VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2023

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Written examination 1

Tuesday 30 May 2023

Reading time: 10.30 am to 10.45 am (15 minutes)
Writing time: 10.45 am to 11.45 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Find the derivative of $y = x\sin(x)$ with respect to x. 1 mark

b. Evaluate $\int_0^{\frac{\pi}{2}} \left(x + \cos(x)\right) dx$. 2 marks

Question 2 (2 marks)

Let $f'(x) = (3 - x)^3$.

Find $f(x)$ given that $f(4) = \frac{5}{4}$.

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Question 3 (5 marks)

A school's first-aid room contains five first-aid kits. Three first-aid kits are red and two are orange.

For three days in a row, one first-aid kit is selected at random for an excursion and returned at the end of the day.

Each of the five first-aid kits has an equal chance of being selected each day.

Let R represent the number of red first-aid kits selected over the three days.

a. Find $\Pr(R = 3)$, the probability that the first-aid kit selected is red on all three days. 1 mark

b. Find the standard deviation of the random variable R. 1 mark

c. Find $\Pr(R = 3 \mid R \geq 2)$. 3 marks

Question 4 (4 marks)

Let $g(x) = 2^{2x} - 9 \times 2^{x} + 20$.

a. Evaluate $g\left(\log_2(3)\right)$, giving your answer as an integer. 2 marks

b. Solve $g(x) = 0$ for x. 2 marks

Question 5 (6 marks)

Consider a continuous random variable X with probability density function f given by

$$f(x)=\begin{cases}\dfrac{x+c}{2} & -d\le x\le d\\ 0 & \text{elsewhere}\end{cases}$$

where $c, d \in R$ and $c \geq 1$.

a. Find d in terms of c. 3 marks

b. Given that $\mathrm{E}(X)=\dfrac{1}{24}$, find the values of c and d. 3 marks

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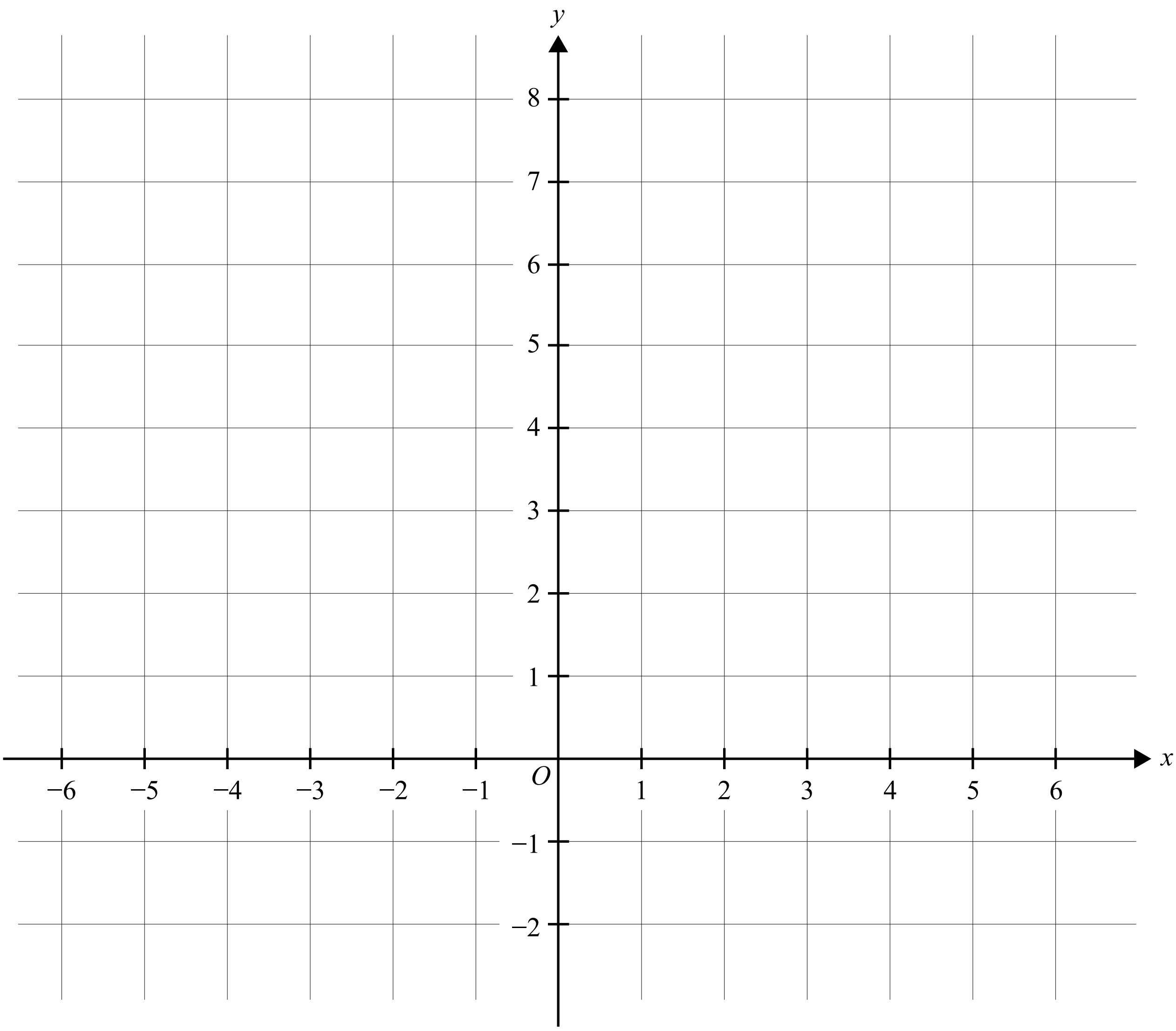
Question 6 (6 marks)

Consider the function f, where $f: D \to R$, $f(x) = 3 + \dfrac{2}{x-1}$, where D is the maximal domain of f.

a. State D, the maximal domain of f. 1 mark

b. Sketch the graph of $y = f(x)$. Label any asymptotes with their equation and the axial intercepts with their coordinates. 3 marks



Question 6 – continued

c. Let $g: R \setminus \{0\} \to R,\ g(x) = \dfrac{1}{x}$.

Determine the rule of the function h such that the graph of $y = h\big(g(x-1)\big)$ is identical to the graph of $y = f(x)$. 2 marks

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Question 7 (6 marks)

Let $f(x) = \log_e(3 + 2x - x^2)$.

a. Find the implied domain of f. 3 marks

b. State the x-coordinate for the local maximum of $f(x)$. 1 mark

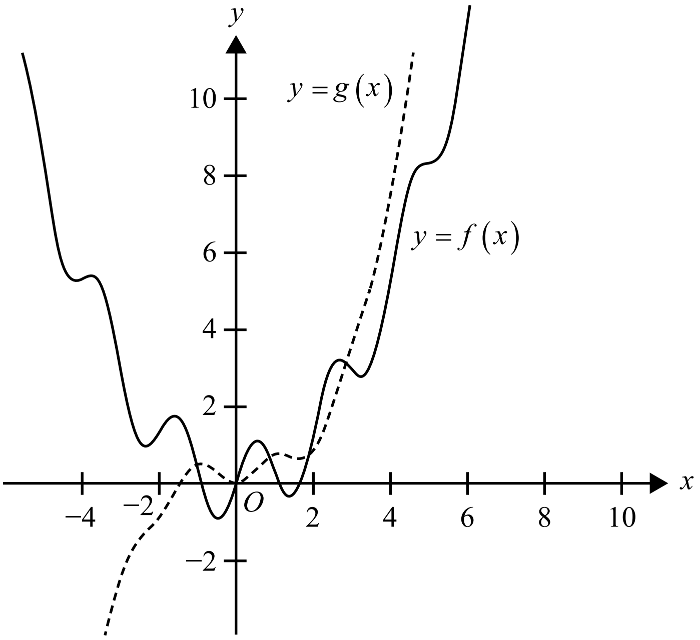
c. Find the values of x when $f(x) = 0$. 2 marks

Question 8 (8 marks)

Consider the function f, where $f: R \to R$, $f(x) = \frac{1}{3}x^2 + \sin(\pi x)$,

and a function g, where the domain of g is R but the equation of g is not known.

Part of the graphs of $y = f(x)$ and $y = g(x)$ are shown below. Both functions pass through the origin.



a. Based on the graphs above, state one reason why the graph of $y = g(x)$ is not the graph of $y = f'(x)$. 1 mark

b. Determine the equation of $g(x)$ given that $g'(x) = f(x)$. 3 marks

Question 8 – continued
TURN OVER

c. Some functions have the property $f(x) = f(-x)$ for all $x \in R$.

Show that when two functions, m and n, have this property, the function $k(x) = m(x) + n(x)$ also has this property. 1 mark

d. Consider the function h, where $h(x) = \frac{1}{3}x^2 + \sin(\pi x - c)$, $c \in R$.

Find all values of c for which $h(x) = h(-x)$, for all $x \in R$. 3 marks

END OF QUESTION AND ANSWER BOOK

DO NOT WRITE IN THIS AREA

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Victorian Certificate of Education
2023

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET