Neap

Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (2 marks)

$$\frac{8p^{-3}q^2}{2p^3q} \div \left(\frac{2p^2q^{-2}}{3p^4q}\right)^2 = \frac{4q}{p^6} \times \frac{9p^8q^2 \times q^4}{4p^4}$$ M1

$$= \frac{9q^7}{p^2}$$ A1

Note: For M1, accept any equivalent.

Question 2 (3 marks)

Method 1:

$$\log_3(2x+3) + \log_3(x-2) = 2$$
$$\log_3\left[(2x+3)(x-2)\right] = 2$$
$$3^2 = (2x+3)(x-2)$$
$$9 = 2x^2 - 4x + 3x - 6$$ M1
$$2x^2 - x - 15 = 0$$
$$(2x+5)(x-3) = 0$$

$x = -\frac{5}{2}$ or $x = 3$ A1

Method 2:

$$\log_3\left[(2x+3)(x-2)\right] = \log_3(3)^2$$
$$3^2 = (2x+3)(x-2)$$
$$9 = 2x^2 - 4x + 3x - 6$$ M1
$$2x^2 - x - 15 = 0$$
$$(2x+5)(x-3) = 0$$

$x = -\frac{5}{2}$ or $x = 3$ A1

For $\log_a(b)$ to exist, $b > 0$. Therefore, $x = -\frac{5}{2}$ cannot be a solution.

$x = 3$ A1

Question 3 (2 marks)

Consider $\left(\frac{1}{3}\right)^x = 27$.

$$\left(\frac{1}{3}\right)^x = 3^3$$
$$3^{-x} = 3^3$$
$$-x = 3$$ M1
$$x = -3$$

$\{x : x \in (-\infty, -3]\}$ **OR** $x \le -3$ A1

Question 4 (3 marks)

The general formula for turning point form is $y = a(x - h)^2 + k$, where (h, k) is the turning point.

The turning point (–2, 6) is given in the graph. Therefore:

$y = a(x + 2)^2 + 6$ M1

Substituting in the other point in the graph to solve for a gives:

$$\begin{aligned} 3 &= a(1+2)^2 + 6 \\ a(3)^2 &= -3 \\ 9a &= -3 \\ a &= -\frac{1}{3} \end{aligned}$$ M1

$y = -\frac{1}{3}(x+2)^2 + 6$ A1

Question 5 (6 marks)

a. **i.** $\frac{\pi}{12} : x$

$\pi : 180$

Using cross-multiplication gives:

$$\begin{aligned} \pi x &= \frac{\pi \times 180}{12} \\ x &= \frac{30}{2} \\ &= 15^\circ \end{aligned}$$ A1

ii. $180 : \pi$

$235 : x$

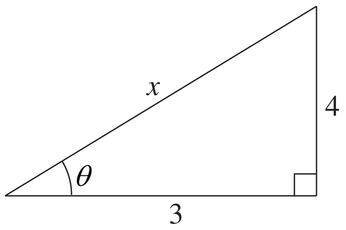
Using cross-multiplication gives:

$$\begin{aligned} 180x &= 235\pi \\ x &= \frac{235\pi}{180} \\ &= \frac{47\pi}{36} \end{aligned}$$ A1

b. **i.** $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ **OR** $\frac{\sqrt{2}}{2}$ A1

ii. $\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$ **OR** $-\frac{\sqrt{3}}{3}$ A1

c. **i.** Given that $\tan(\theta) = \frac{4}{3}$ is in the third quadrant:

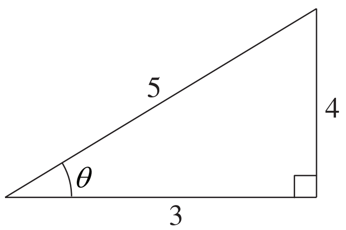


Using Pythagoras' theorem gives:

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = x^2$$
$$x = 5$$

Therefore, $\sin(\theta) = -\frac{4}{5}$. A1

ii. Using the answer from **part c.i.**, in the third quadrant:



$$\frac{1}{\cos(\theta)} = \frac{1}{\left(-\frac{3}{5}\right)}$$
$$= 1 \times \left(-\frac{5}{3}\right)$$
$$= -\frac{5}{3}$$ A1

Note: Consequential on answer to ***Question 5c.i***.

Question 6 (4 marks)

a. $\cos(2\pi - x) - \sin\left(\frac{3\pi}{2} - x\right) = \cos(2x - x)$

$$= \cos(-x)$$
$$= -\cos(x)$$ M1

Using complementary angles gives:

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$$

Hence, $\cos(2\pi - x) - \sin\left(\frac{3\pi}{2} - x\right) = -\cos(x) - (-\cos(x))$

$$= 0$$ A1

b. $\cos(2\propto) = -\frac{\sqrt{3}}{2}$

$$2\propto = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$2\propto = -\frac{5\pi}{6} \text{ and } \frac{5\pi}{6}$$ M1

$$\propto = -\frac{7\pi}{12}, -\frac{5\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$$ A1

Question 7 (3 marks)

When $x = -1$:

$$y = 2(-1)^3 - 4(-1)^2 + (-1) - 1$$
$$= -8$$

Therefore, a coordinate is (–1, –8). A1

$$\frac{dy}{dx} = 6x^2 - 8x + 1$$

When $x = -1$:

$$\frac{dy}{dx} = 6(-1)^2 - 8(-1) + 1$$
$$= 15$$ M1

Therefore, 15 is the gradient of the tangent.

Hence, the equation of the tangent is:

$$y - y_1 = m(x - x_1)$$
$$y - (-8) = 15(x - (-1))$$
$$y + 8 = 15x + 15$$
$$y = 15x + 7$$ A1

Question 8 (3 marks)

$$\int\left(4x + \frac{3}{x^4}\right)dx = \int\left(4x + 3x^{-4}\right)dx$$
$$= \frac{4x^2}{2} + \frac{3x^{-3}}{3} + c$$
$$= 2x^2 + x^{-3} + c$$ M1

Given that $y = f(x)$ passes through the point (–1, 3):

$$y = 2x^2 + x^{-3} + c$$
$$3 = 2(-1)^2 + \left(\frac{1}{-1}\right)^3 + c$$ M1
$$c = 3 - 2 - 1$$
$$= 0$$

Hence:

$$f(x) = 2x^2 + x^{-3}$$ A1

Question 9 (7 marks)

a. $h(x) = a(x+2)(x-1)(x-3)$

Given that $h(0) = -6$:

$-6 = a(0+2)(0-1)(0-3)$

$-6 = a(2)(-1)(-3)$

$-6 = 6a$

$a = -1$

$h(x) = -1(x+2)(x^2-4x+3)$ M1

$= -(x^3-4x^2+3x+2x^2-8x+6)$

$= -(x^3-2x^2-5x+6)$

$= -x^3+2x^2+5x-6$ A1

b. **i.** $h'(x) = -3x^2+4x+5$ A1

Note: Consequential on answer to ***Question 9a.***

ii. $h'(x) = 0$ needs to be solved.

$h'(x) = -3x^2+4x+5$

$-3x^2+4x+5=0$

Using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ gives:

$x = \dfrac{-4 \pm \sqrt{4^2-4(-3)(5)}}{2(-3)}$ M1

$= \dfrac{-4 \pm \sqrt{16+60}}{-6}$

$= \dfrac{-4 \pm \sqrt{76}}{-6}$

$= \dfrac{4 \pm 2\sqrt{19}}{6}$

$= \dfrac{2 \pm \sqrt{19}}{3}$

x-intercepts $= \left(\dfrac{2-\sqrt{19}}{3}, 0\right)$ and $\left(\dfrac{2+\sqrt{19}}{3}, 0\right)$ A1

Note: Consequential on answer to ***Question 9b.i.***

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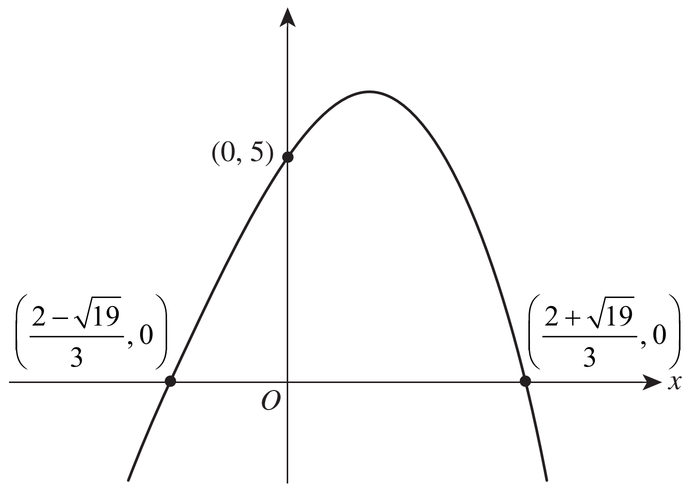
iii.



correct shape A1
correct x- and y-intercepts A1
Note: Consequential on answers to ***Questions 9b.i.*** *and* ***9b.ii.***

Question 10 (7 marks)

a. **i.** $\Pr(B) = 0.7$ A1

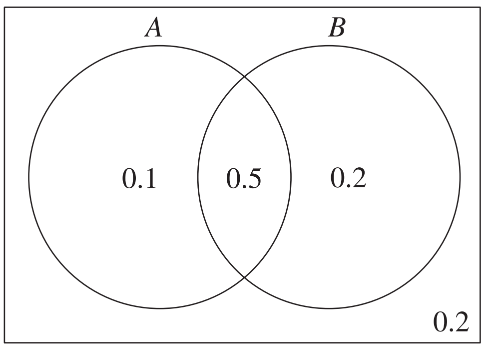
ii.. $\Pr(A \cap B) = 0.5$ A1

iii. $\Pr(A \cup B)' = 0.2$ A1

iv.
$$\Pr(A' \mid B') = \frac{\Pr(A' \cap B')}{\Pr(B')}$$
$$= \frac{0.2}{0.3}$$
$$= \frac{2}{3}$$
A1

Note: A Venn diagram may be used to find the probabilities.

b. **i.**



A2
1 mark per two correct data entries.

ii. Events A and B are not mutually exclusive. The Venn diagram has an intersection, and mutually exclusive events do not have an intersection. A1