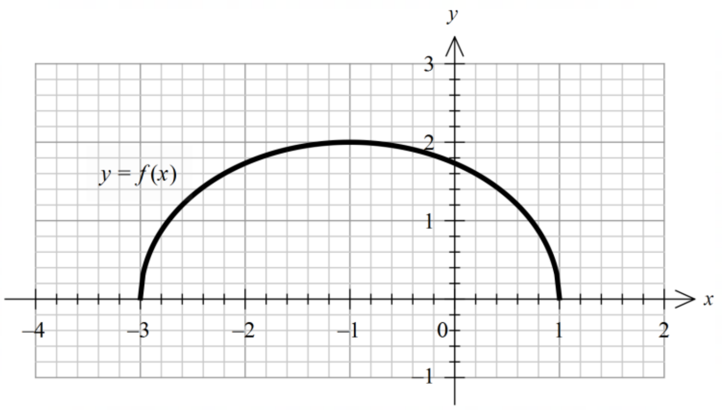
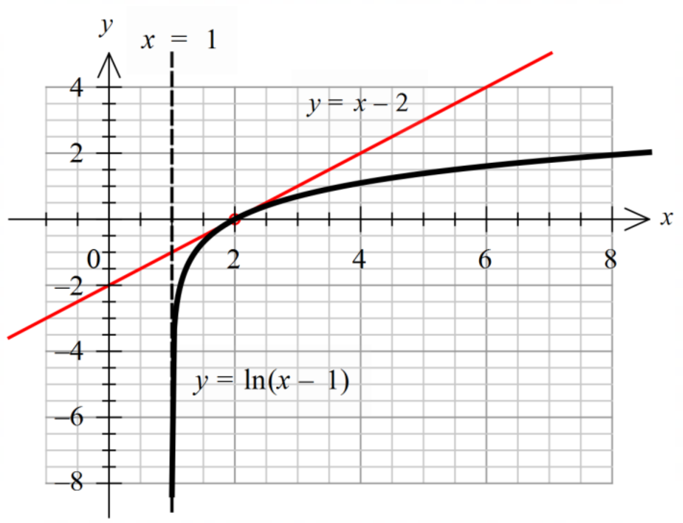
2023 VCE Mathematical Methods 2 (NHT) external assessment report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Correct answer	Comments
1	B	
2	C	
3	D	The maximal domain the function *f*, with rule $f(x)=\sqrt{3-2x-x^2}$, is strictly decreasing is $[-1,1]$. 
4	E	
5	B	
6	E	
7	A	
8	A	
9	C	
10	B	
11	D	$u=g(x)$, $v=e^{g(2x)}$ $\frac{d}{dx}(uv)$

Question	Correct answer	Comments
		$=\frac{d}{dx}\left[g(x)e^{g(2x)}\right]$ $=g'(x)e^{g(2x)}+g(x)\times 2g'(2x)e^{g(2x)}$ using the product and chain rules $=e^{g(2x)}\left(2g(x)g'(2x)+g'(x)\right)$
12	C	
13	D	
14	A	
15	E	
16	B	
17	E	
18	A	$b\neq 10\ 000$ as a fee is deducted from the initial investment $b(1+a)^{10}=12\ 000$ $b(1+a)^{20}=15\ 000$ $a=0.02256\ldots,\ b=9600$
19	D	 The graph of $y=\log_e(x-k)$ has a tangent with a maximum horizontal intercept when $\log_e(x-k)=0$. $x-k=1$ $x=1+k$ An example is shown above for $k=1$.
20	D	

Section B

Question 1a.

$g(x) = x^2 + 4x + 3$

Question 1b.

$p = -1$, $q = -3$ or $p = -3$, $q = -1$

Question 1c.

Method 1:

Solving $g(0) + k = 0$ for k

$k = -3$

Method 2:

$g(x)$ has a y-intercept at (0, 3)

k is a vertical translation, so for $y = g(x) + k$ to pass through the origin $k = -3$

Question 1d.

Method 1:

$g(x)$ has x-intercepts at $(-1, 0)$ and $(-3, 0)$

d is a horizontal translation, so for $y = g(x - d)$ to pass through the origin

$d = 1$ or $d = 3$

Method 2:

Solving $g(0 - d) = 0$ for d gives

$d = 1$ or $d = 3$

Question 1e.

Dilation by a factor of $\frac{1}{3}$ from the y-axis (in the direction of the x-axis)

Question 1f.

$y = g(x) + h(x) = x^2 + 4x + 3 + mx + n = x^2 + (4 + m)x + 3 + n$

At $x = 0$, $\frac{dy}{dx} = 2x + 4 + m = 0$

$m = -4$

Question 1g.

Method 1:

Using the discriminant condition $\Delta = 0$

$$(m+4)^2 - 4(1)(n+3) = 0$$

$$n = \frac{m^2 + 8m + 4}{4}$$

Method 2:

$x_{TP} = \dfrac{-4-m}{2}$, Solving $y(x_{TP}) = 0$

$$n = \frac{m^2 + 8m + 4}{4}$$

Question 1h.

$$y = g(x)h(x) = (x+3)(x+1)(mx+n)$$

$$x = -3,\ x = -1,\ x = -\frac{n}{m}$$

Solving $-1 = -\dfrac{n}{m}$ or $-3 = -\dfrac{n}{m}$

$m = n$ or $n = 3m$

Two pairs (examples)

$m = 1$, $n = 1$

$m = 1$, $n = 3$

$m = 0,\ n \in R \setminus \{0\}$

Question 1i.

$$g\left(h(x)\right) = \left(mx+n+2\right)^2 - 1$$

$$\left(-\frac{n+2}{m}, -1\right)$$

Question 2a.

50.6

Question 2b.

Solving $C(t) = 10$ for t

898 minutes which is 14 hours 58 minutes

Question 2c.

$C'(4) = -\frac{65}{8}e^{-\frac{1}{2}}$

$\frac{a}{b\sqrt{e}} = \frac{65}{8\sqrt{e}}$

Question 2d.

Strictly decreasing

Question 2e.

C_2 is not continuous at $t = 4$ and so is not continuous for $t > 0$

$65e^{-\frac{1}{2}} \neq 65\left(\frac{1-e}{1-\sqrt{e}}\right)e^{-\frac{1}{2}}$, $104.4 \neq 39.4$

Question 2f.

$t = 4$

$C_2(4) = 104.4$

Question 2g.

$$C_2'(t) = \begin{cases} -\frac{65}{8}e^{-\frac{t}{8}} & 0 < t < 4 \\ -\frac{65}{8}\left(\frac{1-e}{1-\sqrt{e}}\right)e^{-\frac{t}{8}} & t > 4 \end{cases}$$

Question 2h.

Solving $C_2'(t) \leq -8$

$(0, 0.1] \cup (4, 7.9]$

Question 3a.

$2\int_{-\sqrt{a}}^{0} f(x)dx$ or $-2\int_{0}^{\sqrt{a}} f(x)dx$ or $\int_{-\sqrt{a}}^{0} f(x)dx - \int_{0}^{\sqrt{a}} f(x)dx$

Area $= \frac{a^2}{2}$

Question 3b.

$$A = 2\sqrt{\frac{4}{3}} \times 4\left(\frac{4}{3}\right)^{\frac{3}{2}}$$

$$= \frac{128}{9}$$

Question 3c.

The graph only has turning points for $a > 0$ and so to form the rectangle connecting the turning points we must have this restricted range of a values.

If $a = 0$, area = 0, $y = x^3$ has one stationary point.

If $a < 0$, $\sqrt{\frac{a}{3}}$ no real solution, $y = x^3 - ax$ has no turning points.

Question 3d.

$$A = 2\sqrt{\frac{a}{3}} \times 4\left(\frac{a}{3}\right)^{\frac{3}{2}}$$

$$= \frac{8a^2}{9}$$

$$= 8\left(\frac{a}{3}\right)^2 \text{ as required}$$

Question 3e.

$$AC = \sqrt{\left(2\sqrt{\frac{a}{3}}\right)^2 + \left(4\left(\sqrt{\frac{a}{3}}\right)^{\frac{3}{2}}\right)^2}$$

$$= \frac{2\sqrt{3\left(4a^3 + 9a\right)}}{9}$$

Question 3f.

$$m_{AC} = -\frac{2a}{3}, \quad m_{\perp AC} = \frac{3}{2a}$$

$$m_{\perp AC} = f'(x), \quad \Rightarrow \frac{3}{2a} = 3x^2 - a$$

$$x = \pm\frac{\sqrt{6\left(2a^2 + 3\right)}}{6\sqrt{a}}$$

Question 4a.

$\frac{7}{10} = 0.7$

Question 4b.

$\frac{\sqrt{21}}{100}$

Question 4c.

The mean will remain unchanged.

The standard deviation will decrease: standard deviation $= \frac{1}{\sqrt{6}} \times \frac{\sqrt{21}}{100} \approx 0.0187$

Question 4d.

0.402

Question 4e.

$\Pr(69 + a < X \le 69 - a) = 0.64$

Method 1:

Solving $\Pr(-\infty < X \le 69 - a) = 0.18$ or $\Pr(-\infty < X \le 69 + a) = 0.82$

$a = 6.408$

Method 2:

$\Pr(X < \mu + a) = 0.82$, $\mu + a = 75.4075...$ or $\Pr(X < \mu - a) = 0.18$, $\mu - a = 62.5924...$

$a = 6.408$

Method 3:

$Z = \frac{\mu + a - \mu}{\sigma} = 0.91536...$

$a = 7 \times 0.915...$

$a = 6.408$

Question 4f.

Let $W \sim \mathrm{Bi}(20, 0.9)$

$\Pr(W > 15) = 0.957$

Question 5a.

2

Question 5b.

$[-1, 3]$

Question 5ci.

$x = -\frac{\pi}{18} + \frac{2\pi k}{3} = \frac{\pi(12k-1)}{18}$ or $x = \frac{7\pi}{18} + \frac{2\pi k}{3} = \frac{\pi(12k+7)}{18}$, where $k \in Z$

Question 5cii.

-10π

Question 5di.

$g_{\max}(x) = \frac{4}{3}$, $a = \frac{\pi}{3}$

Question 5dii.

$g_{\min}(x) = 0$, $a = 0$

Question 5diii.

0.144

Question 5e.

$k = 2n$, $n \in Z^{+}$