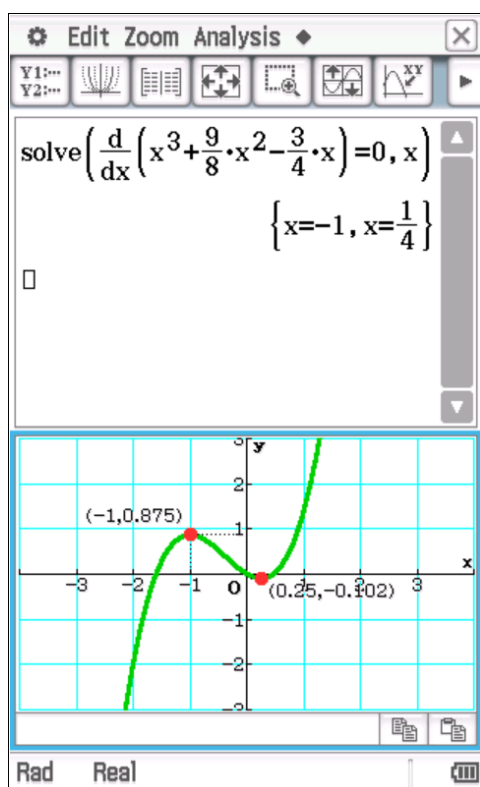
**Question 5**      **Answer B**

$$g(x) = x^3 + \frac{9}{8}x^2 - \frac{3}{4}x$$

$$g'(x) = 3x^2 + \frac{9}{4}x - \frac{3}{4} = 0$$

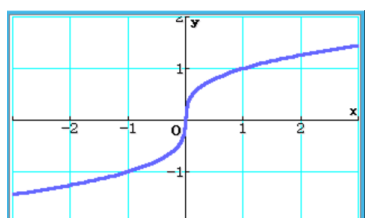
$$\text{Gives } x = -1, x = \frac{1}{4}$$

strictly decreasing for  $\left[-1, \frac{1}{4}\right]$



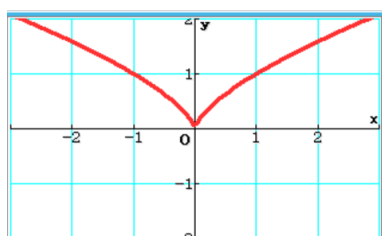
**Question 6**      **Answer D**

**Option A**  $y = x^3, \frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} \neq 0$



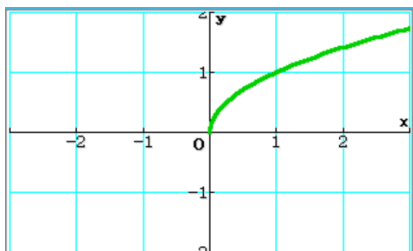
Gradient undefined at  $x = 0$

**Option B**  $y = x^{\frac{2}{3}}, \frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} \neq 0$



Sharp point at  $x = 0$

**Option C**  $y = x^{\frac{1}{2}}, \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \neq 0$

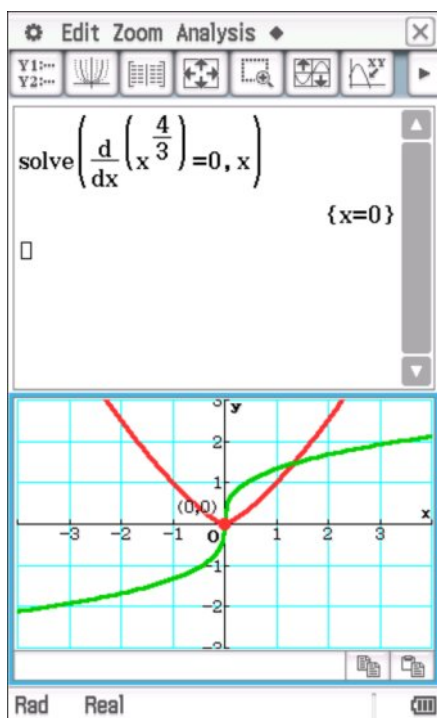


Endpoint at  $(0,0)$

**Option D**  $y = x^{\frac{4}{3}}$ ,  $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} = 0$  for  $x = 0$

Differentiable for all values over its maximal domain.

Gradient graph exists for all  $x \in \mathbb{R}$ .



**Question 7**

**Answer B**

Given  $\int_1^3 f(x)dx = 4$  and  $\int_3^1 g(x)dx = -2$ .

Simplify  $-\int_1^2 g(x)dx + \int_1^3 (2f(x) + 3)dx - \int_2^3 g(x)dx$

$$-\left(\int_1^2 g(x)dx + \int_2^3 g(x)dx\right) + 2\int_1^3 f(x)dx + \int_1^3 (3)dx$$

$$= -\int_1^3 g(x)dx + 2(4) + [3x]_1^3$$

$$= -2 + 8 + 6$$

$$= 12$$

**Question 8**      **Answer D**

Two balls of the same colour selected without replacement.

$$\text{Box A: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6}\right) = \frac{3}{14}$$

$$\text{Box B: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6}\right) = \frac{3}{14}$$

$$\text{Answer: } \frac{3}{14} + \frac{3}{14} = \frac{3}{7}$$

**Question 9**      **Answer C**

$$h: R \setminus \{1\} \rightarrow R, h(x) = \frac{1}{x-1} + 2.$$

$$\text{Average rate of change} = \frac{h(5) - h(2)}{5 - 2} = -\frac{1}{4}$$

$$h'(x) = -\frac{1}{4} \text{ at } x = -1 \text{ or } x = 3$$

The screenshot shows a CAS calculator interface with the following content:

- Toolbar: Edit, Action, Interactive, 0.5, 1/2, undo, redo, dx/dx, dx/dx, Simp, dx/dx, and other symbols.
- Input: `define h(x) = 1/(x-1) + 2`
- Output: `done`
- Input:  $\frac{h(5) - h(2)}{5 - 2}$
- Output:  $-\frac{1}{4}$
- Input: `solve(d/dx(h(x)) = -1/4, x)`
- Output:  $\{x = -1, x = 3\}$

**Question 10**      **Answer D**

$$f(x) = \begin{cases} k \sin\left(\frac{1}{2}x\right) & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^{\pi} k \sin\left(\frac{1}{2}x\right) = 1 \text{ gives } k = \frac{1}{2}$$

$$\int_0^m \frac{1}{2} \sin\left(\frac{1}{2}x\right) = 0.5 \text{ gives } m = \frac{2\pi}{3}$$

$\text{solve}\left(\int_0^{\pi} k \cdot \sin\left(\frac{1}{2} \cdot x\right) dx = 1 \mid 0 < k < \pi, k\right)$   
 $\{k = \frac{1}{2}\}$

$\text{solve}\left(\int_0^m \frac{1}{2} \cdot \sin\left(\frac{1}{2} \cdot x\right) dx = \frac{1}{2} \mid 0 < m < \pi, m\right)$   
 $\{m = \frac{2 \cdot \pi}{3}\}$

**Question 11**      **Answer A**

$$f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}, f(x) = \frac{1}{(x-1)^2} - 2$$

The tangent line at  $x=0$  is  $y=2x-1$ .

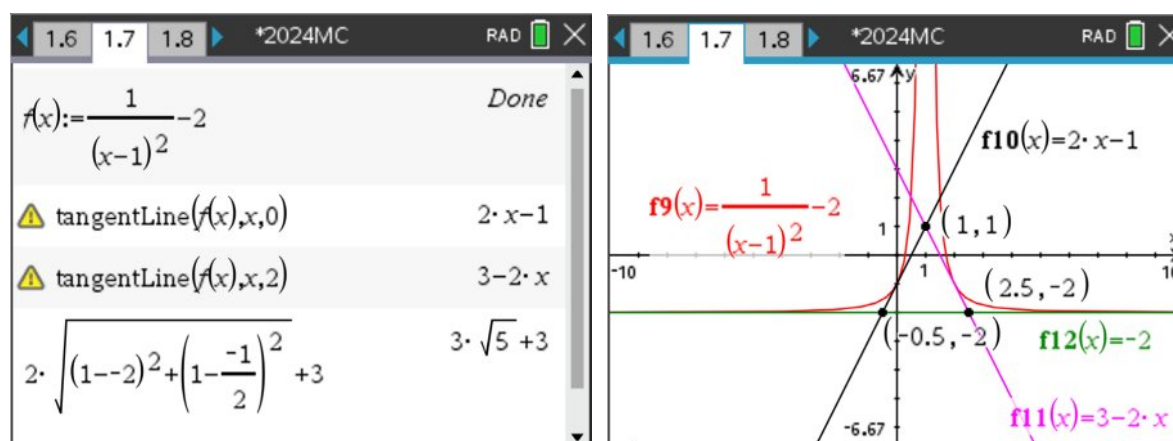
The tangent line at  $x=2$  is  $y=3-2x$ .

The coordinates of the vertices of the triangle are  $(1,1)$ ,  $(-\frac{1}{2}, -2)$  and  $(\frac{5}{2}, -2)$ .

The length of the base is 3.

The length of each of the other two sides is  $\sqrt{\left(1+\frac{1}{2}\right)^2 + (1+2)^2} = \frac{3\sqrt{5}}{2}$ .

$$\text{Perimeter} = 3\sqrt{5} + 3$$

**Question 12**      **Answer D**

$$y = g(x) = 4 \log_2(3x+5), \quad h = 1$$

$$\text{Area of the trapeziums} = \frac{1}{2}(g(0) + 2g(1) + 2g(2) + g(3))$$

$$= 2(\log_2(5) + \log_2(14) + \log_2(64) + \log_2(121))$$

$$= 2(\log_2(5 \times 14 \times 121) + \log_2(64))$$

$$= 2(\log_2(5 \times 14 \times 121) + 6)$$

$$= \log_2(5 \times 14 \times 121)^2 + 12$$

$$\neq \log_2(5 \times 14 \times 121)^2 + 6^2$$

**Question 13**      **Answer C**

$$f(x) = 3 \tan\left(\frac{1}{2}\left(\frac{\pi}{3}x - 1\right)\right) + 5$$

$$\text{Solve } \frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = \frac{\pi}{2}, x = \frac{3(\pi + 1)}{\pi}$$

**OR**

$$\frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = -\frac{\pi}{2}, x = \frac{-3(\pi - 1)}{\pi}$$

$$\text{The period} = \frac{\pi}{\frac{\pi}{6}} = 6$$

$$\text{A general solution is } x = \frac{-3(\pi - 1)}{\pi} + 6k, k \in \mathbb{Z}$$

The image shows two screenshots of a CAS calculator interface. The left screenshot shows the input  $\text{solve}\left(\frac{1}{2} \cdot \left(\frac{\pi}{3} \cdot x - 1\right) = \frac{\pi}{2}, x\right)$  and the output  $x = \frac{3 \cdot (\pi + 1)}{\pi}$ . The right screenshot shows the input  $\text{solve}\left(\frac{1}{2} \cdot \left(\frac{\pi}{3} \cdot x - 1\right) = -\frac{\pi}{2}, x\right)$  and the output  $x = \frac{3 \cdot (\pi - 1)}{\pi}$ .

**Question 14**      **Answer A**

|              |     |     |     |               |
|--------------|-----|-----|-----|---------------|
| $x$          | 0   | 1   | 2   | 3             |
| $\Pr(X = x)$ | 0.2 | 0.1 | $a$ | $\frac{k}{3}$ |

$$\text{Var}(X) = 0.1 + 4a + 3k - (0.1 + 2a + k)^2 = 1.4 \dots(1)$$

$$0.3 + a + \frac{k}{3} = 1 \dots(2)$$

$$a = 0.2, k = 1.5$$

$$E(X) = 0.1 + 2a + k = 2$$

1.11 1.12 1.13 \*2024MC RAD

$$\text{solve}\left(0.2+0.1+a+\frac{k}{3}=1 \text{ and } 0.1+4\cdot a+3\cdot k-(0.1+2\cdot a+k)\right)$$

$$a=-0.8 \text{ and } k=4.5 \text{ or } a=0.2 \text{ and } k=1.5$$

0.1+2\cdot a+k | a=0.2 and k=1.5 2.

Edit Action Interactive

$$\begin{cases} 0.3+a+\frac{k}{3}=1 \\ 0.1+4a+3k-(0.1+2a+k)^2=1.4 \end{cases} \Big|_{a, k}$$

$$\left\{ \left\{ a=-\frac{4}{5}, k=\frac{9}{2} \right\}, \left\{ a=\frac{1}{5}, k=\frac{3}{2} \right\} \right\}$$

$$0.1+2\left(\frac{1}{5}\right)+\frac{3}{2}$$

Alg Standard Real Rad

**Question 15**      **Answer B**

The domain of  $s(x) = 1 - \log_e(1-x)$  is  $(-\infty, -1)$ .

The range  $t(x) = 3\cos(2x-1)+1$  is  $[-2, 4]$ .

The domain of  $s(x) + t^{-1}(x)$  is the intersection of  $(-\infty, -1)$  and  $[-2, 4]$  which is  $[-2, -1)$ .

**Question 16**      **Answer D**

$$f: \mathbb{R} \setminus \left\{ \frac{a}{4} \right\} \rightarrow \mathbb{R}, f(x) = \frac{2}{4x-a} + 3$$

$x_0 = \frac{a}{4}$  will fail as  $x = \frac{a}{4}$  is an asymptote.

etwons method will also fail if the  $x$ -intercept of the tangent line at  $x_n$  is undefined.

ind the equation of the tangent line at any point on the curve. Let the  $x$ -coordinate be  $b$ .

$$y = \frac{3a^2 - 2a(12b+1) + 16b(3b+1)}{(a-4b)^2} - \frac{8x}{(a-4b)^2}$$

$$\text{Solve } y = \frac{3a^2 - 2a(12b+1) + 16b(3b+1)}{(a-4b)^2} - \frac{8x}{(a-4b)^2} = 0 \text{ when } x = \frac{a}{4}$$

$$b = \frac{3a-4}{12}, \text{ hence } x_0 = \frac{3a-4}{12} \text{ will fail.}$$

The  $x_0$  values for any point on the S branch will fail as none of the  $x$ -intercepts of the tangent lines are less than  $\frac{a}{4}$ .

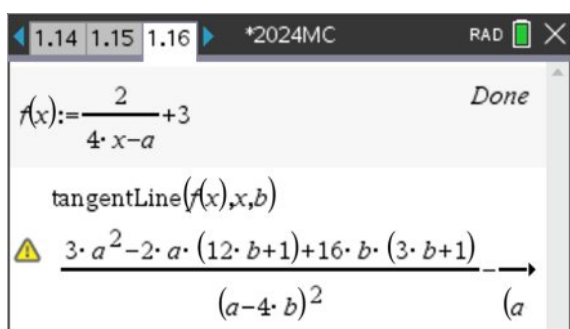
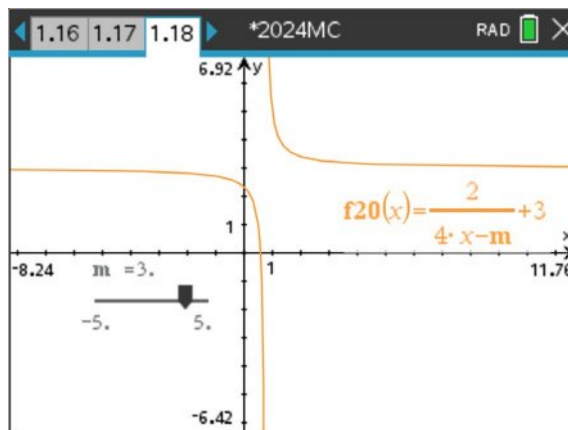
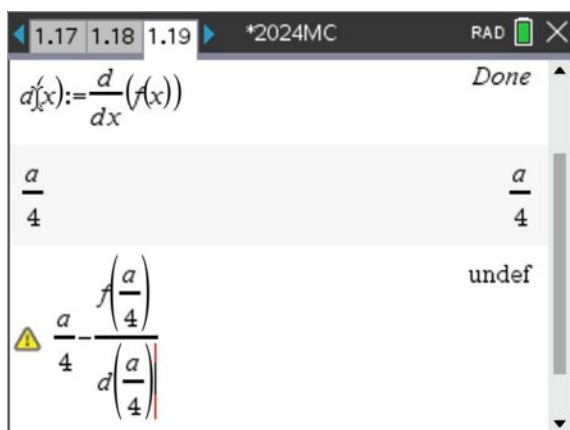
$x_0 < \frac{3a-4}{12}$  will also fail as the  $x$ -intercept of the tangent lines are all greater than  $\frac{a}{4}$ .



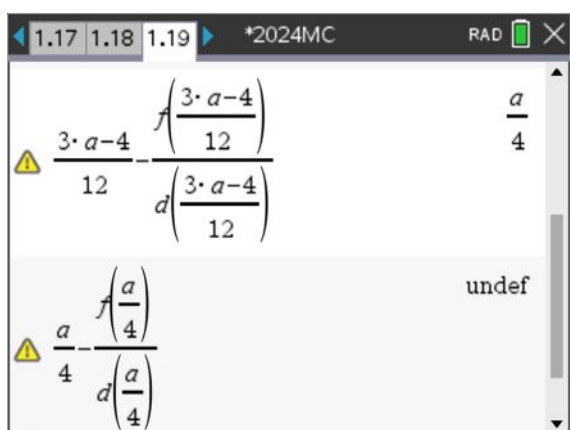
So convergence will only occur if  $\frac{3a-4}{12} < x_0 < \frac{a}{4}$ .

ewtons method fails if  $x_0 \in R \setminus \left(\frac{3a-4}{12}, \frac{a}{4}\right)$ .

The answer can also be found by checking the values in the options to see if they fail when using ewtons method.



$$\left. \begin{aligned} & \left( \frac{2}{(a-4 \cdot b)^2} - \frac{8 \cdot x}{(a-4 \cdot b)^2} = 0, b \right) \Big|_{x = \frac{a}{4}} \\ & \cdot b)^2 \end{aligned} \right\} b = \frac{3 \cdot a - 4}{12}$$



**Question 17**      **Answer A**

$$X \sim \text{Bi}(30, 0.35)$$

$$\Pr(X > 12 \mid X \geq 5)$$

$$\Pr(X \geq 13)$$

$$\Pr(X \geq 5)$$

$$= \frac{0.2197\dots}{0.9925\dots}$$

$$= 0.2215 \text{ correct to four decimal places}$$

**Question 18**      **Answer C**

Let  $A_v$  be the average value of  $f(x) = x^3 + x^2 - x + 1$  for the interval  $[a, 1]$ , where  $a \in (-\infty, 1)$ .

$$A_v = \frac{1}{1-a} \int_a^1 f(x) dx = \frac{3a^3 + 7a^2 + a + 13}{12}$$

$A_v$  is a cubic function.  $y = A_v$  will have 3 solutions between the two turning points.

Solve  $\frac{d}{da} \left( \frac{3a^3 + 7a^2 + a + 13}{12} \right) = 0$  for  $a$ .

$$a = \frac{2\sqrt{10} - 7}{9}, \quad a = \frac{-2\sqrt{10} - 7}{9}$$

$$A_v \left( \frac{2\sqrt{10} - 7}{9} \right) = \frac{-2(20\sqrt{10} - 457)}{729}$$

$$A_v \left( \frac{-2\sqrt{10} - 7}{9} \right) = \frac{2(20\sqrt{10} + 457)}{729}$$

$$A_v \in \left( \frac{-2(20\sqrt{10} - 457)}{729}, \frac{2(20\sqrt{10} + 457)}{729} \right)$$

The answer can also be found by checking the values in the options to see if they give three  $a$  values.

1.22 1.23 1.24 \*2024MC RAD

$f(x) := x^3 + x^2 - x + 1$  Done

$\frac{1}{1-a} \int_a^1 f(x) dx = \frac{3 \cdot a^3 + 7 \cdot a^2 + a + 13}{12}$

solve  $\left( \frac{d}{da} \left( \frac{3 \cdot a^3 + 7 \cdot a^2 + a + 13}{12} \right) = 0, a \right)$

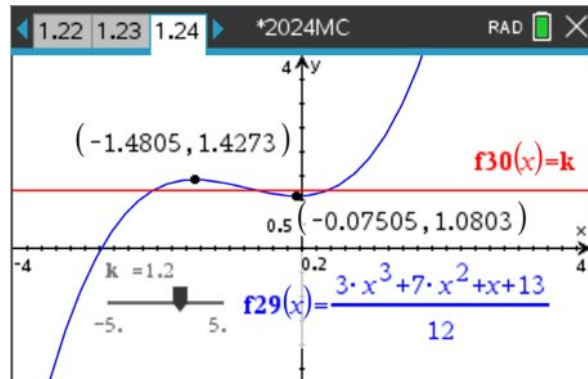
$a = \frac{-(2 \cdot \sqrt{10} + 7)}{9}$  or  $a = \frac{2 \cdot \sqrt{10} - 7}{9}$

---

1.22 1.23 1.24 \*2024MC RAD

$\frac{3 \cdot a^3 + 7 \cdot a^2 + a + 13}{12} \Big|_{a = \frac{-(2 \cdot \sqrt{10} + 7)}{9}} = \frac{2 \cdot (20 \cdot \sqrt{10} + 457)}{729}$

$\frac{3 \cdot a^3 + 7 \cdot a^2 + a + 13}{12} \Big|_{a = \frac{2 \cdot \sqrt{10} - 7}{9}} = \frac{-2 \cdot (20 \cdot \sqrt{10} - 457)}{729}$



Edit Action Interactive

$\text{combine} \left( \frac{1}{1-a} \int_a^1 (x^3 + x^2 - x + 1) dx \right)$

$\frac{3 \cdot a^3 + 7 \cdot a^2 + a + 13}{12}$

**Question 19 Answer B**

$$f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{1}{2} \left( \frac{2x-3}{6} \right)^2} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\frac{3}{2}}{3} \right)^2}$$

$$X \sim N\left(\frac{3}{2}, 3^2\right)$$

- a dilation by a factor of 3 from the x-axis

$$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{2x-3}{6} \right)^2}$$

- a dilation by a factor of  $\frac{1}{3}$  from the y-axis

$$f_2(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{6x-3}{6} \right)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( x-\frac{1}{2} \right)^2}$$

- a translation of  $\frac{1}{2}$  a unit left.

$$f_3(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2}$$

Check:  $(x, y) \rightarrow (x, 3y) \rightarrow \left(\frac{x}{3}, 3y\right) \rightarrow \left(\frac{x}{3} - \frac{1}{2}, 3y\right)$

$$x' = \frac{x}{3} - \frac{1}{2}, x = 3x' + \frac{3}{2}$$

$$y' = 3y, y = \frac{y'}{3}$$

$$\frac{y'}{3} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{3x'+\frac{3}{2}}{3}\right)^2}$$

$$y' = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x')^2}$$

**Question 20      Answer A**

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = e^{x^3+bx}$$

Solve  $f''(x) = 0$  but does not work directly on the T or the CASO .

So find  $f''(x) = (9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx}$  .

There will be no points of inflection when  $f''(x) \geq 0$  for all  $x$  .

Solve  $9x^4 + 6bx^2 + 6x + b^2 = 0$  and  $\frac{d}{dx}(9x^4 + 6bx^2 + 6x + b^2) = 0$

O

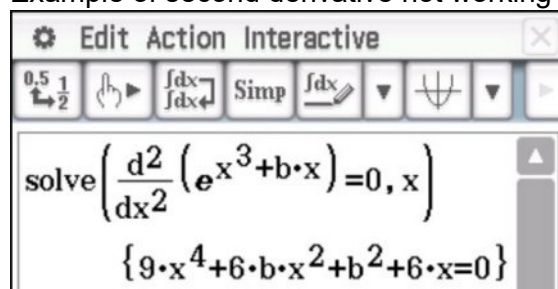
Solve  $(9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx} = 0$  and  $\frac{d}{dx}((9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3+bx}) = 0$

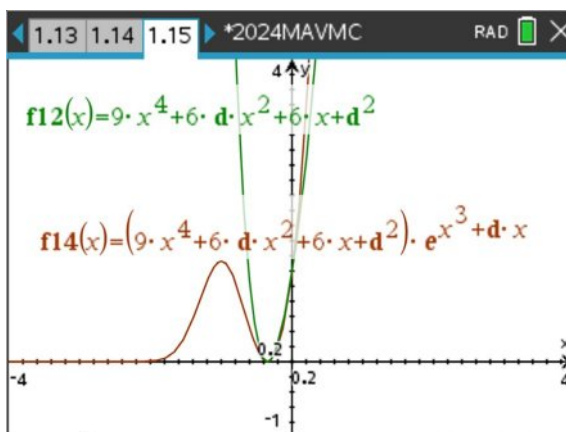
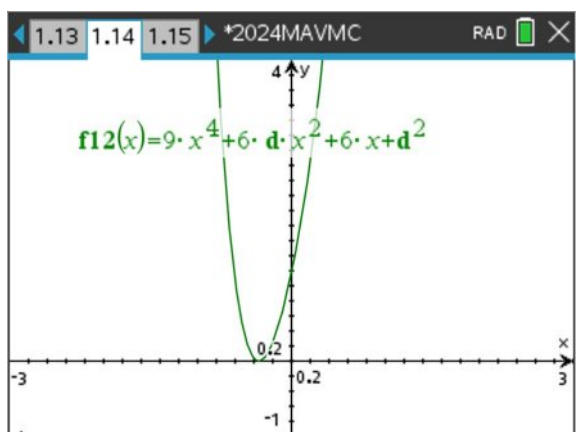
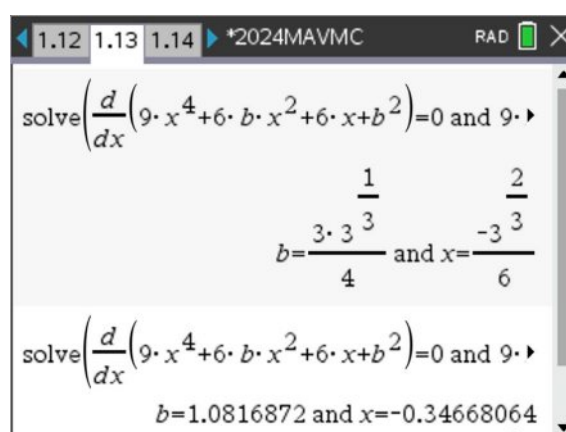
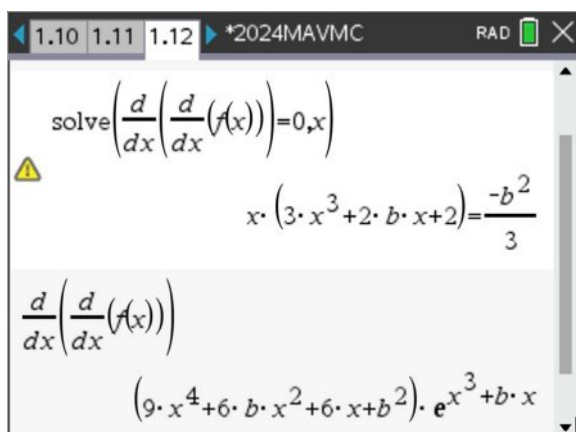
$$b = \frac{3^{\frac{4}{3}}}{4}$$

There will be no points of inflection when  $b \geq \frac{3^{\frac{4}{3}}}{4}$  .

The answer can also be found by checking the values in the options. The easiest way to do this is to graph the function and use a slider. Choose a value of  $b$  that gives two points of inflection and label them with their coordinates. Then use the slider to see when they disappear.

Example of second derivative not working on the CASO .

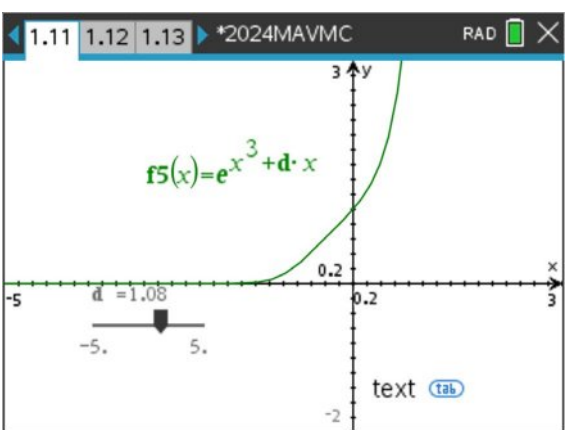
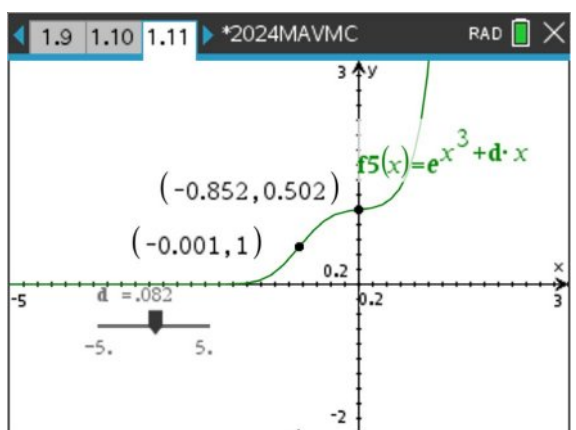




**Examples**

$b < \frac{3^{\frac{4}{3}}}{4}$  (2 points of inflection)

$b = \frac{3^{\frac{4}{3}}}{4}$  (no points of inflection)

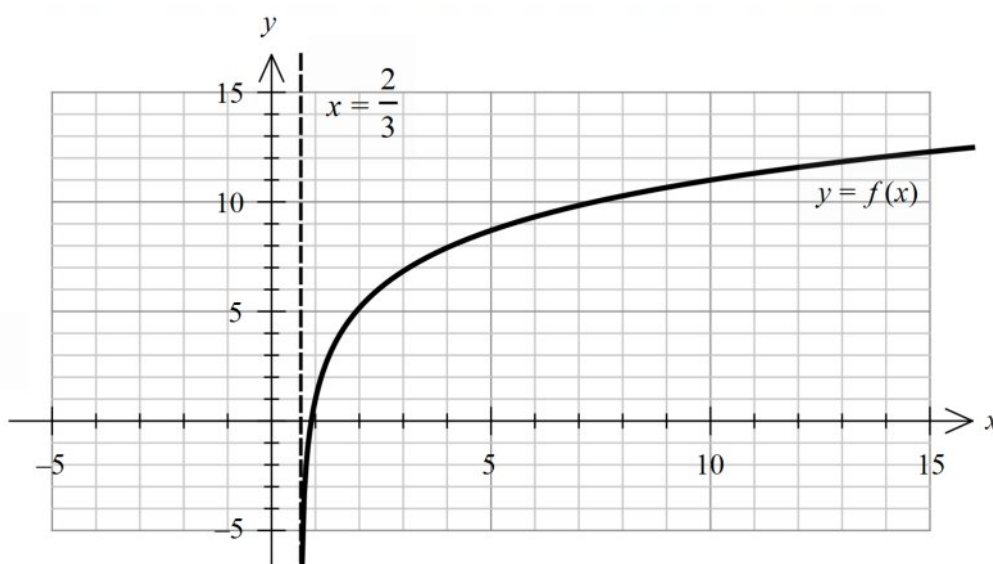


**END OF SECTION A SOLUTIONS**

**SECTION B****Question 1**

$$f: \left(\frac{2}{3}, \infty\right) \rightarrow \mathbb{R}, f(x) = 3 \log_e(3x - 2) + 1$$

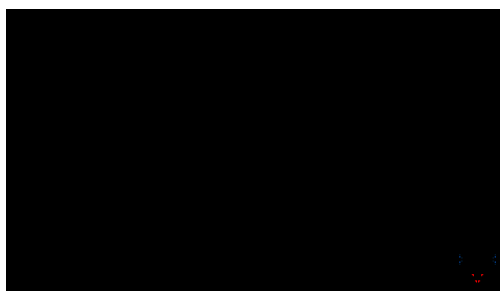
a. Sketch and label asymptote  $x = \frac{2}{3}$

**1A**

b.  $f^{-1}(x) = \frac{1}{3}e^{\frac{x-1}{3}} + \frac{2}{3}$

**1A**

Dom:  $x \in \mathbb{R}$

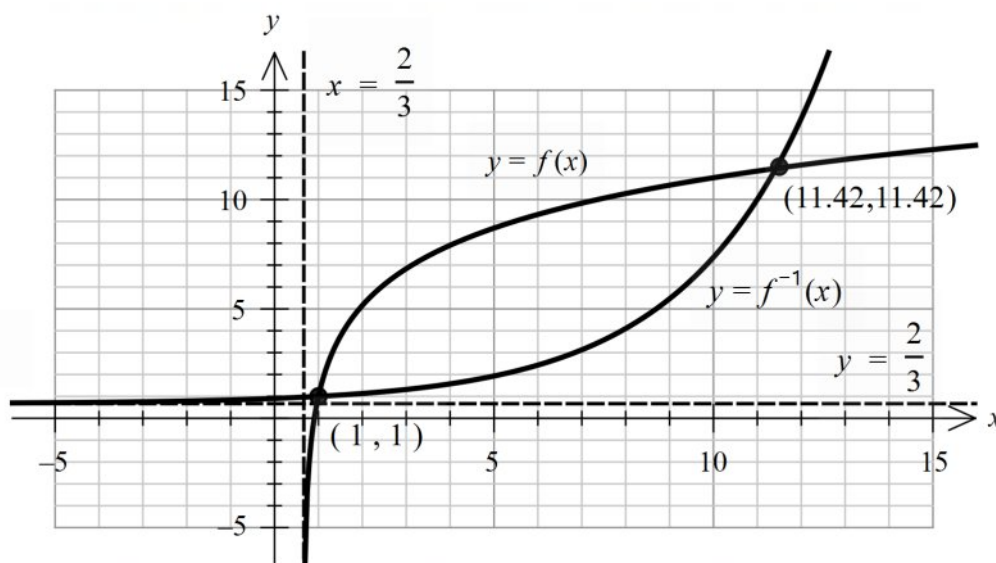
**1A**

c. Sketch  $y = f^{-1}(x)$ . Shape and asymptote  $y = \frac{2}{3}$  **1A**

Points of intersection between  $y = f(x)$  and  $y = f^{-1}(x)$

$(1, 1), (11.42, 11.42)$

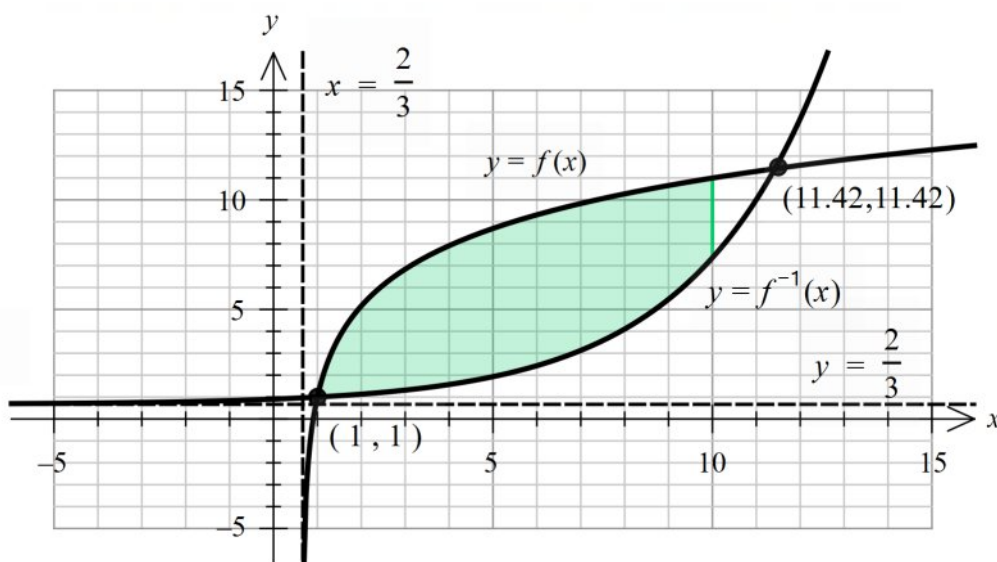
**1A**

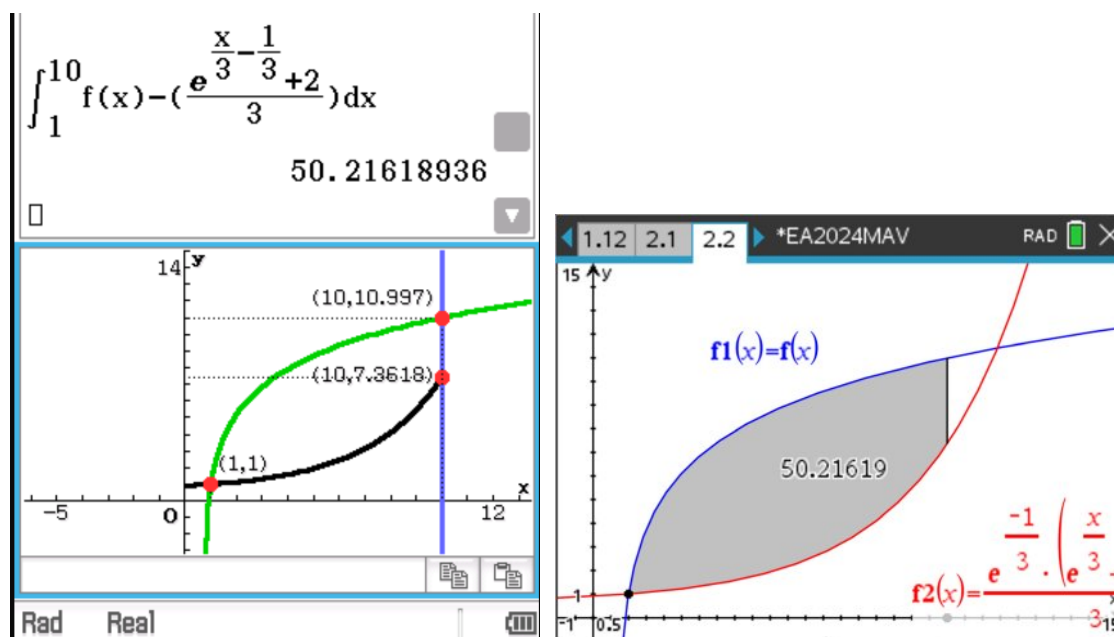


solve  $\left( f(x) = \frac{e^{\frac{x}{3} - \frac{1}{3}} + 2}{3}, x \right)$   
 $\{x=1, x=11.42205843\}$

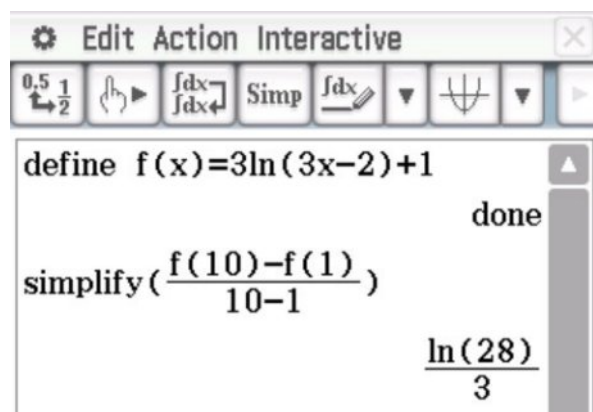
d.i.  $\int_1^{11.42} (f(x) - f^{-1}(x)) dx$  **1A**

d.ii. Area = 50.22 sq units **1A**  
 Shading **1A**





e.i. Average rate of change =  $\frac{f(10) - f(1)}{10 - 1}$   
 $= \frac{\log_e(28)}{3}$ . n correct form  $\frac{\log_e(a)}{b}$  **1A**



e.ii. Solve  $f'(x) = \frac{\log_e(28)}{3}$  for  $x$ .  
 $x = \frac{2}{3} + \frac{9}{\log_e(28)}$  **1A (other forms)**



Edit Action Interactive

define  $f(x)=3\ln(3x-2)+1$  done

solve  $\left(\frac{d}{dx}(f(x))=\frac{\ln(28)}{3}, x\right)$

$$\left\{x=\frac{\ln(784)+27}{3\cdot\ln(28)}\right\}$$

Edit Action Interactive

$$\frac{2\cdot\ln(7)}{3\cdot(\ln(7)+2\cdot\ln(2))} + \frac{4\cdot\ln(2)}{3\cdot(\ln(7))}$$

propFrac  $\left(\frac{\ln(784)+27}{3\cdot\ln(28)}\right)$

$$\frac{2\cdot\ln(7)}{3\cdot(\ln(7)+2\cdot\ln(2))} + \frac{4\cdot\ln(2)}{3\cdot(\ln(7))}$$

simplify  $\left(\frac{2\cdot\ln(7)}{3\cdot(\ln(7)+2\cdot\ln(2))} + \frac{4\cdot\ln(2)}{3\cdot(\ln(7))}\right)$

$$\frac{9}{\ln(7)+2\cdot\ln(2)} + \frac{2}{3}$$

4.1 4.2 4.3 \*EA2024MAV RAD

solve  $\left(\frac{d}{dx}(f(x))=\frac{\ln(28)}{3}, x\right)$

$$x=\frac{2\cdot\ln(28)+27}{3\cdot\ln(28)}$$

propFrac  $\left(\frac{2\cdot\ln(28)+27}{3\cdot\ln(28)}\right)$

$$\frac{9}{\ln(28)} + \frac{2}{3}$$

e.iii. The maximum value of the average rate of change will occur when the gradient of the line passing through  $(a, f(a))$  and  $(b, f(b))$  is steepest. This will occur when  $a=1$  and  $b=2$ .

Maximum average rate of change =  $\frac{f(2)-f(1)}{2-1} = 6\log_e(2)$  **1M**

Solve  $f'(x) = 6\log_e(2)$  for  $x$ .

$$x = \frac{2}{3} + \frac{1}{2\log_e(2)} \quad \mathbf{1A \text{ (other forms)}}$$

Edit Action Interactive

define  $f(x)=3\ln(3x-2)+1$  done

$$\frac{f(2)-f(1)}{2-1}$$

$$6\cdot\ln(2)$$

solve  $\left(\frac{d}{dx}(f(x))=6\cdot\ln(2), x\right)$

$$\left\{x=\frac{1}{2\cdot\ln(2)} + \frac{2}{3}\right\}$$

**e.iv.**  $f$  is continuous over the interval  $[a, b]$  and smooth over the interval  $(a, b)$  but

$f'(x) = \frac{9}{3x-2} \neq 0$  for any  $x$ . Hence,  $f(a) \neq f(b)$ . For the average value to equal zero,  $f(a)$  must equal  $f(b)$ . **1A**

A handwritten note in a box showing the derivative of  $f(x) = \frac{9}{3x-2}$ . The derivative is  $\frac{d}{dx}(f(x)) = \frac{9}{3 \cdot x - 2}$ .

**f.i.**  $h: \left(\frac{2}{b}, \infty\right) \rightarrow \mathbb{R}$ ,  $h(x) = a \log_e (bx - 2) + 1$  where  $h(x) = 3f(5x) - 2$

given  $f(x) = 3 \log_e (3x - 2) + 1$

$$h(x) = 3f(5x) - 2$$

$$h(x) = 9 \log_e (15x - 2) + 1$$

$$a = 9, b = 15$$

**1A**

A handwritten note in a box showing the definition and expansion of  $h(x)$ . It starts with "define  $h(x) = 3f(5x) - 2$ " and "done". Below that, it says "expand( $h(x)$ )" and shows the result " $9 \cdot \ln(15 \cdot x - 2) + 1$ ".

**f.ii.** Solve  $h_1'(x) = f'(x)$

$$x = \frac{3k-1}{3k}$$

As  $k \rightarrow \infty$ ,  $x \rightarrow 1$

As  $k \rightarrow -\infty$ ,  $x \rightarrow 1$

$$x = 1$$

**1A**

A screenshot of a CAS calculator interface. It shows the definition  $h(x) := 3 \cdot f(k \cdot x) - 2$  and the command `solve(d/dx(f(x)) = d/dx(h(x)), x)` resulting in  $x = \frac{3 \cdot k - 1}{3 \cdot k}$ . Below this, it shows the limit  $\lim_{k \rightarrow \infty} \left( \frac{3 \cdot k - 1}{3 \cdot k} \right) = 1$  and  $\lim_{k \rightarrow -\infty} \left( \frac{3 \cdot k - 1}{3 \cdot k} \right) = 1$ .

**Question 2**

$$h(t) = a \sin(b(t-18)) + c$$

a. max 50, min 10, amp = 20

translation vertically:  $-20 + 30 = 10$

$a = 20, c = 30$  **1M** (explanation)

b. One cycle = 18 hours

$$\frac{2\pi}{b} = 18$$

$$2\pi = 18b$$

Gives  $b = \frac{\pi}{9}$  **1M** (show that)

c.  $\frac{1}{18} \int_0^{18} h_A dx - \frac{1}{18} \int_0^{18} h_B dx$  **1M**

$$= \frac{5(\pi + 2)}{\pi} \text{ m} \quad \mathbf{1A}$$

0.5 1/2 f dx f dx Simp f dx

$$\frac{1}{18} \int_0^{18} f(x) dx$$

30

$$\text{combine} \left( \frac{1}{18} \int_0^{18} h(x) dx \right)$$

$$\frac{25 \cdot \pi - 10}{\pi}$$

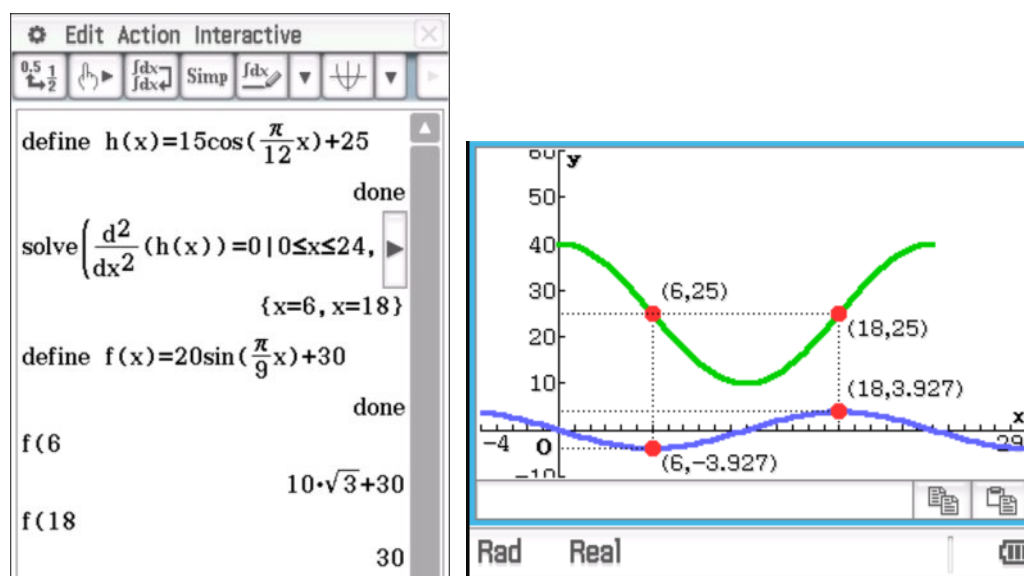
$$\text{combine} \left( 30 - \frac{25 \cdot \pi - 10}{\pi} \right)$$

$$\frac{5 \cdot \pi + 10}{\pi}$$

d.  $h_B(t) = 15 \cos\left(\frac{\pi t}{12}\right) + 25$

The height of the river would be changing fastest at the points of inflection of the graphs of  $h_B$ . So when  $t = 6$  and  $t = 18$ . **1M**

$$h_A(6) = 10\sqrt{3} + 30 \text{ and } h_A(18) = 30 \quad \mathbf{1A}$$



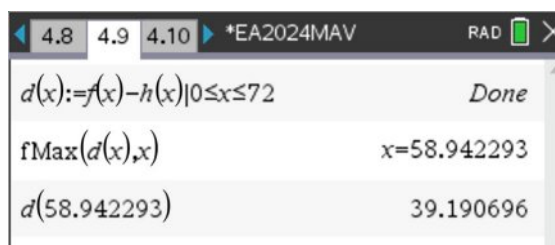
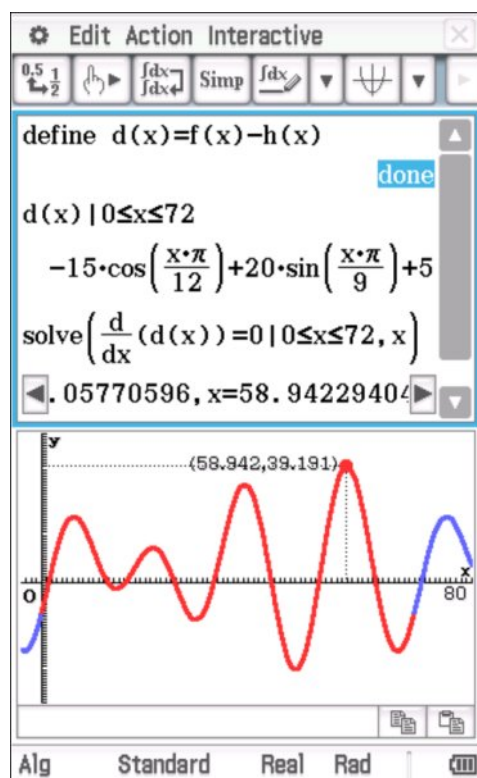
e. Let  $d(x) = h_A(x) - h_B(x)$

The period of the graph of  $d(x)$  is the lowest common multiple of 18 and 24 which is 72 hours.

1A

The maximum difference is 39.19 m.

1A



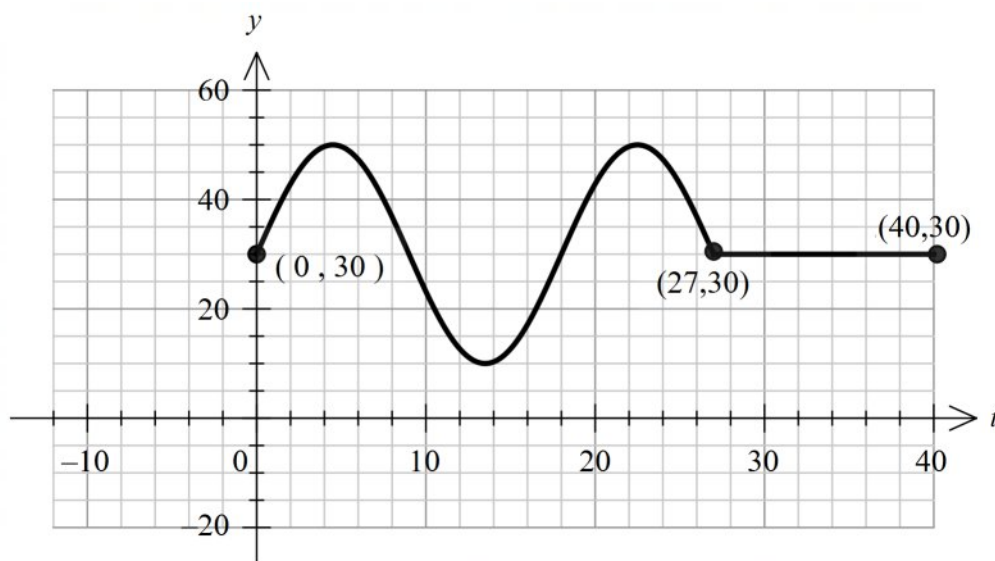
$$f. w(t) = \begin{cases} h_A(t) & 0 \leq t \leq 27 \\ 30 & 27 < t \leq 40 \end{cases}$$

Graph of piecewise function  $w$

1A

Coordinates  $(0, 30)$ ,  $(40, 30)$ ,  $(27, 30)$

1A



**Edit Action Interactive**

$\frac{0.5}{2}$   $\int \frac{dx}{dx}$   $\int \frac{dx}{dx}$   $\text{Simp}$   $\frac{dx}{dx}$   $\int \frac{dx}{dx}$

$\text{solve}\left(\frac{h(p)-h(0)}{p-0}=0.5, p\right)$   
 $\{p=-25.06034947, p=-19.45\}$   
 $h(t) \mid 0 \leq t \leq 27$   
 $20 \cdot \sin\left(\frac{(t-18) \cdot \pi}{9}\right) + 30$   
 $30 \mid 27 < t \leq 40$

Alg Standard Real Rad

g.  $t \in (0, 27) \cup (27, 40)$       **1A**

h. 
$$p(t) = \begin{cases} w(t) & 0 \leq t \leq 40 \\ m \cos(n(t-r)) + s & 40 < t \leq k \end{cases}$$
 am Sunday to pm Tuesday is 0 hours.  
 $k = 60$       **1A**

i. Continuous and smooth at  $t = 40$ . So there is a turning point at  $t = 40$ .

$p = m \cos(n(t-r)) + s$  completes two cycles before recording capacities break, reaching zero height twice. So the range is  $[0,30]$ .

Amplitude = 15,  $m = 15$

Period = 10,  $n = \frac{2\pi}{10} = \frac{\pi}{5}$  **1H**

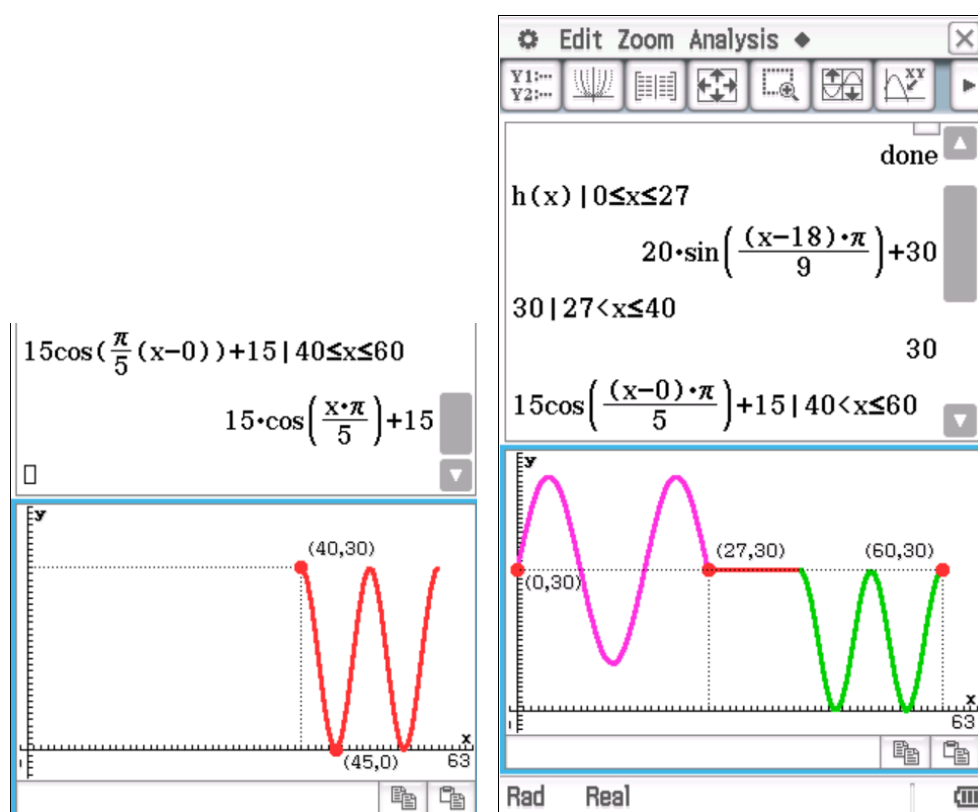
$m = 15, s = 15$ . **1A**

$r = 10q$  where  $q \in Z$  **1A**

**OR**

$m = -15, s = 15$ . **1A**

$r = 5q$  where  $q \in Z$  **1A**

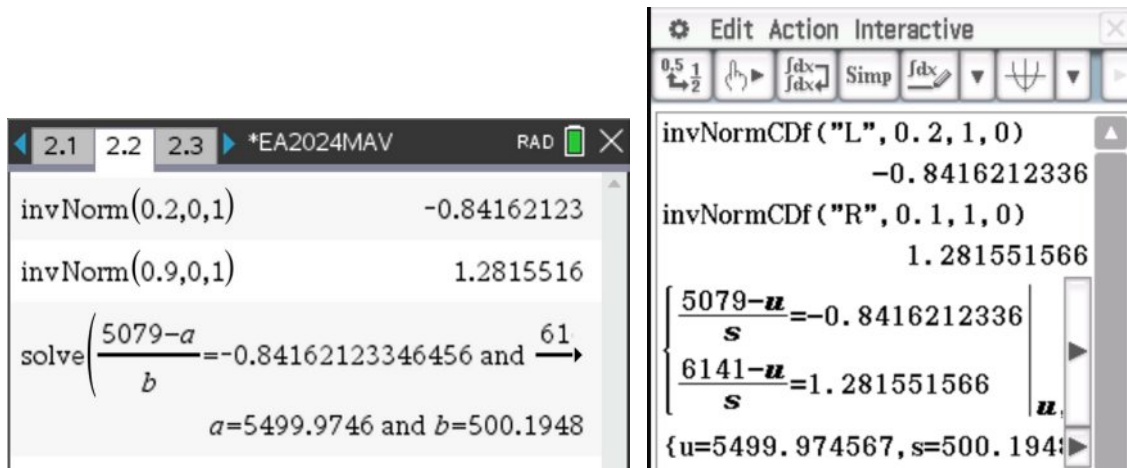


**Question 3**

a.  $X_{AF} \sim N(\mu, \sigma^2)$

Solve  $\frac{5079 - \mu}{\sigma} = -0.841\dots$  and  $\frac{6141 - \mu}{\sigma} = 1.281\dots$  **1M**

$\mu = 5500.0$  kg and  $\sigma = 500.2$  kg **1A**



b.  $X_A \sim N(4085, 445^2)$ ,  $X_{AB} \sim N(5375, 225^2)$

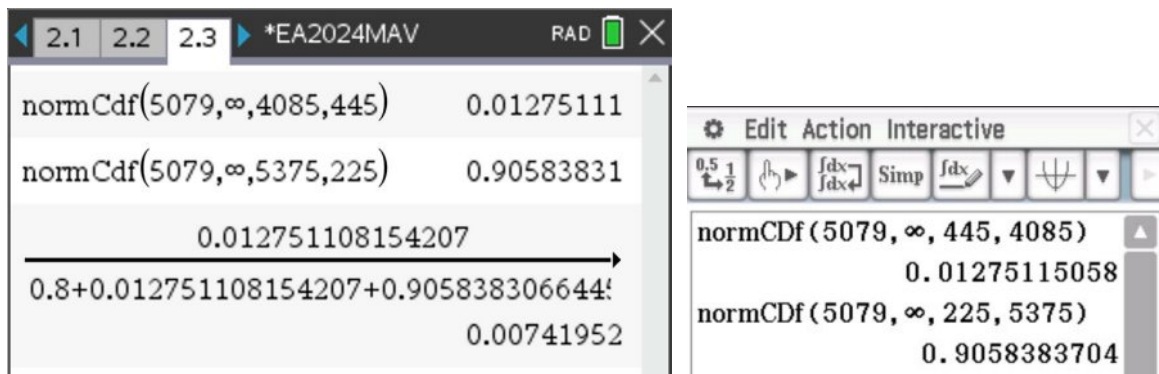
$\Pr(X_A > 5079) = 0.0127\dots$ ,  $\Pr(X_{AB} > 5079) = 0.9058\dots$ ,  $\Pr(X_{AF} > 5079) = 0.8$  **1M**

$\Pr(X_A > 5079 | (X_A > 5079 + X_{AB} > 5079 + X_{AF} > 5079))$

$$= \frac{\frac{1}{3} \times 0.0127\dots}{\frac{1}{3} \times 0.0127\dots + \frac{1}{3} \times 0.9058\dots + \frac{1}{3} \times 0.8}$$

$$= \frac{0.0127\dots}{0.0127\dots + 0.9058\dots + 0.8}$$

**1A**

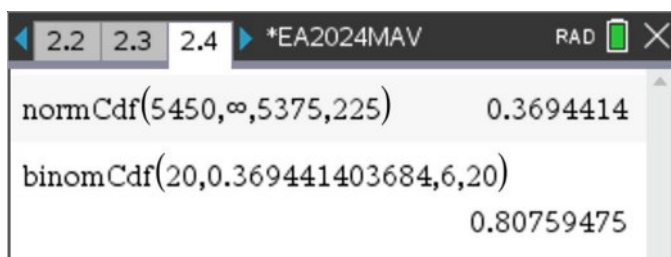


c.  $X_{AB} \sim N(5375, 225^2)$

$\Pr(X_{AB} > 5450) = 0.3694\dots$

$X \sim \text{Bi}(20, 0.3694\dots)$  **1M**

$\Pr(X > 5) = \Pr(X \geq 6) = 0.8076$  **1A**



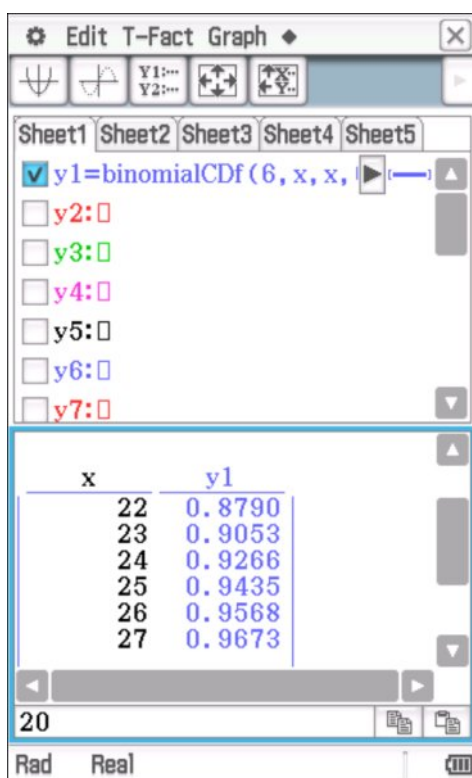
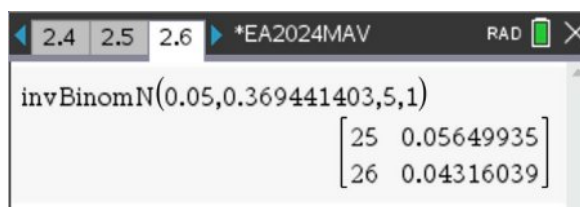
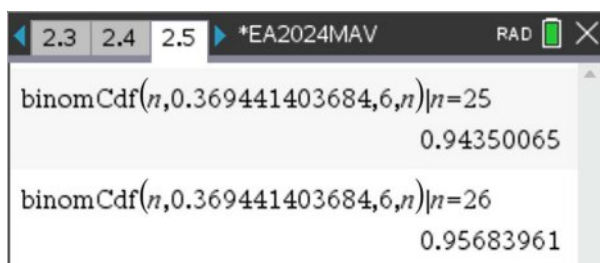
d.  $X_2 \sim \text{Bi}(n, 0.3694\dots)$

Trial and error **1M** (other methods)

|     |                   |
|-----|-------------------|
| $n$ | $\Pr(X_2 \geq 6)$ |
| 25  | 0.9435...         |
| 26  | 0.9568...         |

$n = 26$

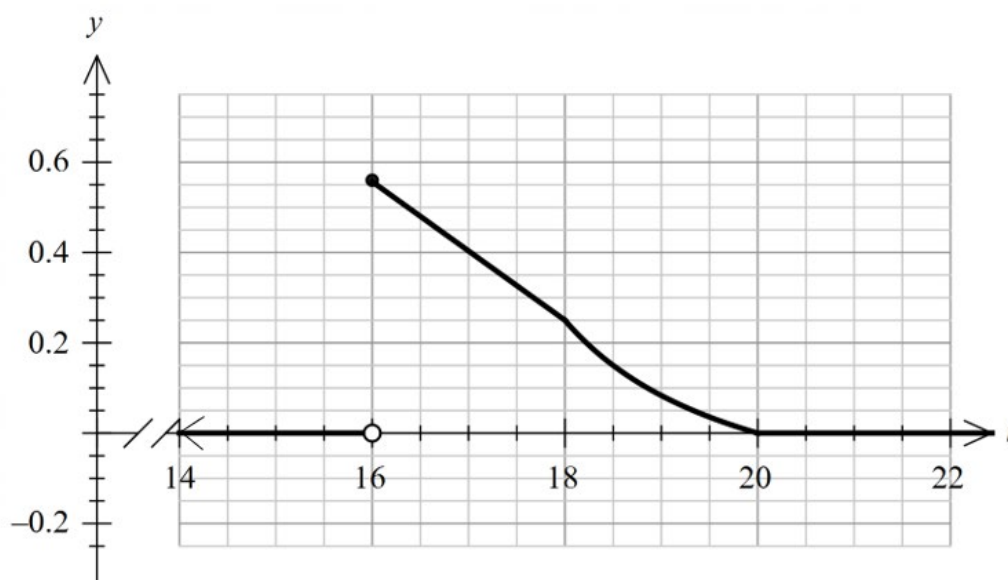
**1A**



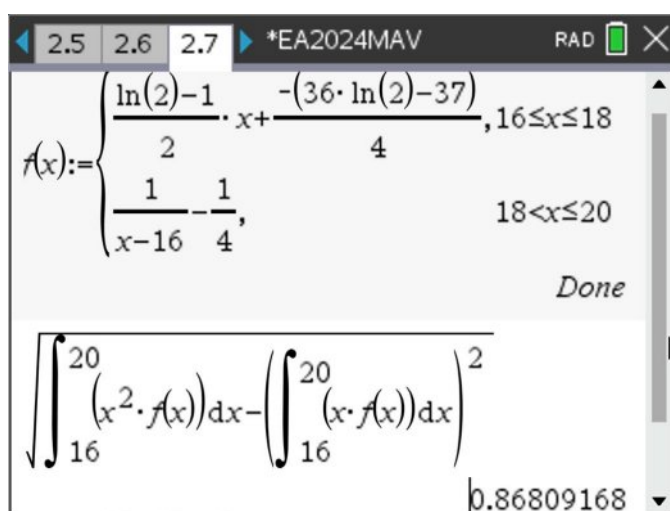
e. Shape, open circle, must draw along the  $x$ -axis **1A**



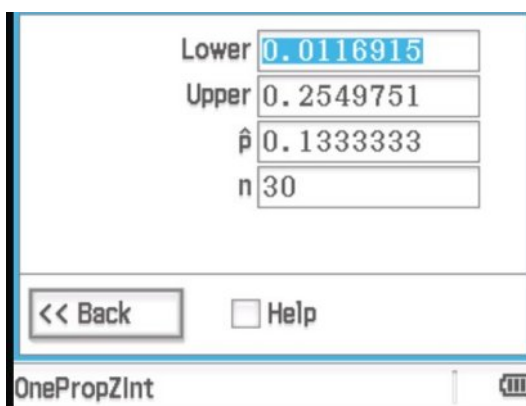
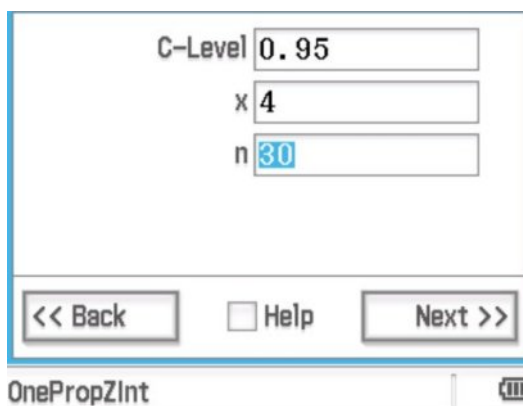
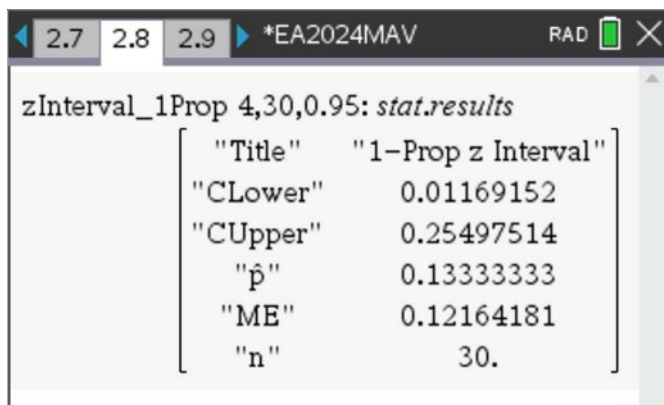
$$f(t) = \begin{cases} \frac{\log_e(2)-1}{2}t + \frac{37-36\log_e(2)}{4} & 16 \leq t \leq 18 \\ \frac{1}{t-16} - \frac{1}{4} & 18 < t \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$



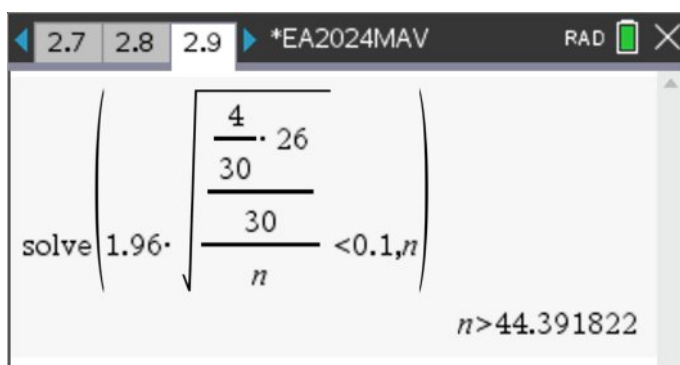
$$\begin{aligned} \text{f. } \text{sd}(T) &= \sqrt{\int_{16}^{20} (t^2 \times f(t)) dt - \left( \int_{16}^{20} (t \times f(t)) dt \right)^2} & \mathbf{1M} \\ &= 0.868 & \mathbf{1A} \end{aligned}$$



$$\text{g. } (0.0117, 0.2550) \quad \mathbf{1A}$$



h. Solve  $1.96\sqrt{\frac{\frac{4}{30} \times \frac{26}{30}}{n}} < 0.1$  for  $n$ .  
 $n = 45$       **1A**



i. Let  $AB$  be the African bush elephant and  $AF$  be the African forest elephant.

$$\Pr(AB \cap AF) = \Pr(AB) \times \Pr(AF) = \frac{2-k^2}{2} \text{ independent events}$$

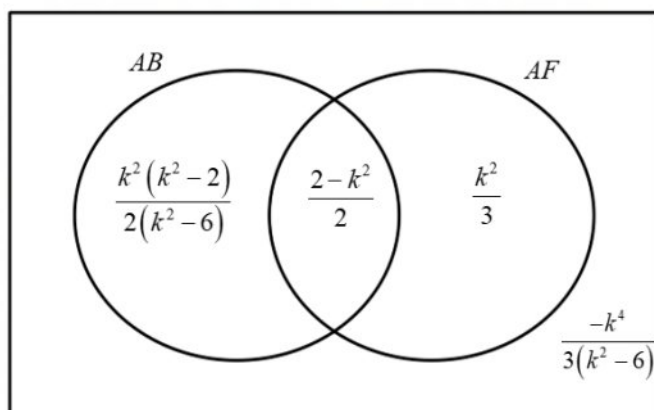
$$\Pr(AB) \times \left( \frac{2-k^2}{2} + \frac{k^2}{3} \right) = \frac{2-k^2}{2} \quad \mathbf{1M}$$

$$\Pr(AB) = \frac{3(k^2 - 2)}{k^2 - 6}$$

$$\Pr(AB \cup AF) + \Pr(AB' \cap AF') = 1$$

$$\frac{3(k^2 - 2)}{k^2 - 6} + \frac{k^2}{3} + \Pr(AB' \cap AF') = 1$$

$$\Pr(AB' \cap AF') = \frac{-k^4}{3(k^2 - 6)} \quad \mathbf{1A}$$



$$\mathbf{j.} \Pr(AB \cap AF') = \frac{3(k^2 - 2)}{k^2 - 6} - \frac{2 - k^2}{2} = \frac{k^2(k^2 - 2)}{2(k^2 - 6)}$$

$$\text{Solve } \frac{d}{dk} \left( \frac{k^2(k^2 - 2)}{2(k^2 - 6)} \right) = 0 \text{ or use fmax}$$

$$k = \sqrt{-2(\sqrt{6} - 3)}$$

$$\text{Maximum probability is } 5 - 2\sqrt{6} \quad \mathbf{1A}$$

3.10 3.11 3.12 \*EA2024MAV RAD

solve  $\left(\frac{d}{dk} \left( \frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} \right) = 0, k \mid 0 < k < \sqrt{2} \right)$

$k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$

$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} \mid k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$   $5 - 2 \cdot \sqrt{6}$

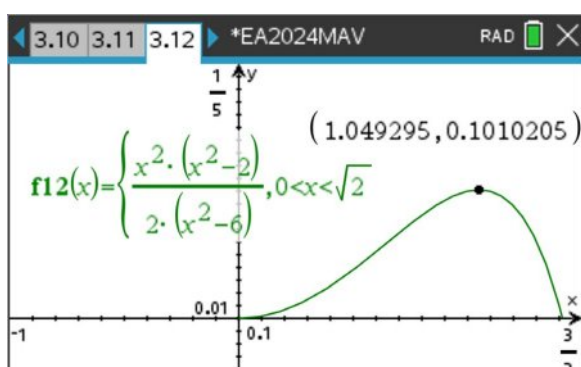
$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} \mid k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$   $0.10102051$

3.10 3.11 3.12 \*EA2024MAV RAD

fMax  $\left( \frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)}, k \mid 0 < k < \sqrt{2} \right)$

$k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$

$\frac{k^2 \cdot (k^2 - 2)}{2 \cdot (k^2 - 6)} \mid k = \sqrt{-2 \cdot (\sqrt{6} - 3)}$   $5 - 2 \cdot \sqrt{6}$



**Question 4**

$h(x) = \sqrt{x^4 - px^2 + 1}$  and  $f(x) = x^4 - px^2 + 1$ , and  $p \in R$

a. Solve  $h(x) = f(x)$  when  $p = 3$

$x = \pm\sqrt{3}, 0, \frac{-\sqrt{5} \pm 1}{2}, \frac{\sqrt{5} \pm 1}{2}$  **1A**

1.1 1.2 1.3 \*EA2024MAV RAD

$f(x) := x^4 - p \cdot x^2 + 1$  Done

$h(x) := \sqrt{x^4 - p \cdot x^2 + 1}$  Done

solve  $(f(x) = h(x), x) \mid p = 3$

$x = -\sqrt{3}$  or  $x = \frac{-(\sqrt{5} + 1)}{2}$  or  $x = \frac{-(\sqrt{5} - 1)}{2}$  or  $x = 0$

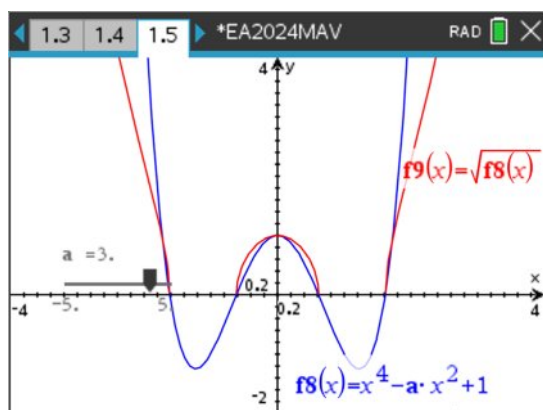
1.1 1.2 1.3 EA2024MAV RAD

$f(x) := x^4 - p \cdot x^2 + 1$  Done

$h(x) := \sqrt{x^4 - p \cdot x^2 + 1}$  Done

solve  $(f(x) = h(x), x) \mid p = 3$

$x = -\sqrt{3}$  or  $x = 0$  or  $x = \frac{\sqrt{5} - 1}{2}$  or  $x = \frac{\sqrt{5} + 1}{2}$  or  $x = \sqrt{3}$

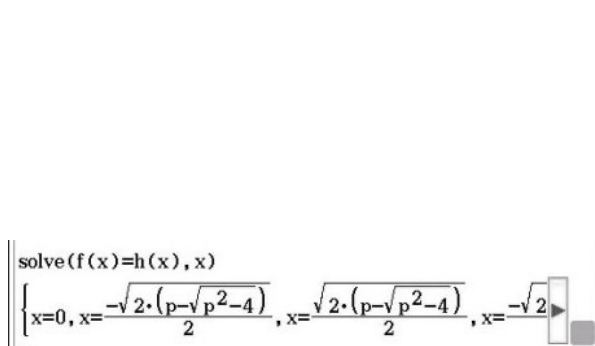
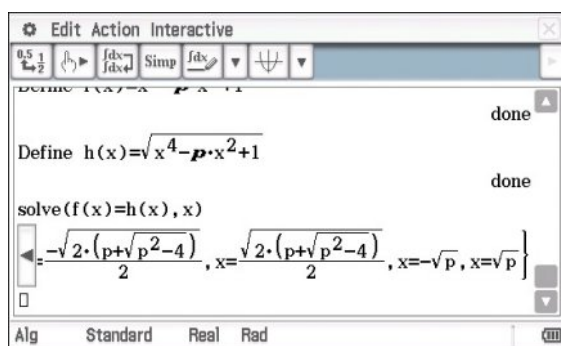
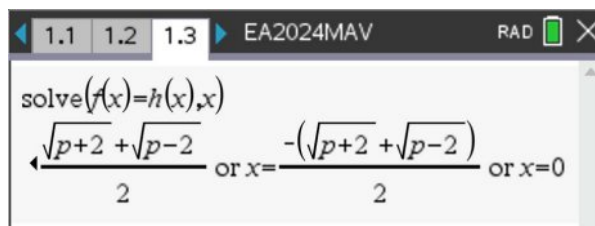
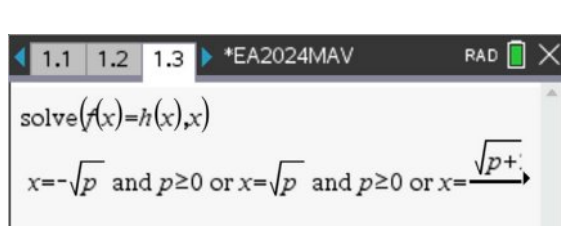


b. Solve  $h(x) = f(x)$  for  $x$ .

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2}, \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} \quad \mathbf{1A}$$

OR

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2}, \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2} \quad \mathbf{1A}$$

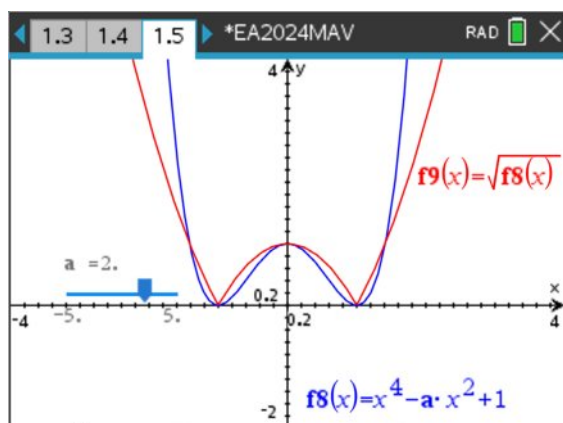


c. If  $p = 2$ ,  $\sqrt{p-2} = 0$ , so  $\frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2}}{2}$

OR

If  $p = 2$ ,  $p^2 - 4 = 0$ , so  $\frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2} = \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2}$

ence onl five solutions.  $\mathbf{1A}$

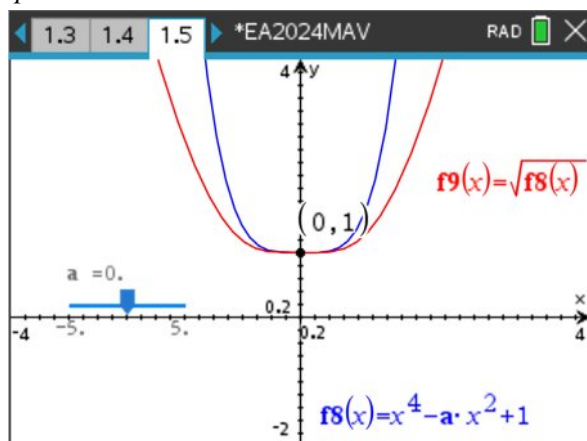


- d. 2 correct 1A  
All correct 2A

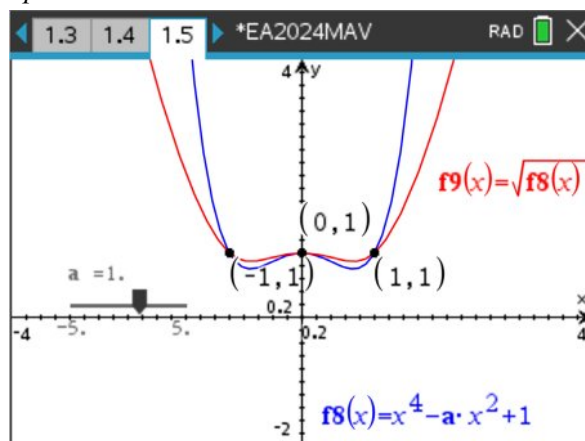
|  |            |                  |
|--|------------|------------------|
| Number of points of intersection           | 1          | 3                |
| $p$ values                                 | $p \leq 0$ | $0 < p < 2$      |
| $x$ -coordinates of points of intersection | 0          | $0, \pm\sqrt{p}$ |

Examples

$p = 0$



$p = 1$



e.  $g(x) = \sqrt{x}$ ,  $f(x) = x^4 - px^2 + 1$

or  $g(f(x))$  to exist the range of  $f$  has to be a subset of, or equal to, the domain of  $g$ .

The domain of  $g$  is  $[0, \infty)$ , the range of  $f$  will be  $[0, \infty)$  for  $p = 2$  and a subset of  $[0, \infty)$  for  $p < 2$ .

The range of  $f$ ,  $\left[1 - \frac{p^2}{4}, \infty\right)$ , is not a subset of  $[0, \infty)$  for  $p > 2$ . 1A

Calculator screenshot showing the solution to the derivative equation:

$$\text{solve}\left(\frac{d}{dx}(f(x))=0, x\right)$$

$$x = \frac{\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } x = \frac{-\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } x = \frac{\sqrt{2 \cdot p}}{2}$$

$$f\left(\frac{\sqrt{2 \cdot p}}{2}\right) = 1 - \frac{p^2}{4}$$

**f. Using the bounded area on the graph**

$$\text{Area} = 2(0.0196238\dots + 0.076381\dots) \quad \mathbf{1M}$$

$$= 0.192 \quad \mathbf{1A}$$

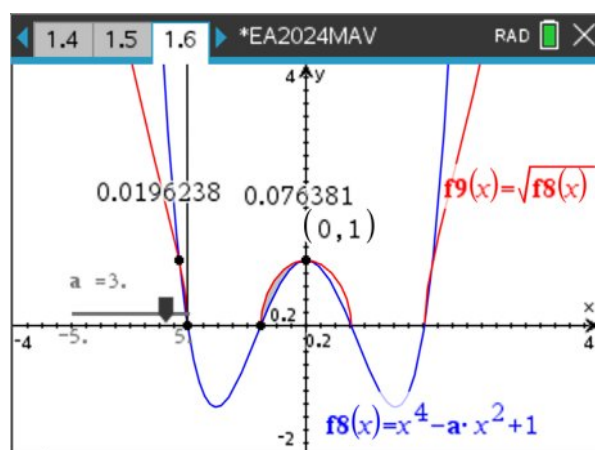
**OR**

**Using definite integrals**

$$\text{Area} = 2 \left( \int_{-\frac{\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{-\frac{\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left( \int_0^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}} (h(x) - f(x)) dx \right) \quad \mathbf{1M (either form)}$$

$$= 0.192 \quad \mathbf{1A}$$



Calculator screenshot showing the definite integral calculation:

$$2 \cdot \left( \int_{-\frac{\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{-\frac{\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right)$$

$$= 0.19200976$$

**g. Area**

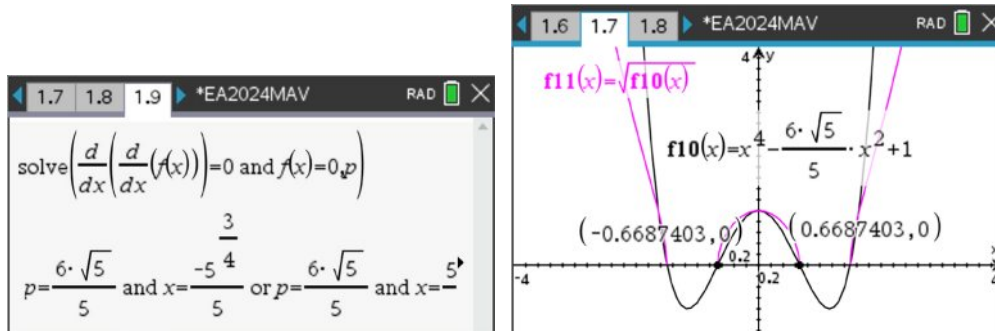
$$= 2 \left( \int_{-\sqrt{p}}^{\frac{-\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{-\sqrt{p+2}+\sqrt{p-2}}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left( \int_0^{\frac{\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{p+2}+\sqrt{p-2}}{2}}^{\sqrt{p}} (h(x) - f(x)) dx \right) \quad \mathbf{OR}$$

$$= 2 \left[ \int_0^{\frac{\sqrt{2(p-\sqrt{p^2-4})}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{2(p+\sqrt{p^2-4})}}{2}}^{\sqrt{p}} (h(x) - f(x)) dx \right] \quad \mathbf{1A}$$

h. Solve  $f''(x) = 0$  and  $f(x) = 0$  for  $p$ . **1M**

$$p = \frac{6\sqrt{5}}{5} \quad \mathbf{1A}$$



i.  $h_v(x) = \sqrt{x^4 - 3x^2 + 1}$  and  $f_v(x) = x^4 - 3x^2 + 1$

$$\text{Cross-sectional area} = \int_{-1.817\dots}^{1.817\dots} (2 - f_v(x)) dx - 0.1920\dots = 7.517\dots \quad \mathbf{1M}$$

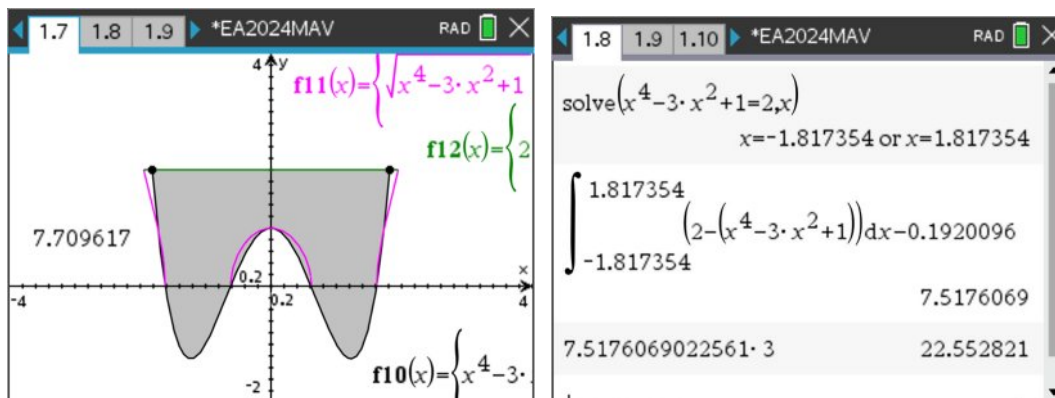
$$\text{Volume} = 7.517\dots \times 3 = 22.552\dots$$

$$\frac{dv}{dt} = 2^t$$

$$\text{Solve } \int_0^b 2^t dt = 22.552\dots \text{ for } b. \quad \mathbf{1H}$$

$$b = 4.06 \text{ seconds} \quad \mathbf{1A}$$

The shaded area on the graph is  $\int_{-1.817\dots}^{1.817\dots} (2 - f_v(x)) dx = 7.709\dots$  and then you need to subtract the bound area found in **part f**.







The screenshot shows a calculator interface with a dark theme. At the top, there are navigation buttons for 1.7, 1.8, and 1.9, along with the text 'EA2024MAV', 'RAD', and a close button. The main display area shows the text 'solve' followed by a mathematical expression:  $\int_0^b 2^t dt = 22.552820706768, b$ . Below this, the result  $b = 4.0559265$  is displayed.

**END OF SOLUTIONS**