Mathematical Methods Units 1 and 2
Written examination 1

September 2023
Reading time: 5 minutes
Writing time: 30 minutes

Name : _____   **Teacher : _____**

Question and Answer Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	7	7	22

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book.
- A formula sheet

Instructions
- Write your **name** and your **teacher's** name in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorized electronic devices into the examination room.

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Question 1 (3 marks)

The line with the equation $(a + 2)x - y = 5$ is perpendicular to the line $2y - x = 3$.

a.   Find the gradient of $2y - x = 3$ 1 mark

$\frac{1}{2}$ (1A)

b.   Find the value of a 2 marks

$a + 2 = -2$ (1M) or $(a + 2) \times \frac{1}{2} = -1$
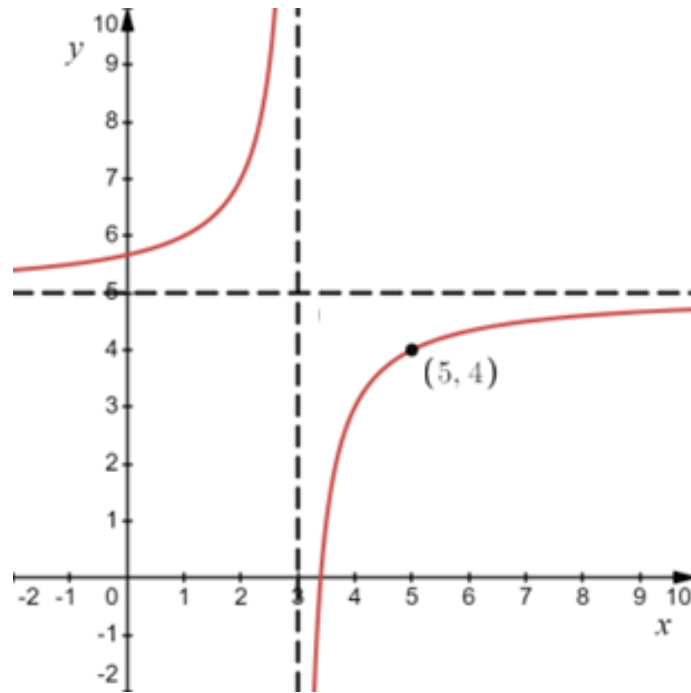
$a = -4$ (1A)

CM to be awarded according to a.

Question 2 (3 marks)

Determine the rule for the following graph in the form of $y = \dfrac{a}{x-h} + k,$

where $a, h, k \in \mathbb{R}$ 3 marks

$(5, 4)$

$y = \dfrac{a}{x-3} + 5$ (1 mark for $h = 3$ and 1mark for $k = 5$)

sub (5,4): $a = -2$

$y = \dfrac{-2}{x-3} + 5\ (1A)$

Question 3 (3 marks)

Two events A and B are such that: Pr(A)=0.6, Pr(B)=0.3 and Pr(A'∩B)=0.1.

a.   Find Pr(A∩B)   (1 mark)

$Pr(A \cap B) = 0.2$ (1 mark)

b.   Find Pr(A' │ B)   (1 mark)

$Pr(A|B') = \frac{1}{3}$ (1 mark)

c.   Are the events A and B independent? Justify your answer   (1 mark)

No. $Pr(A|B') = \frac{1}{3} \neq Pr(A) = 0.6$ (1 mark) any other equivalent relationship

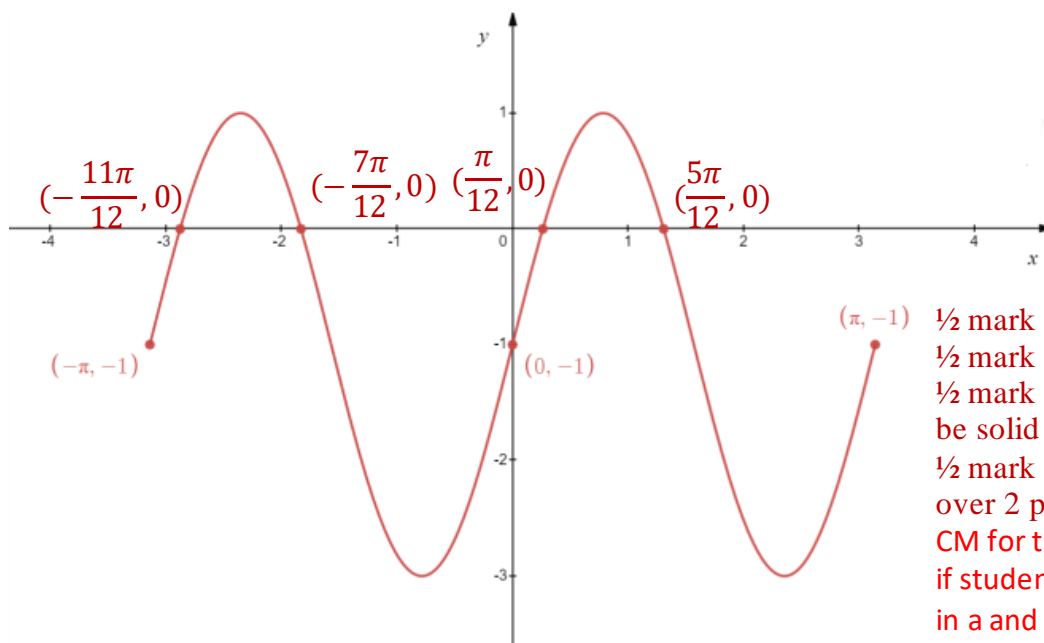accepted as long as justified

Question 4 (5 marks)

a.   Solve the equation $2\sin(2x) - 1 = 0$ for $x \in [-\pi, \pi]$.   (3 marks)

$$2\sin(2x) - 1 = 0 \therefore \sin(2x) = \frac{1}{2} \therefore$$

$2x = \frac{\pi}{6} + 2n\pi$ or $2x = \pi - \frac{\pi}{6} + 2n\pi$ (1 mark for each RA highlighted)

$x \in \left\{ -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{5\pi}{12} \right\}$ (1 mark)

b.   Sketch the graph of $f : [-\pi, \pi] \to R, f(x) = 2\sin(2x) - 1$. Label the axis intercepts and end points with their coordinates.   (2 marks)

$(-\frac{11\pi}{12}, 0)$ $(-\frac{7\pi}{12}, 0)$ $(\frac{\pi}{12}, 0)$ $(\frac{5\pi}{12}, 0)$

$(-\pi, -1)$ $(0, -1)$ $(\pi, -1)$

½ mark for all x-intercepts
½ mark for all y-intercept
½ mark for endpoints ; must be solid dots
½ mark for shape; must be over 2 periods
CM for the x intercepts only if students have 4 solutions in a and they are two positive and two negative solutions.

Question 6 (3 marks)
Evaluate the following:

a. $log_2(16) =$ 1 mark

$log_2(16) = 4$ (1 mark)

b. $2log_2 12 + 3log_2 5 - log_2 15 - log_2 150 =$ 2 marks

$2log_2 12 + 3log_2 5 - log_2 15 - log_2 150 = log_2 \left(\frac{12^2 \times 5^3}{15 \times 150} \right)$ (1 mark)

$= log_2 \left(\frac{2^4 \times 3^2 \times 5^3}{3^2 \times 5^3 \times 2} \right) = log_2 2^3 = 3$ (1 mark)

Question 6 (5 marks)

Solve the following equations for x:

a. $5^{3x} = 25$ 1 mark

$3x = 2, x = \frac{2}{3}$ (1 mark)

b. $3^{2x} = 27 - 6 \times 3^x$ 3 marks

$3^{2x} = 27 - 6 \times 3^x \therefore t^2 = 27 - 6t, t = 3^x \therefore t^2 + 6t - 27 = 0 \,(1\;mark)$
$\therefore (t+9)(t-3) = 0 \therefore t = -9, t = 3 \therefore$
$3^x = -9 \,(no\;sol)(1\;mark), 3^x = 3 \therefore x = 1 (1\;\text{mark})$

c. $log_5(2x - 3) = 2$ 1 mark

$2x - 3 = 5^2 \therefore 2x = 28 \therefore x = 14$ (1 mark)

END OF EXAM 1