The Mathematical Association of Victoria

Trial Exam 2023

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room any technology (calculators or software) or notes of any kind. blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Find $\frac{d}{dx}\left(x^3 \sin(2x)\right)$. 1 mark

b. If $f(x) = \frac{e^{2x}}{2x+1}$ find $f'(2)$. 2 marks

TURN OVER

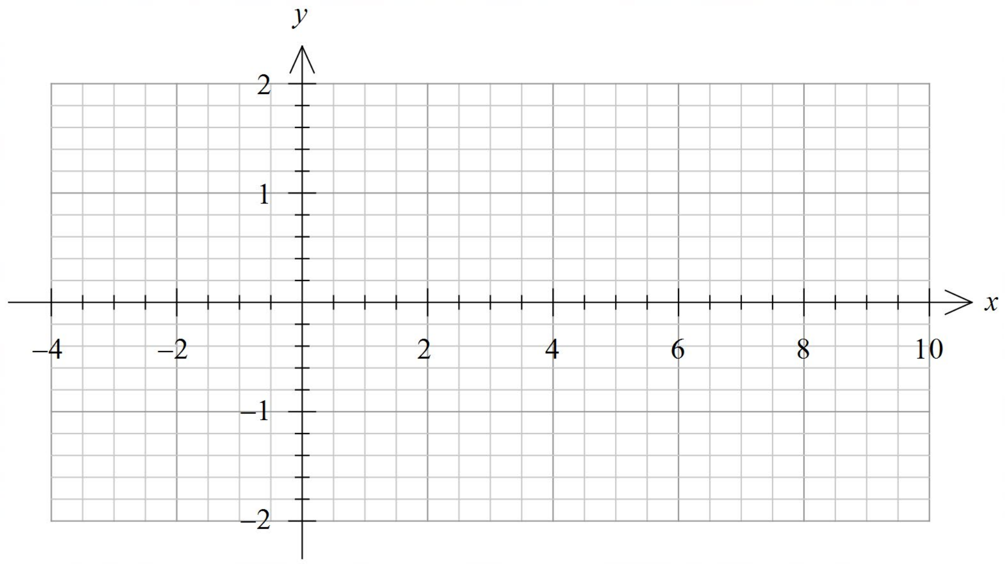
Question 2 (5 marks)

Let $f:(-2,\infty)\to R, f(x)=\dfrac{1}{x+2}$ and $g:(3,\infty)\to R, g(x)=\dfrac{1}{x-3}$.

a. Explain why $f\circ g$ exists. 1 mark

b. Show that $(f\circ g)(x)=\dfrac{1}{2}-\dfrac{1}{4x-10}$. 2 marks

c. Sketch the graph of $f\circ g$ on the set of axes below.
Label any asymptotes with their equations. 2 marks



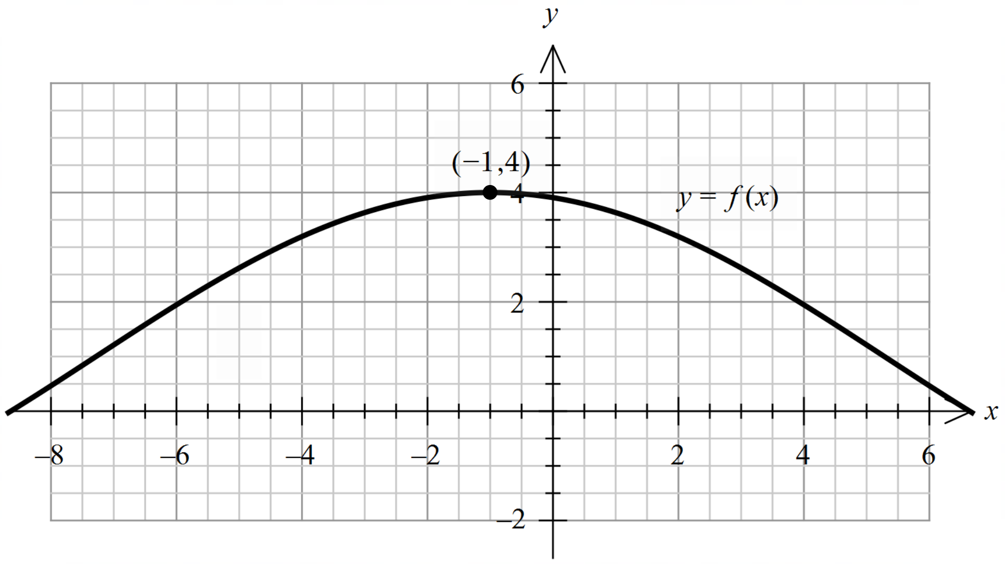
Question 3 (4 marks)

a. Given that $g'(x)=\frac{1}{(3x+1)^2}$, $x \in R \setminus \left\{-\frac{1}{3}\right\}$ find an expression for $g(x)$ if $g(0)=3$. 2 marks

b. Evaluate $\int_{-\frac{\pi}{18}}^{\frac{\pi}{4}} \left(-\frac{1}{2}\sin(6x)\right) dx$. 2 marks

Question 4 (4 marks)

Part of the graph of $y = f(x)$ is shown below. It has a turning point at $(-1, 4)$ and the y-coordinates of the points of inflection are 1. The rule for the graph is $f(x) = A\cos\left(nx + \frac{1}{4}\right) + c$, where $n \in Q$ and $A, c \in Z^+$.



a. **i.** Find the values of A and c. 1 mark

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ii. Find the value of n and give the general solution to the x-coordinates of the points of inflection. 2 marks

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b. State a sequence of transformations that map the graph of $f(x) = A\cos\left(nx + \frac{1}{4}\right) + c$ onto the graph of $g(x) = \cos\left(nx + \frac{1}{4}\right) + c$. 1 mark

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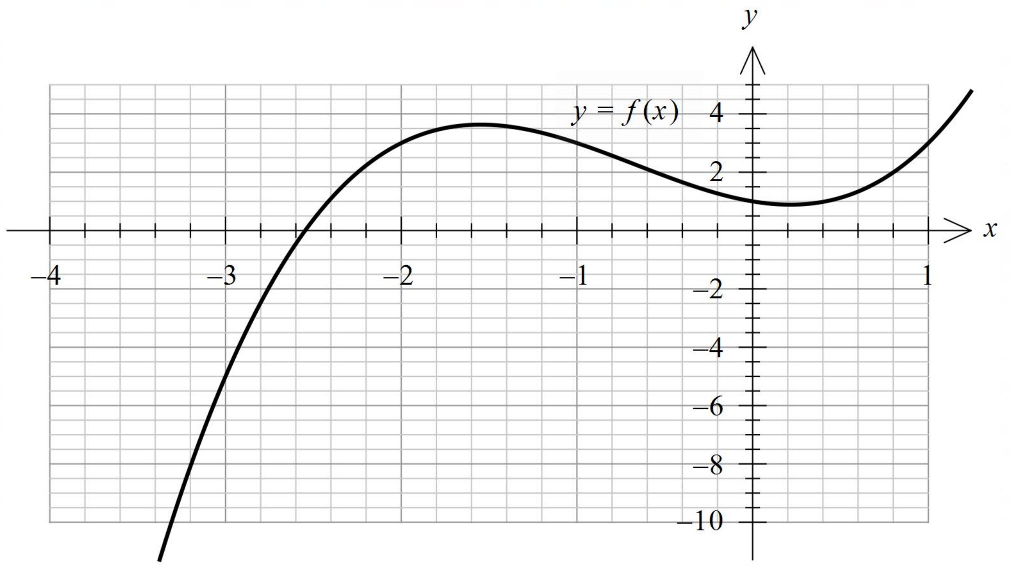
Question 5 (5 marks)

Part of the graph of f where $f(x) = x^3 + 2x^2 - x + 1$ is shown below.



a. Find the x-coordinates of the turning points. 2 marks

The solution to $f(x) = 0$ occurs between $x = -2$ and $x = -3$.

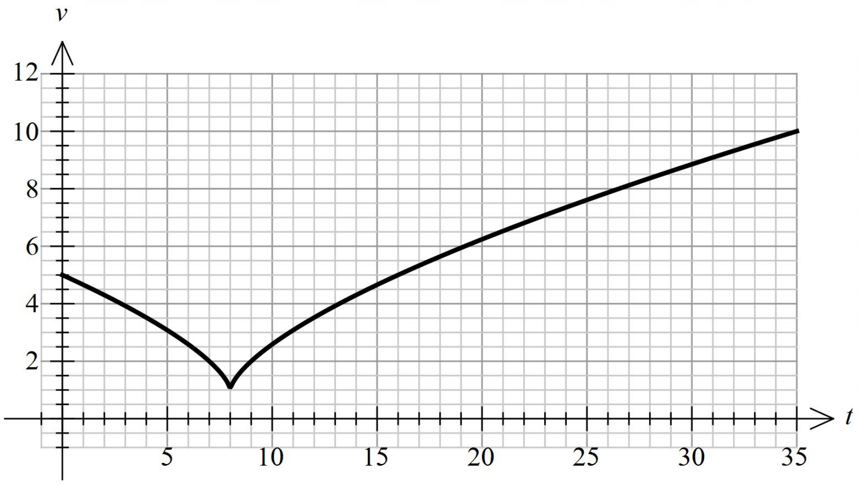
b. Use one iteration of Newton's Method, with $x = -3$ as the initial value, to find a better approximation. 2 marks

c. Draw the tangent line to f at $x = -3$ on the above graph and label the x-axis intercept with its coordinates. 1 mark

TURN OVER

Question 6 (5 marks)

The velocity, v ms^{-1} of a particle at time t seconds has rule $v(t) = (t-8)^{\frac{2}{3}} + 1$ where $t \geq 0$.
Part of the graph of v is shown below.



a. Find the time in seconds, when the acceleration, a ms^{-2}, of the particle is equal to 1 ms^{-2}, given $a = v'(t)$. 2 marks

b. Find the average rate of change of v from $t = 0$ to $t = 35$. 2 marks

A second particle is travelling at v_2 ms^{-1}, where $v_2(t) = -v(t) + 10$. The bounded area between the graphs of v and v_2 represents how much further the second particle has travelled than the first particle over that time period.

c. Write down a definite integral which when evaluated will give this distance. Give your answer in terms of t. Do not evaluate the integral. 1 mark

Question 7 (6 marks)
Hadiza either walks or takes the bus to work, depending on the weather. The probability it is going to rain on a particular morning is 0.2. If it rains on a particular morning the probability Hadiza will walk to work is 0.1. If it does not rain, the probability she will walk to work is 0.8.

a. Given that Hadiza walks to work on a particular morning what is the probability that it rained? 2 marks

Hadiza works for a large accounting company. The duration of her telephone calls to her clients is a random variable T minutes with probability density function

$$f(t) = \begin{cases} \frac{3}{5}\sqrt{t} & 0 \le t \le 1 \\ \frac{3e}{5}e^{-t} & 1 < t < \infty \\ 0 & t < 0 \end{cases}.$$

b. Show that $\Pr(T \le 1) = \frac{2}{5}$. 1 mark

c. Find the 50th percentile of T. 3 marks

TURN OVER

Question 8 (4 marks)
Let $g(x) = a\log_2\left(x+b\right)$ where $a, b \in R$. If $g(2) = 6$ and $g(6) = 9$ find *a* and *b*.

Question 9 (4 marks)

Let $h:(-\infty,a)\rightarrow R, h(x)=\dfrac{1}{(x-a)^2}$ where a is a real constant.

a. Find the rule for h^{-1} in terms of a. 1 mark

b. Find the value of a so that $h=h^{-1}$ has one unique solution. Put your answer in the form $\dfrac{b}{c^d}$ where $b,c\in Z^+$ and $d\in Q$. 3 marks

END OF QUESTION AND ANSWER BOOK