MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

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2023 Trial Examination

SOLUTIONS

Question 1

a. $y = x^{-2}e^{2x}$

$\dfrac{dy}{dx} = -2x^{-3}e^{2x} + 2x^{-2}e^{2x}$

M1

$= -2e^{2x}x^{-3}(1 - x)$

$= \dfrac{2}{x^3}(x - 1)e^{2x}$

A1

b. $f'(x) = (1)\log_e(3x^2) + (x + 1)\frac{6x}{3x^2}$

M1

$= \log_e(3x^2) + \dfrac{2}{x}(x + 1)$

$f'(2) = \log_e(12) + 3$

A1

Question 2

$f(x) = \displaystyle\int \sqrt{x - 1} + 2e^x + \cos(x - 1)\,dx$

$= \dfrac{2}{3}(x - 1)^{\frac{3}{2}} + 2e^x + \sin(x - 1) + c$

M1

$0 = \dfrac{2}{3}(1 - 1)^{\frac{3}{2}} + 2e^1 + \sin(1 - 1) + c$

$c = -2e$

$so\ f(x) = \dfrac{2}{3}(x - 1)^{\frac{3}{2}} + 2e^x + \sin(x - 1) - 2e$

A1

Question 3

$\log_2(2x(x+1)) = 4$

M1

$2x^2 + 2x = 16$
$x^2 + x - 8 = 0$

M1

$x = \frac{-1 \pm \sqrt{33}}{2}$
$x = \frac{-1 + \sqrt{33}}{2}$

A1

Question 4

a. $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

A1

b. $\Pr(X \geq 3) = \binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 + \binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$

M1

$= 4 \times \frac{2}{81} + \frac{1}{81} = \frac{1}{9}$

A1

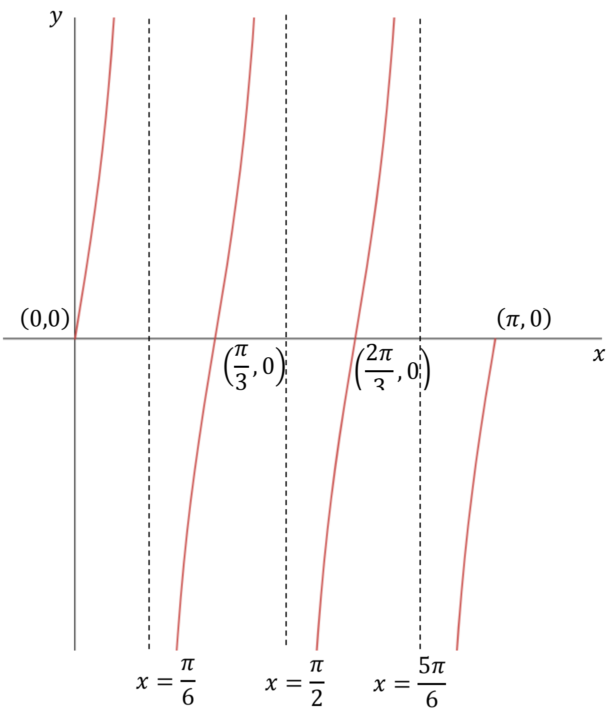
c. $\Pr(X \geq 3 | X > 0) = \frac{\Pr(X \geq 3)}{1 - \Pr(X = 0)}$

$= \frac{\frac{9}{81}}{\frac{16}{81}} = \frac{9}{16}$

A1

Question 5

a.



A1 shape
A1 asymptotes
A1 intercepts and end points

b. $2\tan(3x) + 2 = 0$

$\tan(3x) = -1$

M1

$\theta_R = \dfrac{-\pi}{4}$

M1

$3x = \dfrac{-\pi}{4} + n\pi$

$x = \dfrac{-\pi}{12} + \dfrac{n\pi}{3}$

$x = \dfrac{\pi}{12}(3 + 4n)$

$so\ x = \dfrac{\pi}{4}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$

A1

Question 6

a. $\frac{1}{2}x^2 - 2x + \frac{3}{2}$

$= \dfrac{1}{2}(x^2 - 4x + 3)$

$= \dfrac{1}{2}(x^2 - 4x + 4 - 4 + 3)$

$= \dfrac{1}{2}((x - 2)^2 - 1)$

$= \dfrac{1}{2}(x - 2)^2 - \dfrac{1}{2}$

A1

b. Dilation of factor $\frac{1}{2}$ from the x axis followed by translation of 2 units in the positive x direction and $\frac{1}{2}$ unit in the negative y direction.

A2

c. $a = 2$

A1

d. $h(g(x))$ is defined if $ran\ g \subseteq dom\ h$

$ran\ g = dom\ h = \left[-\frac{1}{2}, \infty\right)$ hence $h(g(x))$ is defined.

A1

e. $h(g(x)) = \sqrt{1 + 2\left(\frac{1}{2}(x - 2)^2 - \frac{1}{2}\right)} = \sqrt{(x - 2)^2} = x - 2$

A1

Question 7

a. Translation of 2 units in the negative x direction

A1

b. $y_2(x) = -3x^2(x^2-4)$

$$\frac{dy_2}{dx} = -6x(x^2-4) - 6x^3 = -12x^3 + 24x$$
$$= -12x(x^2-2) = 0$$

M1

$$x = 0, \pm\sqrt{2}$$
$$y_2(0) = 0$$
$$y_2(\sqrt{2}) = y_2(-\sqrt{2}) = 12$$

So stationary points at $(0,0)$, $(-\sqrt{2}, 12)$ and $(\sqrt{2}, 12)$

A1

c. Area bounded by $y_1(x)$ and the axes is given by $\int_0^2 y_1(x)\,dx + \int_2^4 y_1(x)\,dx$

Which is equivalent to $\int_{-2}^0 y_2(x)\,dx + \int_0^2 y_2(x)\,dx$

And due to symmetry is equivalent to $2\int_0^2 y_2(x)\,dx$

Hence $a = b = 2$

A2

d. $Area = 2\int_0^2 -3x^4 + 12x^2\,dx = 6\int_0^2 -x^4 + 4x^2\,dx$

$$= 6\left[-\frac{1}{5}x^5 + \frac{4}{3}x^3\right]_0^2$$

M1

$$= 6\left(-\frac{1}{5}(2)^5 + \frac{4}{3}(2)^3\right)$$
$$= 64 - \frac{192}{5} = \frac{128}{5} \text{ sq units}$$

A1

Question 8

a.

$$\frac{d}{dx}(x^n \log_e(x))$$

$$let\ u = x^n, u' = nx^{n-1}, \quad v = \log_e(x), \quad v' = \frac{1}{x}$$

$$\frac{d}{dx}(x^n \log_e(x)) = (x^n)\left(\frac{1}{x}\right) + (nx^{n-1})(\log_e(x))$$

M1

$= x^{n-1} + nx^{n-1}\log_e(x)$
$= x^{n-1}(nlog_e(x) + 1)$ M1

Hence $\int x^{n-1}(nlog_e(x) + 1)dx = x^n \log_e(x)$

b. $\int x^{n-1}(nlog_e(x) + 1)dx = x^n \log_e(x)$

$$\int x^{n-1} nlog_e(x)dx + \int x^{n-1}dx = x^n \log_e(x)$$

$$\int x^{n-1} nlog_e(x)dx = x^n \log_e(x) - \frac{1}{n}x^n$$

$$\int x^{n-1} log_e(x)dx = \frac{1}{n}x^n \log_e(x) - \frac{1}{n^2}x^n$$

M1

$$\int_0^1 ax^2 \log_e(x)\, dx = 1, \quad so\ n = 3$$

$$= a\left[\frac{1}{3}x^3 \log_e(x) - \frac{1}{9}x^3\right]_0^1 = 1$$

M1

$$a\left(\left(0 - \frac{1}{9}\right) - (0 - 0)\right) = 1$$

$a = -9$

A1

c. $E(X) = \int_0^1 -9x^3 \log_e(x)\, dx$

M1

$$\int x^{n-1} log_e(x)dx = \frac{1}{n}x^n \log_e(x) - \frac{1}{n^2}x^n \qquad where\ n = 4,$$

$$\int x^3 log_e(x)dx = \frac{1}{4}x^4 \log_e(x) - \frac{1}{16}x^4$$

So $E(X) = -9\left[\frac{1}{4}x^4 \log_e(x) - \frac{1}{16}x^4\right]_0^1$

M1

$$= -9\left(\left(0 - \frac{1}{16}\right) - (0 - 0)\right)$$

$$E(X) = \frac{9}{16}$$

A1