2023 VCE Mathematical Methods Year 12 Trial Examination 2

Detailed Answers

Kilbaha Education

Quality educational content

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SECTION A

ANSWERS

1	A	B	**C**	D	E
2	A	B	C	**D**	E
3	A	**B**	C	D	E
4	A	B	C	D	**E**
5	A	B	C	D	**E**
6	**A**	B	C	D	E
7	**A**	B	C	D	E
8	A	B	**C**	D	E
9	A	B	**C**	D	E
10	A	**B**	C	D	E
11	A	**B**	C	D	E
12	**A**	B	C	D	E
13	A	B	C	D	**E**
14	A	B	C	D	**E**
15	**A**	B	C	D	E
16	A	B	C	**D**	E
17	A	**B**	C	D	E
18	A	B	C	**D**	E
19	A	B	C	**D**	E
20	A	B	**C**	D	E

SECTION A

Question 1 **Answer C**

$y=\sin(2\pi cx)$ has a period $T=\dfrac{2\pi}{2\pi c}=\dfrac{1}{c}$

Question 2 **Answer D**

All of **A. B. C. E.** are correct,

The maximal domain is $(c\ \infty)$

Question 3 **Answer B**

$\dfrac{dy}{dx}=4\sin\left(\dfrac{x}{2}\right)$

$y=\int 4\sin\left(\dfrac{x}{2}\right)dx$

$y=-8\cos\left(\dfrac{x}{2}\right)+c$

$y=0,\ x=0,\ \Rightarrow 0=-8\cos(0)+c \quad c=8$

$y=8\left(1-\cos\left(\dfrac{x}{2}\right)\right)$

$\int\left(4\cdot\sin\left(\dfrac{x}{2}\right)\right)dx \qquad -8\cdot\cos\left(\dfrac{x}{2}\right)$

$\text{solve}\left(y=\int\left(4\cdot\sin\left(\dfrac{x}{2}\right)\right)dx+c,c\right)|x=0 \text{ and } y=0$

$c=8$

Question 4 **Answer E**

$f(x)=2\log_e(x-a)$

domain $f=(a\ \infty)$

$g(x)=\log_e(x-a)^2$,

domain $g=(-\infty\ a)\cup(a\ \infty)$

Define $f(x)=2\cdot\ln(x-a)$ *Done*

Define $g(x)=\ln\left((x-a)^2\right)$ *Done*

domain$(f(x),x)$ $\quad a<x<\infty$

domain$(g(x),x)$ $\quad -\infty<x<a$ or $a<x<\infty$

Alan and Ben are both incorrect, both Colin and David are correct.

Question 5 **Answer E**

$f(x)=4x^3-6x^2$

$f\ (x)=m(x)=12x^2-12x$

$f\ \ (x)=\dfrac{dm}{dx}=24x-12=12(2x-1)<0$

$x<\dfrac{1}{2} \quad \Rightarrow \quad x\in\left(-\infty\ \dfrac{1}{2}\right)$

$\text{Define } f(x)=4\cdot x^3-6\cdot x^2$ Done

$\text{solve}\left(\dfrac{d^2}{dx^2}(f(x))<0,x\right)$ $x<\dfrac{1}{2}$

Question 6 **Answer A**

x	0	1	2	3
$f(x)=\sqrt{2x+3}$	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{7}$	3

$$A_T=\frac{1}{2}\left(f(0)+2\left(f(1)+f(2)\right)+f(3)\right)=\frac{1}{2}\left(\sqrt{3}+2\left(\sqrt{5}+\sqrt{7}\right)+3\right)$$

Question 7 **Answer A**

BRR or RBR or RRB, 3 ways of drawing two red and one blue, with replacement

$$\Pr(2R \text{ and } 1B)=\frac{3br^2}{(b+r)^3}$$

Question 8 **Answer C**

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & \text{otherwise}\end{cases}$$

$$\int_0^{\infty} x\,f(x)\,dx=2 \quad \Rightarrow \lambda=\frac{1}{2}$$

$\Pr(X>2)=0.368$

$\text{Define } f(x)=\lambda\cdot e^{-\lambda\cdot x}$ Done

$\text{solve}\left(\int_0^{\infty}(x\cdot f(x))\,dx=2,\lambda\right)\Big|\lambda>0$ $\lambda=0.500000$

$\int_2^{\infty} f(x)\,dx\Big|\lambda=\dfrac{1}{2}$ 0.3679

Question 9 **Answer C**

$$\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\ ,\ \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)=(0.889,1.031)$$

$$\hat{p}=\frac{0.889+1.031}{2}=\frac{24}{25}=0.96$$

$$z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=z\sqrt{\frac{\frac{24}{25}\times\frac{1}{25}}{25}}$$

$$=\frac{2z\sqrt{6}}{125}=\frac{1.031-0.889}{2}=0.0710$$

$z=1.812, \quad Z\overset{d}{=}N(0,1)$

$\Pr(-1.812<Z<1.812)=0.9299 \qquad C=93$

$\dfrac{1.031-0.889}{2}$ 0.0710

$\dfrac{0.071}{\frac{2\cdot\sqrt{6}}{125}}$ 1.8116

$\text{normCdf}(-1.8112,1.8112,0,1)$ 0.9299

Question 10 **Answer B**

$X \overset{d}{=} Bi\left(n=15,\ p=0.6\right)$

$E\left(X\right)=np=15\times 0.6=9$

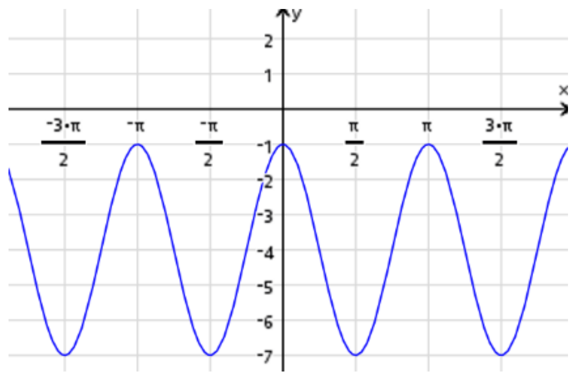
$\text{Var}\left(X\right)=np\left(1-p\right)=15\times 0.6\times 0.4=3.6$

$\Pr\left(X>9 \mid X\geq 3.6\right)=\dfrac{\Pr\left(X\geq 10\right)}{\Pr\left(X\geq 4\right)}=0.404$

n:=15	15
p:=0.6	0.6000
ex:=$n\cdot p$	9.0000
$varx$:=$n\cdot p\cdot(1-p)$	3.6000
$\dfrac{\text{binomCdf}(n,p,10,n)}{\text{binomCdf}(n,p,4,n)}$	0.4040

Question 11 **Answer B**

The graph of the function does not cross the x-axis. The graph has an infinite number of maximum turning points and also an infinite number of minimum turning points. The graph also has an infinite number of points of inflexion.
Mia is incorrect, both Jack and Zach are correct.



Question 12 **Answer A**

$f\left(x\right)=\log_e\left(\sqrt{b-\sqrt{x-a}}\right)$

$\sqrt{x-a}\geq 0 \qquad x\geq a$

$b-\sqrt{x-a}>0 \qquad \sqrt{x-a}<b$

$x-a<b^2$

$x<a+b^2$

$\left[a \ \ a+b^2\right)$

Define $f(x)=\ln\left(\sqrt{b-\sqrt{x-a}}\right)$	*Done*
domain$(f(x),x)\mid b>0$	$a\leq x<a+b^2$ and $b>0$

Question 13 **Answer E**

$P \overset{d}{=} N\left(14.5 \qquad {}^2\right)$ 0.

$\Pr\left(P>148.6\right)+\Pr\left(P<148.4\right)$

$=2\left(\Pr\left(P<148.4\right)\right)=0.6171$

$500\times 0.6171=308.54$

so 309 pieces of paper

2· normCdf(−∞,148.4,148.5,0.2)	0.6171
500· 0.6170750644	308.5375

Question 14 **Answer E**

$(1)\ x-y+z=1 \quad (2)\ x+2y-z=3$

A. Let $x=k$ then $\begin{array}{l}(1)\ -y+z=1-k \\ (2)\ 2y-z=3-k\end{array}$ adding $y=4-2k,\ z=y+1-k=5-3k$

B. Let $x=2k$, then $\begin{array}{l}(1)\ -y+z=1-2k \\ (2)\ 2y-z=3-2k\end{array}$ adding $y=4-4k,\ z=y+1-2k=5-6k$

C. Let $y=k$ then $\begin{array}{l}(1)\ x+z=1+k \\ (2)\ x-z=3-2k\end{array}$ adding $x=\frac{1}{2}(4-k),\ z=1+k-x=\frac{1}{2}(3k-2)$

D. Let $y=2k$, then $\begin{array}{l}(1)\ x+z=1+2k \\ (2)\ x-z=3-4k\end{array}$ adding $x=2-k,\ z=1+2k-x=-1+3k$

E. Let $z=k$ then $\begin{array}{l}(1)\ x-y=1-k \\ (2)\ x+2y=3+k\end{array}$ subtracting $y=\frac{2}{3}(1+k)\neq\frac{1}{3}(3+k),\ x=1-k+y=\frac{1}{3}(5-k)$

A. B. C. D. are all correct **E.** is incorrect.

Question 15 **Answer A**

$$f(x)=\begin{cases}4-2x & \text{for } 0\le x\le 3\\ -2 & \text{for } 3<x\le 5\\ x-7 & \text{for } 5<x\le 8\end{cases}$$

$$\bar{f}=\frac{1}{8}\int_0^8 f(x)\,dx=-\frac{5}{16}$$

Define $f1(x)=\begin{cases}4-2\cdot x, & 0<x\le 3\\ -2, & 3\le x\le 5\\ x-7, & 5\le x\end{cases}$ Done

$\frac{1}{8}\cdot\int_0^8 f1(x)\,dx$ $\quad\frac{-5}{16}$

solve$(\log_3(y)=4\cdot\log_2(x)+1,y)$

$y=3\cdot x^{\frac{4\cdot\ln(3)}{\ln(2)}}$ and $x\ge 0$

$3\cdot x^{\frac{4\cdot\ln(3)}{\ln(2)}}=3\cdot x^{4\cdot\log_2(3)}$ true

Question 16 **Answer D**

$$\log_3(y)=4\log_2(x)+1$$

$$\log_3(y)-1=\log_2(x^4)$$

$$\log_3(y)-\log_3(3)=\log_3\left(\frac{y}{3}\right)=\log_2(x^4)$$

$$\frac{\log_e\left(\frac{y}{3}\right)}{\log_e(3)}=\frac{\log_e(x^4)}{\log_e(2)}$$ using change of base rule for logs $\log_a(b)=\frac{\log_e(b)}{\log_e(a)}$

$$\log_e\left(\frac{y}{3}\right)=\frac{\log_e(3)}{\log_e(2)}\log_e(x^4)=\log_2(3)\log_e(x^4)=\log_e\left(x^{4\log_2 3}\right)$$

$$y=3x^{4\log_2 3}$$

Question 17 **Answer B**

A. When $a=-6$ and $b=-12$ there is a unique solution is true.

B. When $a \in R\backslash\{3\}$ and $b \in R$ there is an infinite number of solutions is false.

C. When $a=-8$ and $b=-12$ there is an infinite number of solutions is true.

D. When $a=3$ and $b=4$ there is no solution is true.

E. When $a=3$ and $b \neq -12$ there is no solution is true.

When $a=3$

$$3x-4y=6 \quad \Rightarrow 3x-4y=6$$

$$-6x+8y=b \quad \Rightarrow 3x-4y=-\frac{b}{2}$$

When $a=-8$

$$-8x-4y=-16 \quad \Rightarrow \quad 2x+y=4$$

$$-6x-3y=b \quad \Rightarrow \quad 2x+y=-\frac{b}{3}$$

$eq1 := a\cdot x-4\cdot y=2\cdot a$ $\qquad a\cdot x-4\cdot y=2\cdot a$

$eq2 := -6\cdot x+(a+5)\cdot y=b$ $\qquad (a+5)\cdot y-6\cdot x=b$

$\text{solve}\left(\det\left(\begin{bmatrix} a & -4 \\ -6 & a+5 \end{bmatrix}\right)=0, a\right)$ $\qquad a=-8 \text{ or } a=3$

$\text{solve}\left(\begin{cases} eq1 \\ eq2 \end{cases}, \{x,y\}\right)|a=-8 \text{ and } b=-12$ $\qquad x=\frac{-(c3-4)}{2} \text{ and } y=c3$

$\text{solve}\left(\begin{cases} eq1 \\ eq2 \end{cases}, \{x,y\}\right)|a=3 \text{ and } b=4$ $\qquad \text{false}$

$\text{solve}\left(\begin{cases} eq1 \\ eq2 \end{cases}, \{x,y\}\right)|a=3 \text{ and } b=-12$ $\qquad 2\cdot(2\cdot c4+3)$

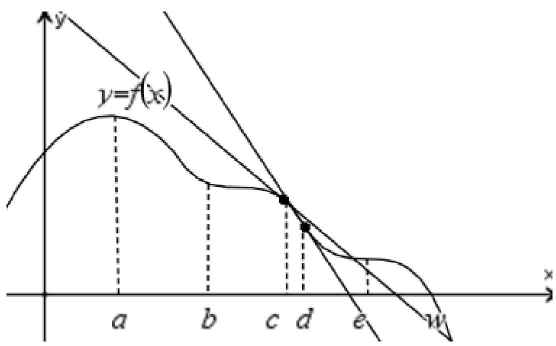
Question 18 **Answer D**

The function *f* is derivative of the function *g*, $g(x)=\int f(x)dx, \quad f(x)=g'(x),$

Question 19 **Answer D**

Question 20 **Answer C**

Although the point $x_0=e$ is closer to the actual root at $x=w$, Newton's method will fail at $x_0=e$, since the derivative at $x_0=e$ is zero, there is a stationary point of inflexion at $x_0=e$. If we draw tangents at both $x_0=c$ and $x_0=d$ we see that the tangent at $x_0=c$ crosses the *x*-axis closer to $x_0=w$, so use $x_0=c$ as our initial starting value.



END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a. $f(x) = -x^3 + bx^2 + cx$

$f\;(x) = -3x^2 + 2bx + c$

since there is a turning point at $(3,0)$

$f(3) = -27 + 9b + 3c = 0 \quad (1)$

and the gradient is also zero at this point $f\;(3) = -27 + 6b + c = 0 \quad (2)$

solving (1) and (2) gives

$b = 6, \; c = -9$

Define $f(x)=-x^3+b\cdot x^2+c\cdot x$	*Done*
$f(3)$	$9\cdot b+3\cdot c-27$
Define $fd(x)=\frac{d}{dx}(f(x))$	*Done*
$fd(3)$	$6\cdot b+c-27$
solve$(f(3)=0$ and $fd(3)=0,\{b,c\})$	$b=6$ and $c=-9$

M1

A1

b. since there is a turning point at $x = 4$

$f\;(4) = -48 + 8b + c = 0 \quad (3)$

the area $\int_0^6 f(x)\,dx = 144$

$72b + 18c - 324 = 144 \quad (4)$

solving (3) and (4) gives

$b = \frac{11}{2}, \quad c = 4$

$fd(4)$	$8\cdot b+c-48$
$\int_0^6 f(x)\,dx$	$72\cdot b+18\cdot c-324$
solve$\left(fd(4)=0 \text{ and } \int_0^6 f(x)\,dx=144,\{b,c\}\right)$	$b=\frac{11}{2}$ and $c=4$

A1

A1

c. at $x = 2$ $\;f\;(2) = 4b + c - 12$

the tangent line at $x = 2$ is

$y = (4b + c - 12)x - 4(b - 4)$

Now this tangent passes through the origin so $b = 4$

since this is parallel to $y = 2x - 5$

$f\;(2) = 4b + c - 12 = 2 \quad (5)$

then $c = -2$

$f(2)$	$4\cdot b+2\cdot c-8$
$fd(2)$	$4\cdot b+c-12$
tangentLine$(f(x),x,2)$	$(4\cdot b+c-12)\cdot x-4\cdot(b-4)$
solve$(4\cdot b+c-12=2,c)\vert b=4$	$c=-2$

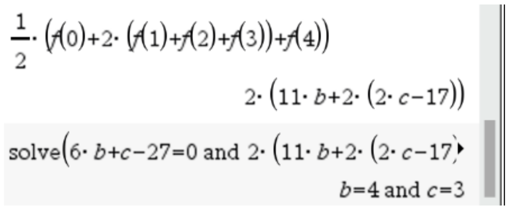
A1

A1

d. since there is a turning point at $x = 3$

$f(3) = -27 + 6b + c = 0 \quad (6)$ and

the area using the trapezium rule



A1

$$A_T = \frac{1}{2}\left(f(0) + 2\left(f(1) + f(2) + f(3)\right) + f(4)\right)$$

$$A_T = 2\left(11b + 2(2c - 17)\right) = 44 \quad (7)$$

solving (6) and (7) gives

$b = 4, \quad c = 3$ A1

e. since $y = -8 - (x-2)^3 = -8 - \left(x^3 - 6x^2 + 12x - 8\right) = -x^3 + 6x^2 - 12x$

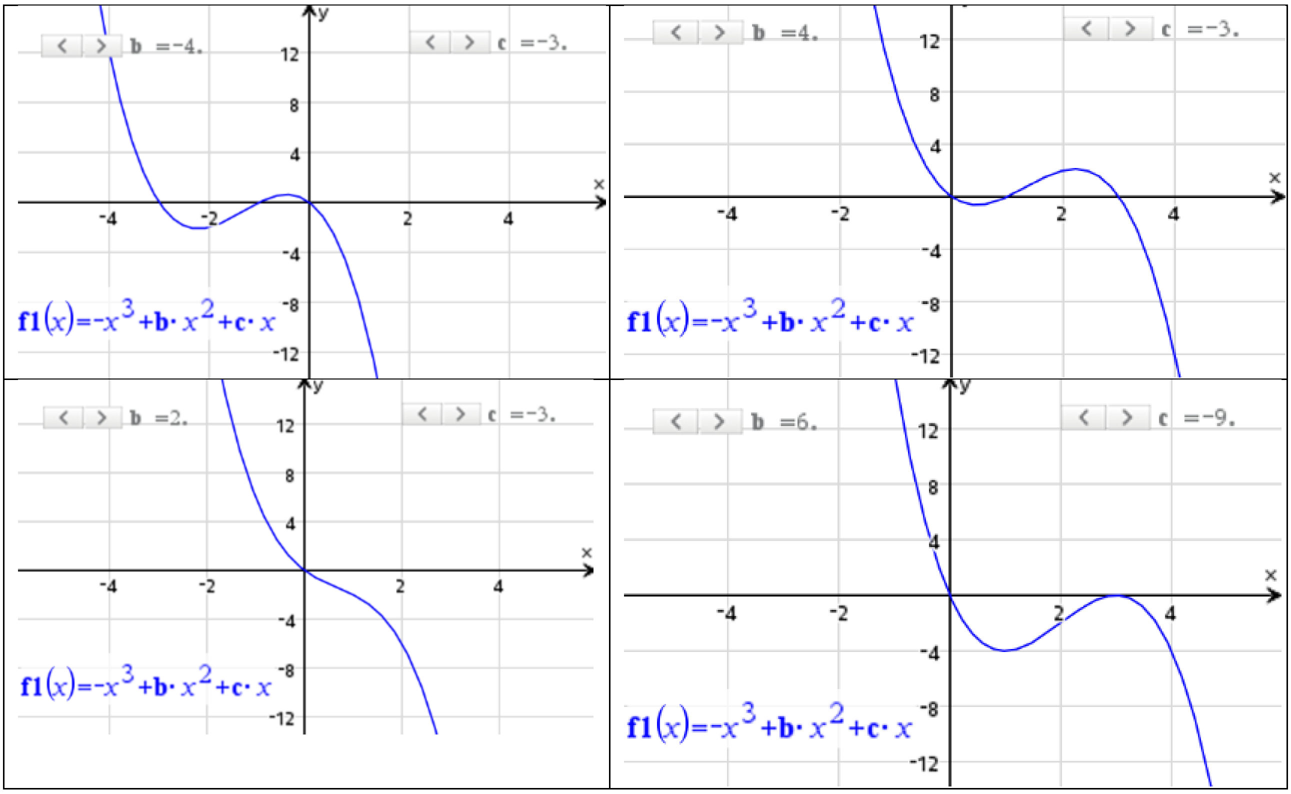
so $b = 6, \quad c = -12$, the stationary point of inflexion is the point $(2, -8)$ A2

f. $f(x) = -x^3 + bx^2 + cx = -x\left(x^2 - bx - c\right)$ the graph always passes through the origin,

and when $b = c = 0$ there is a stationary point of inflexion at the origin.

For only one other solution $\Delta = b^2 + 4c = 0$ so $b^2 = -4c$

$c < 0$ and $b = \pm\sqrt{-4c}$ or when $c = 0$ $f(x) = -x^2(x - b)$ so $b \in R \backslash \{0\}$ A1



Question 2

a.i. $g:[0,9]\to R,\ g(x)=px^3+qx^2+s$

$P(0,6):\ g(0)=6=s$ A1

$D(9,0):\ g(9)=0\ \Rightarrow\ 729p+81q+6=0\ \ (1)$

$g\ (x)=3px^2+2qx$

$g'(0)=0,\ \ g'(9)=0\ \Rightarrow 243p+18q=0\ \ (2)$ A1

$\Rightarrow\ q=-\dfrac{27p}{2}$ into (1)

$p\left(729-\dfrac{81\times 27}{2}\right)=-6$ M1

solving gives $p=\dfrac{4}{243}\quad q=-\dfrac{2}{9}$ and $s=6$

ii.

Define $g(x)=p\cdot x^3+q\cdot x^2+6$	*Done*
$g(9)=0$	$729\cdot p+81\cdot q+6=0$
Define $mg(x)=\dfrac{d}{dx}(g(x))$	*Done*
$mg(0)$	0
$mg(9)=0$	$243\cdot p+18\cdot q=0$
solve$(g(9)=0$ and $mg(9)=0,\{p,q\})$	$p=\dfrac{4}{243}$ and $q=\dfrac{^-2}{9}$

$g(x)=\dfrac{4}{243}x^3-\dfrac{2}{9}x^2+6=\dfrac{2}{243}\left(2x^3-27x^2+729\right)$

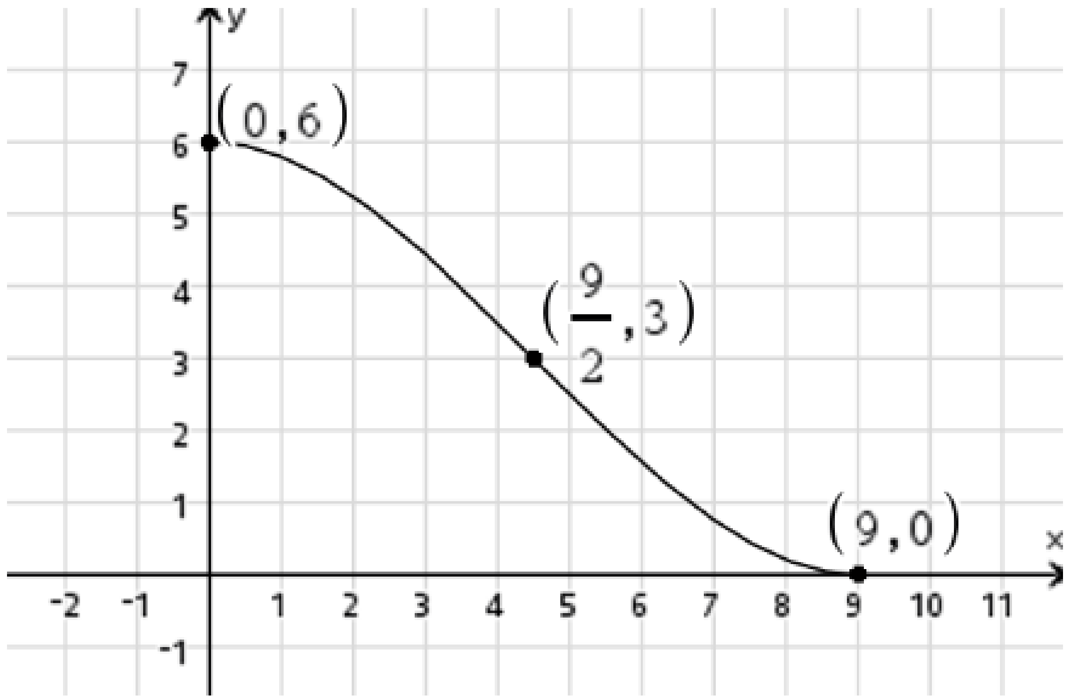
$mg(x)=g\ (x)=\dfrac{2}{243}\left(6x^2-54x\right)$

$mg\ (x)=\dfrac{2}{243}(12x-54)=0,\ \Rightarrow\ x=\dfrac{54}{12}=\dfrac{9}{2}\qquad g\left(\dfrac{9}{2}\right)=3$ M1

$\left(\dfrac{9}{2},3\right)$ A1

Define $g(x)=p\cdot x^3+q\cdot x^2+6 \mid p=\frac{4}{243}$ and $q=\frac{-2}{9}$	*Done*
$\text{solve}\left(\frac{d^2}{dx^2}(g(x))=0,x\right)$	$x=\frac{9}{2}$
$g\left(\frac{9}{2}\right)$	3
$\frac{d}{dx}(g(x)) \mid x=\frac{9}{2}$	$^{-}1$

iii. G1



$\frac{1}{9-0}\cdot\int_0^9 g(x)\,dx$	3

iv. $\bar{g}=\frac{1}{b-a}\int_a^b g(x)\,dx$

$\bar{g}=\frac{1}{9-0}\int_0^9\left(\frac{4}{243}x^3-\frac{2}{9}x^2+6\right)dx=3$ A1

b. Consider the line segment which passes through the points $B(2,5)$ and $C(5,2)$

$m_{BC}=\frac{2-5}{5-2}=-1=m$, the line BC has the equation

$y-5=-1(x-2)=-x+2,\quad y=-x+7,\quad k=7$ A1

Consider the parabolic section, since it has a minimum turning point at $D(9,0)$

its equations is $y=a(x-9)^2$ and since it also passes through the point $C(5,2)$

$2=a(5-9)^2=16a,\quad a=\frac{1}{8}$ also $\frac{dy}{dx}=2a(x-9),\quad \left.\frac{dy}{dx}\right|_{x=5}=-8a=-1,\quad a=\frac{1}{8}$

$y=\frac{1}{8}(x-9)^2=\frac{1}{8}\left(x^2-18x+81\right)=\frac{x^2}{8}-\frac{9x}{4}+\frac{81}{8},\quad a=\frac{1}{8},\quad b=-\frac{9}{4},\quad c=\frac{81}{8}$ A1

Consider the trigonometric section, let $f(x)=R\cos(nx)+5$

at $B(2,5)$ $\quad f(2)=5=R\cos(2n)+5 \quad\Rightarrow\quad R\cos(2n)=0$ since $R\neq 0$

$2n=\frac{\pi}{2},\quad n=\frac{\pi}{4},\quad f'(x)=-nR\sin(nx)$ and since the join is smooth at B M1

$f'(2)=-nR\sin(2n)=-nR\sin\left(\frac{\pi}{2}\right)=-nR=-1,\quad R=\frac{1}{n}=\frac{4}{\pi}$ A1

Define $f1(x)=r\cdot\cos(n\cdot x)+5$	*Done*
Define $f2(x)=7-x$	*Done*
Define $f3(x)=a\cdot(x-9)^2$	*Done*
solve$(f3(5)=2,a)$	$a=\frac{1}{8}$
expand$(f3(x))\vert a=\frac{1}{8}$	$\frac{x^2}{8}-\frac{9\cdot x}{4}+\frac{81}{8}$

c. For design A, $g'\left(\frac{9}{2}\right)=-1$ and this is also the steepest slope for design B,

since $m=-1$, so both designs have equal maximum slopes of -1 A1

Question 3

a. $\Pr(A \mid \oplus) = \dfrac{\Pr(A\oplus)}{\Pr(\oplus)} = \dfrac{31}{81}$ A1

b. $B \stackrel{d}{=} Bi(n = 20, p = 0.1)$, 10% of 20 is 2

binomCdf(20,0.1,3,20)	0.3231

$\Pr(B > 2) = \Pr(B \geq 3) = 0.3231$ A1

c. $T \stackrel{d}{=} N(\mu = ?, \sigma^2 = ?)$

$\Pr(T > 12) = 0.31, \quad \Pr(T < 12) = 0.69 \quad \Rightarrow \quad (1) \quad \dfrac{12 - \mu}{\sigma} = 0.4959$ M1

$\Pr(T < 9) = 0.16 \quad \Rightarrow \quad (2) \quad \dfrac{9 - \mu}{\sigma} = -0.9945$ A1

solving (1) and (2) gives $\mu = 10$ and $\sigma = 2.0$ A1

$eq1 := \frac{12-m}{s} = \text{invNorm}(0.69,0,1)$	$\frac{12-m}{s} = 0.4959$
$eq2 := \frac{9-m}{s} = \text{invNorm}(0.16,0,1)$	$\frac{9-m}{s} = -0.9945$
solve($eq1$ and $eq2$,{m,s})	$s=2.0130$ and $m=11.0019$

d.i. $\hat{P} = \dfrac{X}{n}, \quad n = 50, \quad \hat{P} > 0.3 \Rightarrow X > 50 \times 0.3 = 15$ M1

$X \stackrel{d}{=} Bi\left(n = 50, p = \dfrac{1}{3}\right)$

binomCdf$\left(50,\frac{1}{3},16,50\right)$	0.6310

$\Pr(X > 15) = \Pr(X \geq 16) = 0.631$ A1

ii. $n = 30, \quad p = \dfrac{1}{3}, \quad 95\%, \quad z = 1.96$

$\dfrac{1}{3} \pm 1.96\sqrt{\dfrac{\frac{1}{3} \times \frac{2}{3}}{30}} = (0.165, 0.502)$ A1

$\frac{1}{3} - 1.96 \cdot \sqrt{\frac{\frac{1}{3} \cdot \frac{2}{3}}{30}}$	0.1646
$\frac{1}{3} + 1.96 \cdot \sqrt{\frac{\frac{1}{3} \cdot \frac{2}{3}}{30}}$	0.5020

zInterval_1Prop 10,30,0.95: *stat.results*

"Title"	"1−Prop z Interval"
"CLower"	0.1646
"CUpper"	0.5020
"p̂"	0.3333
"ME"	0.1687
"n"	30.0000

e. $\hat{Q} = \dfrac{X}{n} > \dfrac{1}{n}, \quad n = ? \quad \Rightarrow \; X > 1$

$X \overset{d}{=} Bi\left(n = ?, p = 0.3\right)$

$\Pr\left(X > 1\right) > 0.95$

$1 - \left[\Pr\left(X = 0\right) + \Pr\left(X = 1\right)\right] > 0.95$

$\Pr\left(X = 0\right) + \Pr\left(X = 1\right) < 0.05$ M1

$0.7^{n} + n \times 0.7^{n-} \times 0.3 = 0.05$ solving gives $n = 138$

so we need $n = 14$ A1

$$\text{solve}\left(\left(0.7\right)^{n} + n \cdot \left(0.7\right)^{n-1} \cdot 0.3 = 0.05, n\right) \mid n > 0$$
$$n = 13.8245$$

f.i. $\displaystyle\int_{4}^{5}\left(1 - \frac{k}{x^2}\right)dx + \int_{5}^{6}\left(1 - \frac{k}{\left(x-10\right)^2}\right)dx = 1$

$\left[x + \dfrac{k}{x}\right]_{4}^{5} + \left[x + \dfrac{k}{x-10}\right]_{5}^{6} = 1$ M1

$5 + \dfrac{k}{5} - 4 - \dfrac{k}{4} + 6 - \dfrac{k}{4} - 5 + \dfrac{k}{5} = 1$

A1

$2k\left(\dfrac{1}{4} - \dfrac{1}{5}\right) = \dfrac{k}{10} = 1 \quad \Rightarrow \quad k = 10$

ii. $E\left(X\right) = 5, \quad E\left(X^2\right) = 25.2954$

$\text{Var}\left(X\right) = E\left(X^2\right) - \left(E\left(X\right)\right)^2 = 0.2954, \quad \text{sd}\left(X\right) = \sqrt{0.2954} = 0.5435$ A1

$\Pr\left(5 - 0.5435 < X < 5 + 0.5435\right) = \displaystyle\int_{4.4565}^{5.5435} B\left(x\right)dx = 0.5992$ A1

$$\text{Define } b(x) = \begin{cases} 1 - \dfrac{10}{x^2}, & 4 \le x \le 5 \\ 1 - \dfrac{10}{(x-10)^2}, & 5 \le x < 6 \end{cases} \qquad \textit{Done}$$

$$\int_{4}^{6}\left(x \cdot b(x)\right)dx \qquad 5.0000$$

$$\int_{4}^{6}\left(x^2 \cdot b(x)\right)dx \qquad 25.2954$$

$$sd := \sqrt{25.295376929508 - 25} \qquad 0.5435$$

$$\int_{5-sd}^{5+sd} b(x)\, dx \qquad 0.5992$$

Question 4

a. $f:[0,8]\to R,\quad f(x)=4+4\cos\left(\dfrac{\pi x}{8}\right)$

Define $f(x)=\left\{4\cdot\cos\left(\frac{\pi\cdot x}{8}\right)+4, 0\le x\le 8\right.$	Done
Define $s(x)=\sqrt{x^2+(f(x))^2}$	Done
solve$\left(\frac{d}{dx}(s(x))=0,x\right)\mid 0<x<8$	$x=4.6221$
$f(4.6220731601924)$	3.033
$s(4.6220731601924)$	5.528

Let $s(x)=\sqrt{x^2+\left(f(x)\right)^2}$ M1

$$s(x)=\sqrt{x^2+16+32\cos\left(\frac{\pi x}{8}\right)+16\cos^2\left(\frac{\pi x}{8}\right)}$$

solving $\dfrac{ds}{dx}=0$ gives $x=4622$

$f(4.622)=3.033$ so $(4.622,3.033)$ and $s_{\min}=s(4.622)=5.528$ A1

b. the gradient function $m(x)=f\ (x)=-\dfrac{\pi}{2}\sin\left(\dfrac{\pi x}{8}\right)$

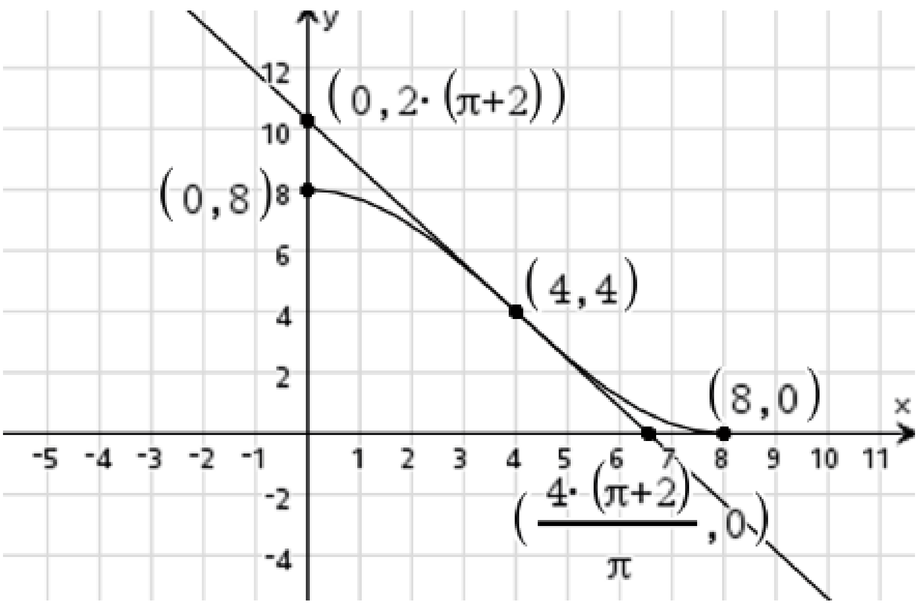
the derivative of the gradient function

$m'(x)=f''(x)=-\dfrac{\pi^2}{16}\cos\left(\dfrac{\pi x}{8}\right)=0$ for inflexion points, M1

$\dfrac{\pi x}{8}=\dfrac{\pi}{2},\quad x=4\quad f(4)=4+4\cos\left(\dfrac{\pi}{2}\right)=4$ inflexion point at $(4,4)$

c. $f\ (4)=-\dfrac{\pi}{2}\sin\left(\dfrac{\pi}{2}\right)=-\dfrac{\pi}{2}\quad (4,4)$, the equation of the tangent at the point of inflexion is $y-4=-\dfrac{\pi}{2}(x-4),\quad y=-\dfrac{\pi x}{2}+2(\pi+2)$ M1

d. G2



e. The tangent to the curve at the point crosses the x-axis at the point $\left(\frac{4(\pi+2)}{\pi},0\right)$

the required area is three regions $A = A_1 + A_2 + A_3$,

$$A_1 = \int_0^4 \left(\left(-\frac{\pi x}{2}+2(\pi+2)\right)-\left(4+4\cos\left(\frac{\pi x}{8}\right)\right)\right)dx$$

$$A_2 = \int_4^{\frac{4\pi+2}{\pi}} \left(\left(4+4\cos\left(\frac{\pi x}{8}\right)\right)-\left(-\frac{\pi x}{2}+2(\pi+2)\right)\right)dx \;,\; A_3 = \int_{\frac{4\pi+2}{\pi}}^{8} \left(4+4\cos\left(\frac{\pi x}{8}\right)\right)dx \quad \text{A1}$$

$A = 3.102$ A1

Define $t(x)=\text{tangentLine}(f(x),x,4)$ Done

$t(x)$ $\quad 2\cdot(\pi+2)-\frac{\pi\cdot x}{2}$

$\text{solve}\left(2\cdot(\pi+2)-\frac{\pi\cdot x}{2}=0,x\right)$ $\quad x=\frac{4\cdot(\pi+2)}{\pi}$

$xa:=\frac{4\cdot(\pi+2)}{\pi}$ $\quad \frac{4\cdot(\pi+2)}{\pi}$

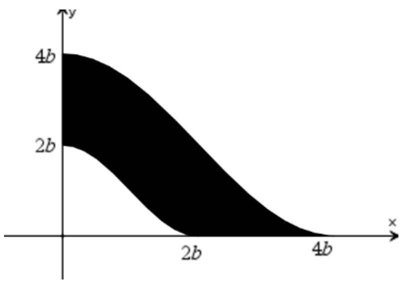
$a1:=\int_0^4 (t(x)-f(x))\,dx$ $\quad 4\cdot\pi-\frac{32}{\pi}$

$a2:=\int_4^{xa} (f(x)-t(x))\,dx$ $\quad \frac{32\cdot\cos(1)}{\pi}-\frac{16}{\pi}$

$a3:=\int_{xa}^{8} f(x)\,dx$ $\quad \frac{-16\cdot(2\cdot\cos(1)-\pi+2)}{\pi}$

$a1+a2+a3$ $\quad 3.1016$

f. the required area is two regions



Define $a(b)=\int_0^{2\cdot b} (g1(x)-g2(x))\,dx+\int_{2\cdot b}^{4\cdot b} g1(x)\,dx$ Done

$a(b)$ $\quad 6\cdot b^2$

$$A_1 = \int_0^{2b}\left(g_1(x)-g_2(x)\right)dx$$

$$A_2 = \int_{2b}^{4b} g_1(x)\,dx$$

$A = A_1 + A_2 = 6b^2$ A1

g.i. $g'(x) = \begin{cases} -1 & \text{for } x<0 \\ -\frac{\pi}{2}\sin\left(\frac{\pi x}{2a}\right) & \text{for } 0<x<2a \\ 1 & \text{for } x>2a \end{cases}$ A1

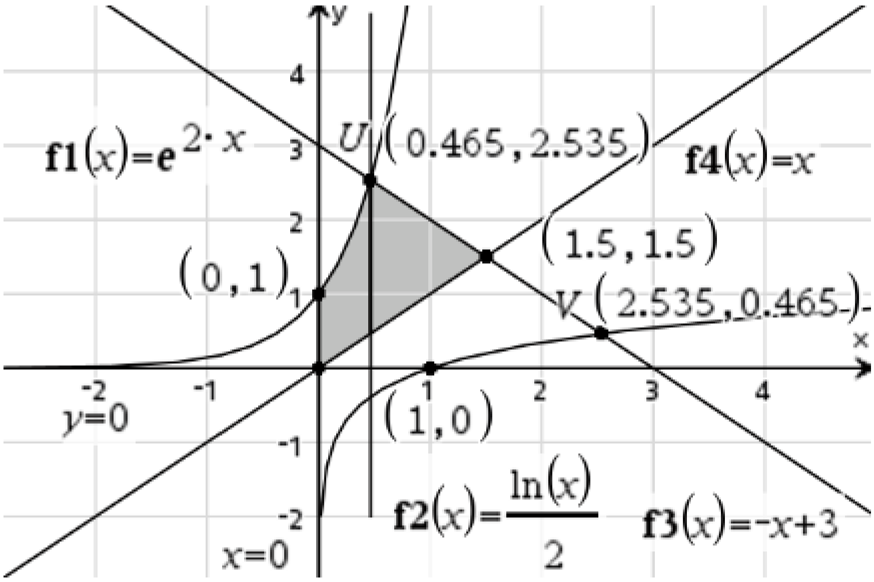
ii. strictly increasing for $(2a,\infty)$ A1

Question 5

a.i. $f : R \to R,\ f(x) = e^{2x}$

$f^{-1} : (0, \infty) \to R,\ f^{-1}(x) = \frac{1}{2}\log_e(x)$ A1

G1



ii. solving $e^{2x} = -x + 3$ gives $x = 0465$,
solving $\frac{1}{2}\log_e(x) = -x + 3$ gives
$x = 2535$
$U(0.465, 2.535)$, $V(2.535, 0.465)$

Define $f1(x)=e^{2\cdot x}$	*Done*
solve$(f1(y)=x,y)$	$y=\frac{\ln(x)}{2}$ and $x>0$
Define $f2(x)=\frac{\ln(x)}{2}$	*Done*
⚠ solve$(f1(x)=3-x,x)$	$x=0.4651$

A1

xu:=0.46508086797604	0.4651
⚠ solve$(f2(x)=3-x,x)$	$x=2.5349$
xv:=2.534919132024	2.5349

iii. The line $y = x$ intersects the line at the point $W(1.5 \quad)$

$A = 2\left[\int_0^{xu}\left(f(x) - x\right)dx + \int_{xu}^{1.5}\left((3 - x) - x\right)dx\right]$ other methods are valid A1

$A = 2\left[\int_0^{0.465}\left(e^{2x} - x\right)dx + \int_{2.535}^{1.5}(3 - 2x)\,dx\right]$

$2\cdot\left(\int_0^{xu}(f1(x)-x)\,dx + \int_{xu}^{1.5}(3-2\cdot x)\,dx\right)$ 3.4607

$A = 3.4607$ A1

b.i. $g:(0,\infty)\rightarrow R,\ \ g(x)=3\log_e(x)$

$g^{-1}:R\rightarrow R,\ \ g^{-1}(x)=e^{\frac{x}{3}}$

Define $f5(x)=3\cdot\ln(x)$ *Done*

solve$(f5(y)=x,y)$ $y=e^{\frac{x}{3}}$ A1

ii. solving $3\log_e(x)=e^{\frac{x}{3}}$ gives

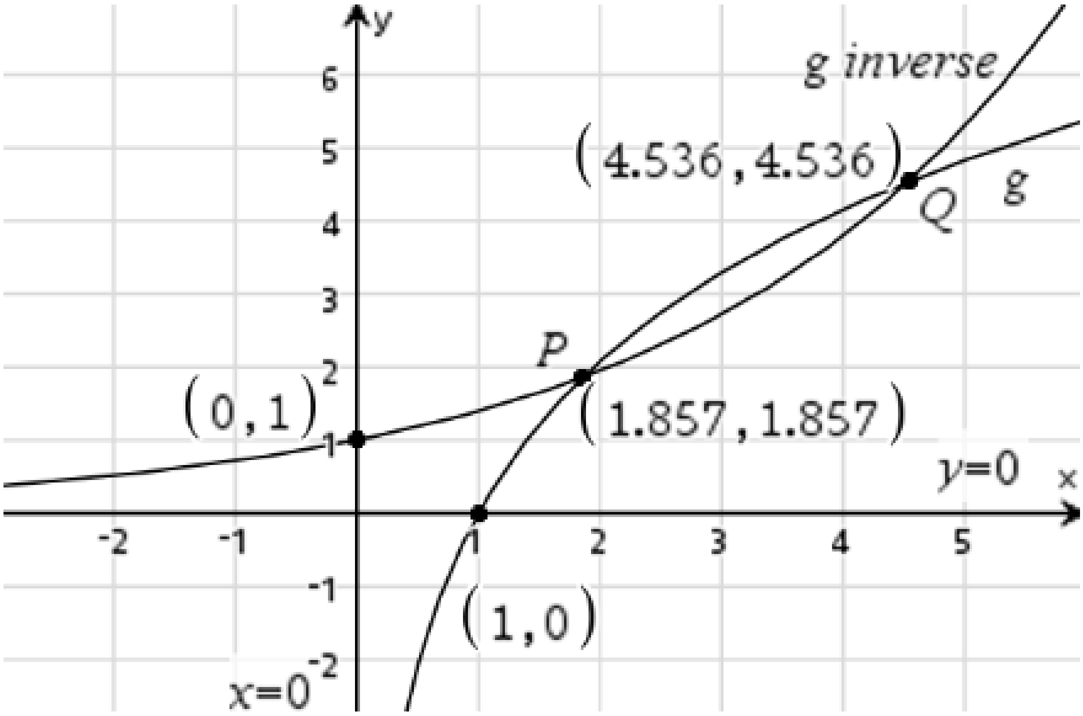
$x=1.857, 4.536$,

$p=1.857,\ \ q=4.536$

Define $f6(x)=e^{\frac{x}{3}}$ *Done*

solve$(f5(x)=f6(x),x)$ $x=1.8572$ or $x=4.5364$ A1

iii. G1

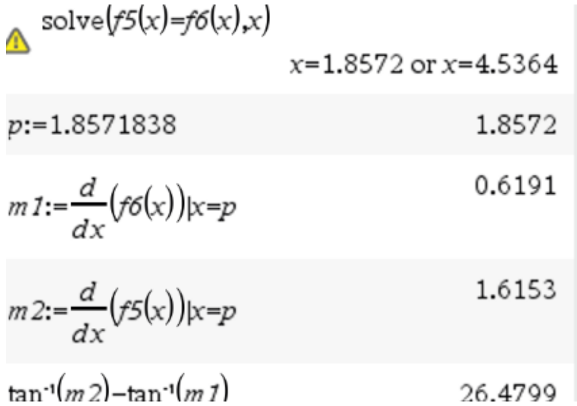


iv. $m_1 = g^{-}(p) = 0.6191 = \tan(\theta_1)$

$m_2 = g(p) = 1.6153 = \tan(\theta_2)$

$\theta_2 - \theta_1 = \tan^{-1}(m_2) - \tan^{-1}(m_1)$

$= 248\ ^{0}$



A1

A1

c. $h(x) = e^{kx}$, $h^{-}(x) = \frac{1}{k}\ln(x)$

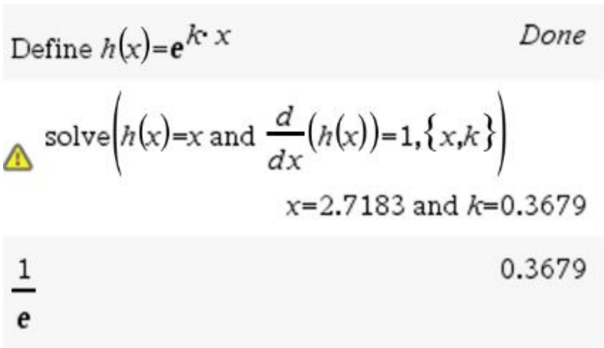
this touches the line $y = x$

when $h(x) = e^{kx} = x$ and

$h(x) = ke^{kx} = 1$

solving gives $x = e \quad k = \frac{1}{e}$

when $k = \frac{1}{e} \quad h^{-}(e) = h(e) = e$



M1

A2

function h and h^{-}	values of k
do not intersect.	$k > \frac{1}{e}$
have only one point of intersection.	$-e \le k < 0$ or $k = \frac{1}{e}$
have two points of intersection.	$0 < k < \frac{1}{e}$
have three points of intersection	$k < -e$

END OF SECTION B SUGGESTED ANSWERS

End of detailed answers for the
2023 Kilbaha VCE Mathematical Methods Trial Examination 2

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