TSSM
Creating VCE Success

2023 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

Materials supplied
- Question and answer book of 8 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. If $y = \frac{e^{2x}}{x^2}$ find $\frac{dy}{dx}$ in factorized form.

2 marks

b. Consider $f(x) = (x + 1)log_e(3x^2)$. Find $f'(2)$

2 marks

Question 2

Given $f'(x) = \sqrt{x - 1} + 2e^x + \cos(x - 1)$ and $f(1) = 0$, find $f(x)$.

2 marks

TURN OVER

Question 3

Solve $\log_2(2x) + \log_2(x + 1) = 4$

3 marks

Question 4

David has a $\frac{1}{3}$ probability of catching the early train to work every morning, otherwise he catches the later train.

a. What is the probability David misses the early train four days in a row?

1 mark

b. What is the probability David catches the early train on at least three of the next four days?

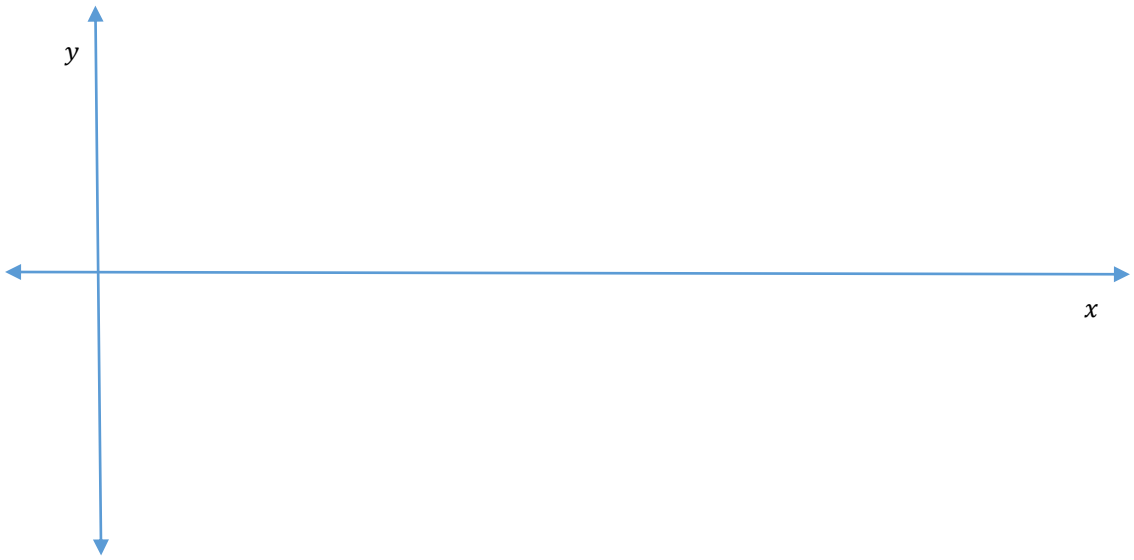
2 marks

c. What is the probability David catches the early train on at least three of the next four days, given that he caught the early train at least once?

1 mark

Question 5

a. Consider $y(x) = 2\tan(3x)$ for $x \in [0, \pi]$ labelling all intercepts and asymptotes.



3 marks

b. Solve $y(x) + 2 = 0$ over $x \in [0, \pi]$

3 marks

TURN OVER

Question 6

Consider $g: [a, \infty) \to R, g(x) = \frac{1}{2}x^2 - 2x + \frac{3}{2}$

a. Complete the square for $g(x)$.

_____

_____

1 mark

b. Hence or otherwise, state a series of transformations that takes the graph of $y = x^2$ to $y = g(x)$.

_____

_____

_____

_____

2 marks

c. State the smallest value of a such that $g(x)$ is a 1:1 function.

_____

_____

1 mark

Let $h(x) = \sqrt{1 + 2x}$

d. Show that $h\big(g(x)\big)$ is defined.

_____

_____

1 mark

e. State the rule for $h\big(g(x)\big)$, assuming $x \geq 2$.

_____

_____

1 mark

Question 7

Consider $y_1(x) = -3x(x-2)^2(x-4)$ and $y_2(x) = -3x^2(x-2)(x+2)$.

a. State a single transformation that takes the graph of $y = y_1(x)$ to $y = y_2(x)$.

_____

_____

1 mark

b. Find the co-ordinates of the stationary points of $y_2(x)$.

_____

_____

_____

_____

2 marks

The area bounded by the x and y axes and the curve $y_1(x)$ is equivalent to $a\int_0^b y_2(x)\,dx$.

c. State the values of a and b.

_____

_____

2 marks

d. Hence find the area bounded by the x and y axes and the curve $y_1(x)$.

_____

_____

_____

_____

2 marks

TURN OVER

Question 8

Let $p(x)$ be a probability density function such that:

$$p(x) = \begin{cases} ax^2 \log_e(x) & 0 \leq x \leq 1 \\ 0 & \text{elswhere} \end{cases}$$

a. Show by differentiation that the antiderivative of $x^{n-1}(n\log_e(x) + 1)$ is $x^n \log_e(x) + c$, where $c \epsilon R$.

2 marks

b. Hence or otherwise, find the value of a such that $p(x)$ is a probability density function.

3 marks

c. Determine $E(X)$

3 marks

END OF QUESTION AND ANSWER BOOK