Neap

Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 10 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (2 marks)

Simplify $\frac{8p^{-3}q^{2}}{2p^{3}q} \div \left(\frac{2p^{2}q^{-2}}{3p^{4}q}\right)^{2}$. Express your answer using positive indices.

Question 2 (3 marks)

Solve $\log_3(2x + 3) + \log_3(x - 2) = 2$ for x.

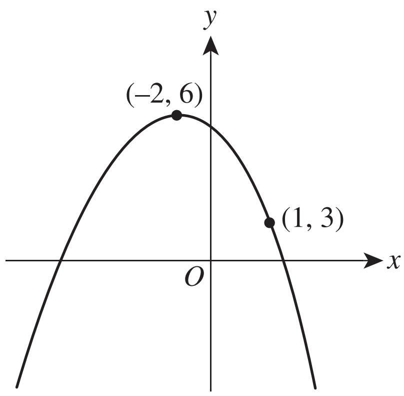
Question 3 (2 marks)

Let $\left\{x : \left(\frac{1}{3}\right)^{x} \geq 27\right\}$.

Solve for x.

Question 4 (3 marks)

Consider the following parabola.



Find the equation of the parabola in turning point form.

Question 5 (6 marks)

a. **i.** Express $\frac{\pi}{12}$ radians in degrees. 1 mark

ii. Express 235° in radians. 1 mark

b. **i.** State the exact value of $\cos\left(-\frac{\pi}{4}\right)$. 1 mark

ii. State the exact value of $\tan\left(\frac{5\pi}{6}\right)$. 1 mark

c. Let $\tan(\theta) = \frac{4}{3}$, for $\pi < \theta < \frac{3\pi}{2}$.

i. Find $\sin(\theta)$. 1 mark

ii.. Find $\frac{1}{\cos(\theta)}$. 1 mark

Question 6 (4 marks)

a. Simplify $\cos(2\pi - x) - \sin\left(\frac{3\pi}{2} - x\right)$. 2 marks

b. Solve $-\cos(2\propto) = \frac{\sqrt{3}}{2}$, for $\propto \in [-\pi, \pi]$. 2 marks

Question 7 (3 marks)

Find the equation of the tangent to the curve $y = 2x^3 - 4x^2 + x - 1$ at the point where $x = -1$.

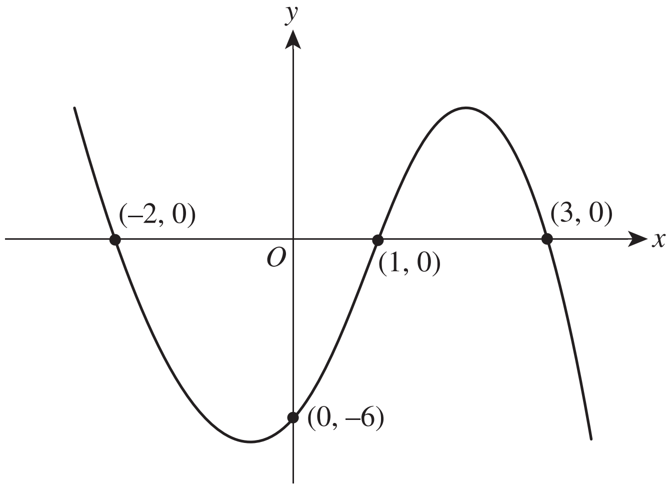
Question 8 (3 marks)

A curve with the equation $y = f(x)$ passes through the point $(-1, 3)$.

Given that $f'(x) = 4x + \frac{3}{x^4}$, find $f(x)$.

Question 9 (7 marks)

Consider the following graph of $y = h(x)$.



a. Find the equation of $h(x)$ in the form $h(x) = ax^3 + bx^2 + cx + d$. 2 marks

b. **i.** Find $h'(x)$.

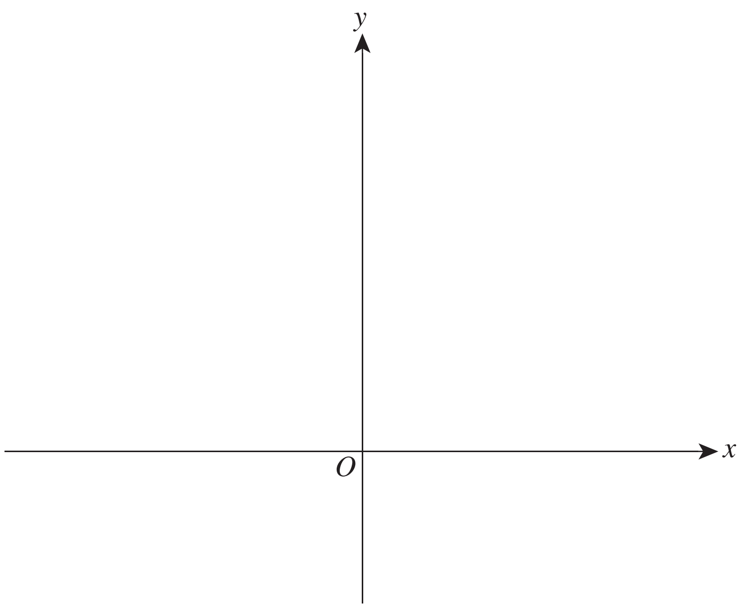
1 mark

ii. Find the x-intercepts of $h'(x)$.

2 marks

iii. On the axes below, sketch the graph of $y = h'(x)$. Label the axial intercepts with their coordinates.

2 marks



Question 10 (7 marks)

A and B are events such that $\Pr(A) = 0.6$, $\Pr(A' \cap B) = 0.2$ and $\Pr(A \cap B') = 0.1$.

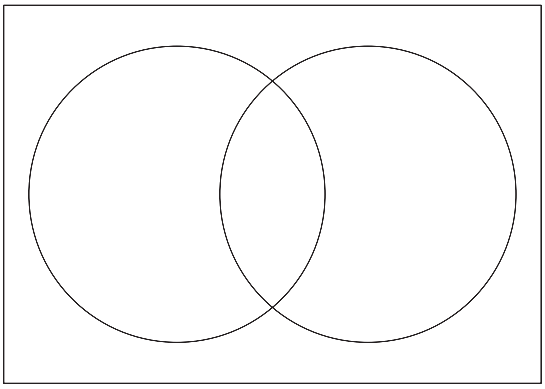
a. **i.** Find $\Pr(B)$. 1 mark

ii. Find $\Pr(A \cap B)$. 1 mark

iii. Find $\Pr(A \cup B)'$. 1 mark

iv. Find $\Pr(A' \mid B')$. 1 mark

b. **i.** Present the information found in **parts a.i.–a.iv.** on the Venn diagram below. 2 marks



ii. Explain whether events A and B are mutually exclusive with reference to the Venn diagram from **part b.i.** 1 mark

END OF QUESTION AND ANSWER BOOKLET

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VCE Mathematical Methods Units 1&2

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	Area $\approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

MMU12_FS_2023

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}\left((X - \mu)^2\right) = \mathrm{E}\left(X^2\right) - \mu^2$
binomial coefficient	$\dbinom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
binomial	$\Pr(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\, dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}\left(\hat{P}\right) = p$
standard deviation	$\mathrm{sd}\left(\hat{P}\right) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}}\right)$

END OF FORMULA SHEET