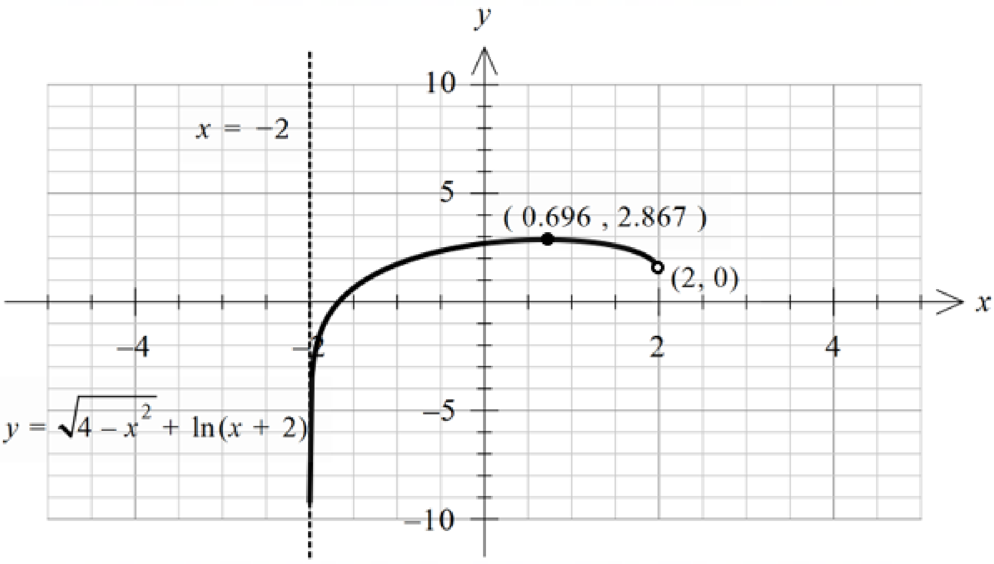
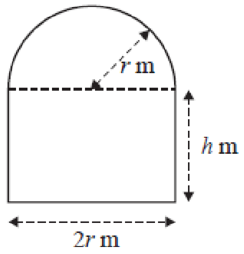
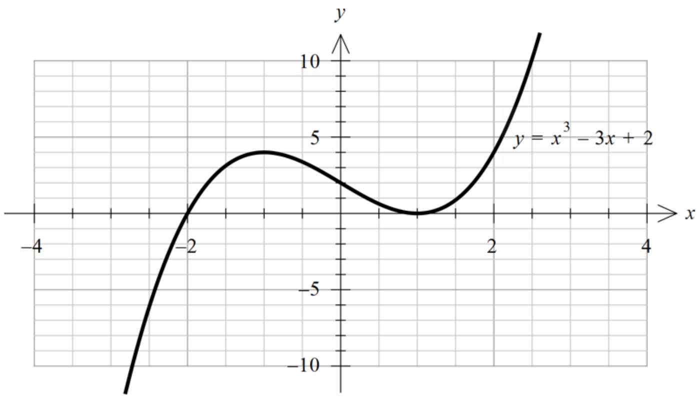
# 2022 VCE Mathematical Methods 2 (NHT) external assessment report

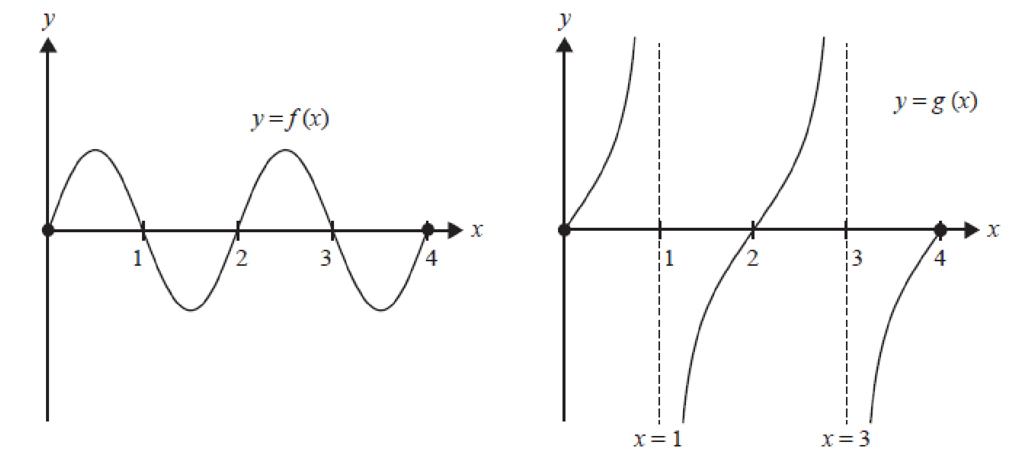
## Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

## Section A – Multiple-choice questions

| Question | Correct answer | Comments |
|---|---|---|
| 1 | B | |
| 2 | E | |
| 3 | E | |
| 4 | B | |
| 5 | C | |
| 6 | D | |
| 7 | C | |
| 8 | D |   The range is $\left(-\infty, 2.866...\right]$. So it is contained within the interval $\left(-\infty, 2.9\right)$. |
| 9 | A | |
| 10 | D | |

| Question | Correct answer | Comments |
|---|---|---|
| 11 | E |  <br>$A=2rh+\frac{\pi r^2}{2}$, $P=\pi r+2h+2r=8$, $h=4-\frac{\pi r}{2}-r$, $A=8r-2r^2-\frac{\pi r^2}{2}$ |
| 12 | B | |
| 13 | D | |
| 14 | D | |
| 15 | E | |
| 16 | A | |
| 17 | C | |
| 18 | C | $x^2+(y-1)^2=1$, $y=-\sqrt{1-x^2}+1$, $\tan(135°)=-1$, solve $\frac{d}{dx}\left(-\sqrt{1-x^2}+1\right)=-1$ at $x=k$, $k=-\frac{\sqrt{2}}{2}$ |
| 19 | A |  <br>$x^3-px+2=0$ has three distinct real solutions for $p\in(3,\infty)$. When $x=3$, there are two distinct real solutions as shown. For values of $p$ greater than three, the $y$-coordinate of the local maximum turning point is positive and the $y$-coordinate of the local minimum turning point is negative, which means there will be three distinct real solutions. |

| Question | Correct answer | Comments |
|---|---|---|
| 20 | A | $h = f \times g$ has three $x$-intercepts on the interval $x \in [0,4]$. The $x$-intercepts are at $x = 0$, $x = 2$ and $x = 4$.   |

# Section B

## Question 1a.

$f'(x) = -\frac{6}{5}(x-2)^2$ or $f'(x) = -\frac{6}{5}x^2 + \frac{24x}{5} - \frac{24}{5}$

## Question 1b.

$\left(2, \frac{3}{5}\right)$

## Question 1ci.

$f'(x) = -\frac{6}{5}(x-2)^2 = -\frac{6}{5}$,

$(x-2)^2 = 1$,

$x - 2 = \pm 1$,

$x = 1$ or $x = 3$, since $x = 1$ is given, the other point $D$ is at $x = 3$.

## Question 1cii.

$y = \frac{19}{5} - \frac{6x}{5}$

## Question 1ciii.

Solving $f(x)=\frac{19}{5}-\frac{6x}{5}$

$\left(0,\frac{19}{5}\right)$ or $(0,0.38)$

## Question 1civ.

$129.8°$

## Question 1cv.

$$A_{Total}=\int_0^3\left(\frac{19}{5}-\frac{6x}{5}-f(x)\right)dx+\int_3^{\frac{19}{6}}\left(\frac{19}{5}-\frac{6x}{5}\right)dx-\int_3^{\frac{3^{\frac{1}{3}}\times2^{\frac{2}{3}}+4}{2}}f(x)dx=2.7015,$$

or

$$A_{Total}=\int_0^{\frac{3^{\frac{1}{3}}\times2^{\frac{2}{3}}+4}{2}}\left(\frac{19}{5}-\frac{6x}{5}-f(x)\right)dx+\int_{\frac{3^{\frac{1}{3}}\times2^{\frac{2}{3}}+4}{2}}^{\frac{19}{6}}\left(\frac{19}{5}-\frac{6x}{5}\right)dx=2.7015$$

## Question 2ai.

$x\in(-\infty,2]$

## Question 2aii.

$h_1(x)=\sqrt{2+x}$

## Question 2bi.

Dilation by a factor 2 from the *x*-axis, translation of 1 unit to the right, or

Dilation by a factor of $\frac{1}{4}$ from the *y*-axis, translation by $\frac{5}{2}$ units to the right.

## Question 2bii.

$a=1,\ b=2,\ c=1,\ d=0$ or $a=\frac{1}{4},\ b=1,\ c=\frac{5}{2},\ d=0$

## Question 2biii.

$x=\frac{5}{3}$

## Question 2c.

$h_3(x) = a(x-4)^2 + 6$,

$h_3(x) = -(x-4)^2 + 6$ or $h_3(x) = -x^2 + 8x - 10$

## Question 2d.

$x \in [0, \infty)$, $h^{-1}(x) = -x^2 + 2$

## Question 2e.

Method 1:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ performs the inverse transformation to get $y = 2 - x^2$ for $x \geq 0$, adding $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ translates the graph to $y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give $h_3$.

Method 2:

Let $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $x' = y + 4,\ y = x' - 4$, $y' = x + 4,\ x = y' - 4$, substitute into $y = \sqrt{2 - x}$, $y = -(x-4)^2 + 6$ for $x \geq 4$, although the rule is correct, because the maximal domain of the image is $x \geq 4$, the transformation cannot give $h_3$.

## Question 3a.

$A = \int_0^1 (q(x) - p(x))\,dx$ or $2\int_0^1 (q(x) - 2)\,dx$,

$A = \dfrac{4(e-1)}{e}$ or $A = -4e^{-1} + 4$

## Question 3b.

*p* is strictly increasing and *q* is strictly decreasing.

## Question 3c.

$p^{-1}(x) = \log_e\left(\dfrac{2}{2-x}\right)$ or $p^{-1}(x) = \log_e\left(\dfrac{-2}{x-2}\right)$ or $p^{-1}(x) = \log_e\left(\dfrac{-1}{x-2}\right) + \log_e(2)$

$q^{-1}(x) = \log_e\left(\dfrac{2}{x-2}\right)$ or $q^{-1}(x) = -\log_e(x-2) + \log_e(2)$

## Question 3di.

(1.805, 2.329)

## Question 3dii.

$y = -x + 4.134$

## Question 3ei.

$$r(x) = p(x)q(x)$$
$$= 2\left(1-e^{-x}\right)\times 2\left(1+e^{-x}\right)$$
$$= 4\left(1^2 - \left(e^{-x}\right)^2\right)$$
$$= 4\left(1-e^{-2x}\right)$$

## Question 3eii.

Domain $R$, Range $(-\infty, 4)$

## Question 3eiii.

$$r^{-1}(x) = \frac{1}{2}\log_e\left(\frac{4}{4-x}\right)$$
$$r^{-1}(x) = \frac{1}{2}\log_e\left(\frac{2^2}{2^2-x}\right)$$

## Question 3fi.

$y = x$

## Question 3fii.

(0, 0), (3.999, 3.999)

## Question 4ai.

$$\int_0^6 xf(x)\,dx$$
$$= \frac{18}{5}$$

## Question 4aii.

$\frac{6}{5}$ or 1.2

## Question 4bi.

3.6856

## Question 4bii.

$$\Pr(X > 2 \mid X < 3.685634...)$$

$$= \frac{\Pr(2 < X < 3.685634)}{\Pr(X < 3.685634...)}$$

$$= \frac{\int_{2}^{3.685634...} f(x)\,dx}{0.5}$$

$$= 0.78$$

## Question 4c.

0.5319

## Question 4d.

$$\Pr(J \geq 15.1) = 0.95, \ \ -1.64485... = \frac{15.1 - \mu}{\sigma},$$

$$\Pr(J > 23.9) = 0.10, \ 1.28155... = \frac{23.9 - \mu}{\sigma},$$

$$\mu = 20, \ \sigma = 3$$

## Question 4ei.

11

## Question 4eii.

0.0773

## Question 4f.

35

## Question 4gi.

0.0624

## Question 4gii.

(0.3465, 0.5910)

## Question 4giii.

For approximately 95% of all randomly selected samples from the population, the confidence interval calculated in this manner will capture the population proportion of customers who said the doughnuts are delicious.

## Question 5a.

$b_2(t) = -20e^{-\frac{t}{5}} + 20$

## Question 5b.

$\frac{1}{10-0}\int_0^{10} h(t)dt$

= 20.01

## Question 5ci.

$h'(t) = -4e^{-\frac{t}{5}}\left(\cos(2\pi t) + 10\pi\sin(2\pi t)\right)$

## Question 5cii.

Max of $h'(t)$, (0.7, 18.9)

## Question 5d.

Method 1:

$(40-1.894...)+(36.382...-1.894...)+(36.382...-5.176...)+\ (33.413...-5.176...)+(33.413...-33.406...)$ =
132

Method 2:

$\int_0^2 \left|h'(t)\right| dt$ = 132