Neap

Trial Examination 2022

VCE Mathematical Methods Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	**D**	E
2	**A**	B	C	D	E
3	A	**B**	C	D	E
4	A	B	C	**D**	E
5	A	B	C	**D**	E
6	A	B	**C**	D	E
7	A	**B**	C	D	E
8	A	B	C	D	**E**
9	A	B	C	D	**E**
10	A	B	**C**	D	E
11	**A**	B	C	D	E
12	A	B	C	**D**	E
13	A	B	C	D	**E**
14	A	B	C	**D**	E
15	A	B	**C**	D	E
16	A	**B**	C	D	E
17	**A**	B	C	D	E
18	A	**B**	C	D	E
19	A	B	C	**D**	E
20	A	B	**C**	D	E

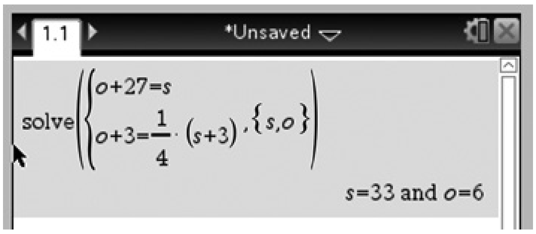
Question 1 D

Let S = Sam's age and O = Oma's age.

equation 1: $O+27=S$

equation 2: $O+3=\frac{1}{4}(S+3)$

Solving the simultaneous equations using a graphics calculator gives:



Oma is 6 years old and Sam is 33 years old.

Question 2 A

equation 1: $y=4x-3$

Rearranging equation 2 gives:

$$y-ax+2x+b-1=0$$
$$y=ax-2x-b+1$$
$$y=(a-2)x-b+1$$

For an infinite number of solutions to exist, the lines must lie on top of each other. Therefore, as the equations use the form $y = mx + c$, the gradient and c-value must be the same.

$a-2=4$ and $-b+1=-3$

$a=6$ and $b=4$

Question 3 B

B is correct. The general form of a rectangular hyperbola is $y=\frac{p}{x-m}+n$, where $\frac{1}{p}$ is a dilation from the x-axis, m is a translation in the positive direction of the x-axis and n is a translation in the positive direction of the y-axis. The asymptotes are located at $x = m$ and $y = n$.

From the graph, the asymptotes are located at $x = b$ and $y = -c$. Therefore, the equation is $y=\frac{p}{x-b}-c$.

A and **D** are incorrect. These equations represent graphs that have horizontal asymptotes at $y = c$.

C is incorrect. This equation represents a graph with a vertical asymptote at $x = -b$.

E is incorrect. The equation represents a graph with a horizontal asymptote at $y = -a$.

Question 4 D

D is correct. There is a reflection in the x-axis, a translation of 2 units in the negative direction of the x-axis, and a translation of 3 units in the positive direction of the y-axis.

A and **B** are incorrect. There are no dilations from $f(x)$ to $h(x)$.

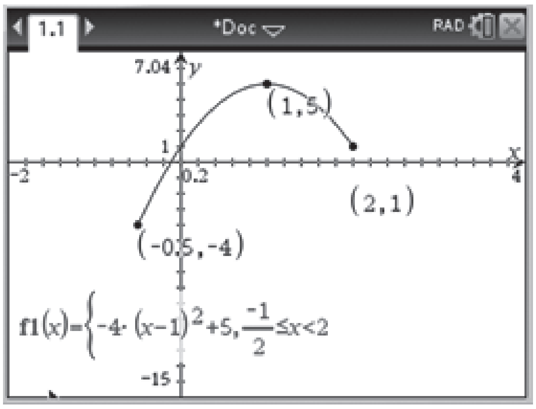
C is incorrect. $h(x)$ is reflected in the x-axis, not the y-axis.

E is incorrect. $h(x)$ is translated 2 units in the negative, not positive, x-axis direction.

Question 5 D

Method 1:

Entering $y = -4(x - 1)^2 + 5$ with the domain restriction into a graphics calculator gives:



From the graph, the starting point is at $\left(-\frac{1}{2}, -4\right)$ and this point is included (solid circle). The end point is at (2, 1) and this point is not included (open circle) as the domain is $\left[-\frac{1}{2}, 2\right)$.

Note: The calculator will show both end points as solid circles.

Method 2:

D is correct. $y = -4(x-1)^2 + 5$ has a turning point at (1, 5) and an endpoint of (2, 1). This should be an open circle, as the domain of the graph does not include this point.

A and **B** are incorrect. These graphs have solid circles at the endpoint (2, 1).

C and **E** are incorrect. These graphs have the turning point at (–1, 5), which is not possible as the turning point would be further left than the known starting point at $\left(-\frac{1}{2}, -4\right)$.

Question 6 C

Using the identity $\sin^2(x) + \cos^2(x) = 1$:

$$
\begin{aligned}
(0.02)^2 + \cos^2(a) &= 1 \\
\cos^2(a) &= 1 - (0.02)^2 \\
\cos(a) &= \pm\sqrt{1 - (0.02)^2}
\end{aligned}
$$

As $\sin(a)$ is positive in quadrants 1 and 2, $\cos(a)$ can be either positive (quadrant 1) or negative (quadrant 2).

Question 7 B

Let $y = f(x)$.

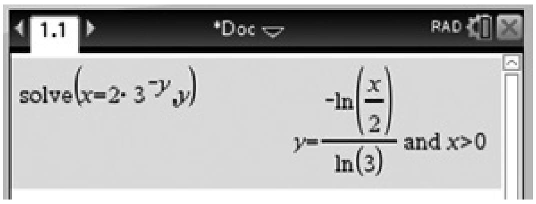
Swapping the x and y components gives:

$x = 2 \times 3^{-y}$

Rearranging the equation in terms of y gives:

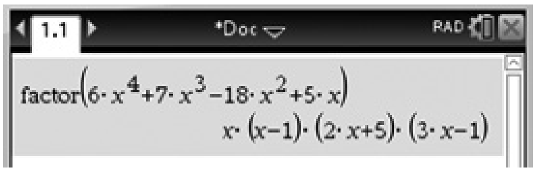
$$\begin{aligned} x &= 2 \times 3^{-y} \\ \frac{x}{2} &= 3^{-y} \\ \log_3\left(\frac{x}{2}\right) &= -y \\ y &= -\log_3\left(\frac{x}{2}\right) \end{aligned}$$

Note: Solving for y using a graphics calculator will not provide a response that matches any of the options. Hence, the question should be solved manually.



Question 8 E
Method 1:

Factorising $P(x)$ using a graphics calculator gives:



Hence, the rational solutions are $x = 0, 1, -\frac{5}{2}, \frac{1}{3}$. Therefore, $x = \frac{5}{2}$ is not a rational solution of $P(x)$.

Method 2:

To find the rational solutions of $P(x)$, find the factors of $P(x)$.

Since $P(x) = x(6x^3 + 7x^2 - 18x + 5)$, one solution is $x = 0$.

To find the other solutions, find the factors of $Q(x) = 6x^3 + 7x^2 - 18x + 5$.

Substituting $x = 1$ gives:

$$\begin{aligned} Q(1) &= 6(1)^3 + 7(1)^2 - 18(1) + 5 \\ &= 6 + 7 - 18 + 5 \\ &= 0 \end{aligned}$$

Hence, $x = 1$ is a solution.

Substituting $x = \frac{5}{2}$ gives:

$$\begin{aligned} Q\left(\frac{5}{2}\right) &= 6\left(\frac{5}{2}\right)^3 + 7\left(\frac{5}{2}\right)^2 - 18\left(\frac{5}{2}\right) + 5 \\ &\neq 0 \end{aligned}$$

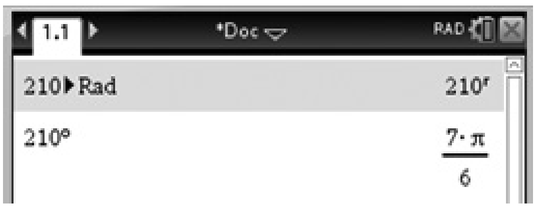
Hence, $x - \frac{5}{2}$ is not a factor of $Q(x)$ or $P(x)$. Therefore, $x = \frac{5}{2}$ is not a rational solution of $P(x)$.

Question 9 E
Method 1:

$$\begin{aligned} 360x &= 2\pi \times 210 \\ x &= \frac{2\pi \times 210}{360} \\ x &= \frac{7\pi}{6} \end{aligned}$$

Method 2:

Using a graphics calculator:



Question 10 C

Given $\int_{6}^{12} f(x)\cdot dx = 3$, $\int_{3}^{6} f(2x)\cdot dx$ has been dilated by a factor of $\frac{1}{2}$ from the y-axis. Hence,

$\int_{3}^{6} f(2x)\cdot dx = \frac{3}{2}$.

Question 11 A

$$\begin{aligned}\Pr(A \cup B) &= \text{union of events } A \text{ and } B \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}\end{aligned}$$

Question 12 D

${}^{7}C_{5}$ is not equivalent to $\frac{7!}{5!}$. All the other options give the correct number of combinations.

Question 13 E
Method 1:

The total number of combinations of 8 people from 13 people is ${}^{13}C_{8} = \frac{13!}{8!\times 5!} = 1287$.

From 5 men, 3 are chosen: ${}^{5}C_{3} = \frac{5!}{3!\times 2!} = 10$

From 8 women, 5 are chosen: ${}^{8}C_{5} = \frac{8!}{5!\times 3!} = 56$

Therefore, $\Pr(\text{3 men and 5 women chosen}) = \frac{10\times 56}{1287} = \frac{560}{1287}$.

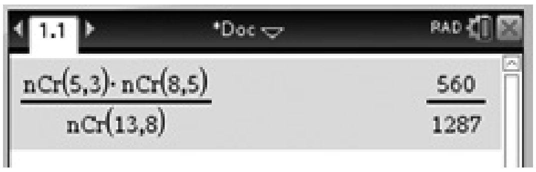
Method 2:

Using the graphics calculator:

The total number of combinations of 8 people from the 13 people is ${}^{13}C_{8}$.

From 5 men, 3 are chosen: ${}^{5}C_{3}$

From 8 women, 5 are chosen: ${}^{8}C_{5}$



Therefore, $\Pr(\text{3 men and 5 women chosen}) = \frac{560}{1287}$.

Question 14 D

Method 1:

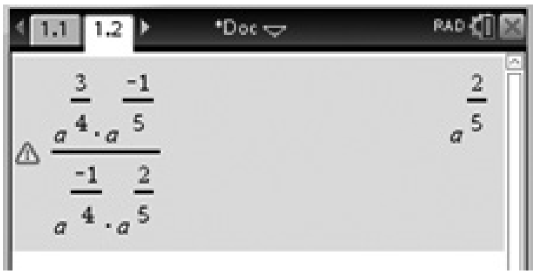
$$\frac{a^{\frac{3}{4}} \times a^{-\frac{1}{5}}}{a^{-\frac{1}{4}} \times a^{\frac{2}{5}}} = a^{\frac{3}{4} - \frac{1}{5} + \frac{1}{4} - \frac{2}{5}}$$

$$= a^{1 - \frac{3}{5}}$$

$$= a^{\frac{2}{5}}$$

Method 2:

Entering the expression into a graphics calculator:



Question 15 C

The period of the graph $y = a\tan(bx)$ is given by $P = \frac{\pi}{n}$ where $n = b$.

Hence the period is $\frac{\pi}{b}$.

Question 16 B

Method 1:

$$\frac{\log_a(32)}{\log_a(4)} = \frac{\log_a(2)^5}{\log_a(2)^2}$$

$$= \frac{5\log_a(2)}{2\log_a(2)}$$

$$= \frac{5}{2}$$

Method 2:

Entering the expression into a graphics calculator:

Question 17 A
Method 1:

For the average rate of change, find the gradient between the points where $x = 0$ and $x = \frac{1}{2}$.

$$\begin{aligned} f(0) &= 3\times 0^3 - 4\times 0^2 - 0 + 2 \\ &= 2 \end{aligned}$$

Hence, the coordinates are (0, 2).

$$\begin{aligned} f\left(\frac{1}{2}\right) &= 3\times\left(\frac{1}{2}\right)^3 - 4\times\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 2 \\ &= \frac{3}{8} - 1 + \frac{3}{2} \\ &= \frac{7}{8} \end{aligned}$$

Hence, the coordinates are $\left(\frac{1}{2}, \frac{7}{8}\right)$.

The average rate of change is:

$$\begin{aligned} \frac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0} &= \frac{\frac{7}{8} - 2}{\frac{1}{2} - 0} \\ &= \frac{-\frac{9}{8}}{\frac{1}{2}} \\ &= -\frac{9}{4} \end{aligned}$$

The instantaneous rate of change is given by finding the gradient function of the curve, then finding the gradient when $x = \frac{1}{2}$.

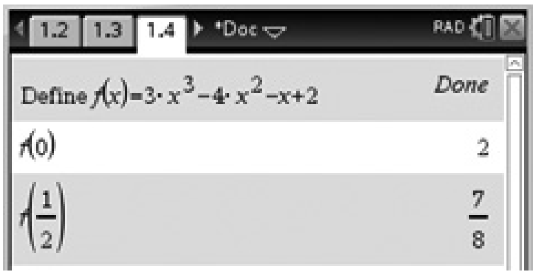
$f(x) = 3x^3 - 4x^2 - x + 2$

$f'(x) = 9x^2 - 8x - 1$

$$\begin{aligned} f'\left(\frac{1}{2}\right) &= 9\left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) - 1 \\ &= \frac{9}{4} - 4 - 1 \\ &= -\frac{11}{4} \end{aligned}$$

Method 2:

Using a graphics calculator:



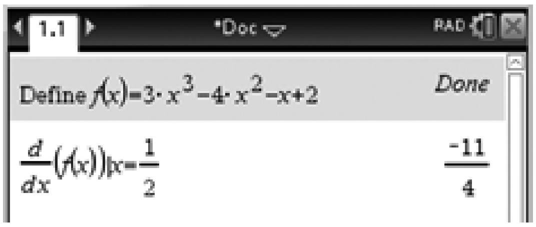
Find the gradient for the two points:

$$\frac{f\left(\frac{1}{2}\right)-f(0)}{\frac{1}{2}-0}=\frac{\frac{7}{8}-2}{\frac{1}{2}-0}$$

$$=\frac{-\frac{9}{8}}{\frac{1}{2}}$$

$$=-\frac{9}{4}$$

The instantaneous rate of change is given by finding $f'\left(\frac{1}{2}\right)$ using a graphics calculator:



Question 18 B

Let the original amount of uranium (U_0) be 100%.

Therefore, when half of the material has decayed, $U = 50\%$.

$t = 4.5$ billion years $= 4.5 \times 10^9$

Thus, the equation is $50\% = 100\% \times 2^{-k \times 4.5 \times 10^9}$ years.

Method 1:

$$\begin{aligned} 0.5 &= 1 \times 2^{-k \times 4.5 \times 10^9} \\ \log_2(0.5) &= -k \times 4.5 \times 10^9 \\ k &= -\frac{\log_2(0.5)}{4.5 \times 10^9} \\ &= 2.22 \times 10^{-10} \end{aligned}$$

Method 2:

For the equation $0.5 = 1 \times 2^{-k \times 4.5 \times 10^9}$, solve for k using the graphics calculator:

Hence, $k = 2.22 \times 10^{-10}$.

Question 19 D

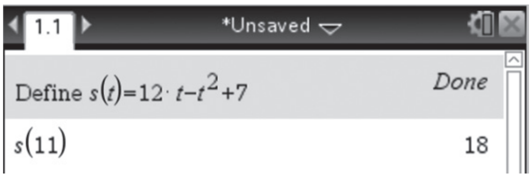
Method 1:

For $s(t) = 12t - t^2 + 7$, find $s(11)$.

$$\begin{aligned} s(11) &= 12 \times 11 - 11^2 + 7 \\ &= 18 \end{aligned}$$

Method 2:

Using a graphics calculator, define the function and then find $s(11)$.



Question 20 C

$\{x: h'(x) < 0\}$ is the set where the gradient is negative. From the graph, this occurs for $x \in (-\infty, -4)$.

SECTION B

Question 1 (15 marks)

a. For the midpoint of two points:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{\frac{3}{2}-2}{2}, \frac{5-\frac{5}{2}}{2}\right)$$

$$= \left(-\frac{1}{4}, \frac{5}{4}\right) \qquad \text{A1}$$

Using a graphics calculator:

b. The distance between two points is given by the equation:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

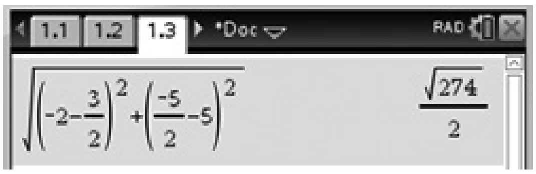
$$= \sqrt{\left(-2-\frac{3}{2}\right)^2+\left(-\frac{5}{2}-5\right)^2} \qquad \text{M1}$$

$$= \sqrt{\left(-\frac{7}{2}\right)^2+\left(-\frac{15}{2}\right)^2}$$

$$= \sqrt{\frac{274}{4}}$$

$$= \frac{\sqrt{274}}{2} \qquad \text{A1}$$

Using a graphics calculator:



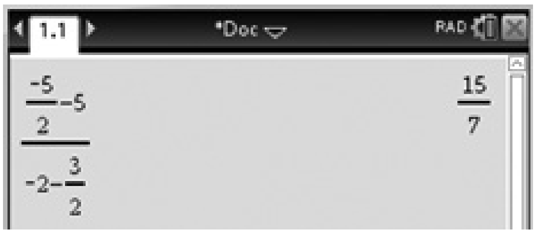
c. The gradient of the line segment that passes through these two points is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-\frac{5}{2} - 5}{-2 - \frac{3}{2}}$$

$$= \frac{15}{7}$$ M1

Using a graphics calculator:



Using the equation $y - y_1 = m(x - x_1)$, substitute one point into the equation.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{15}{7}\left(x - \frac{3}{2}\right)$$ M1

$$y = \frac{15}{7}x - \frac{45}{14} + 5$$

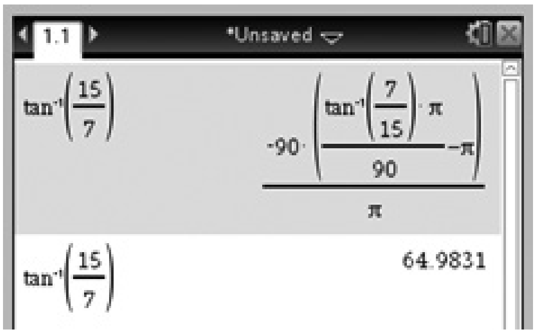
$$y = \frac{15}{7}x + \frac{25}{14}$$ M1

$$= f(x)$$

d. The gradient of $f(x)$ is $\frac{15}{7}$.

$$m = \tan(\theta)$$
$$\frac{15}{7} = \tan(\theta)$$
$$\theta = \tan^{-1}\left(\frac{15}{7}\right)$$
$$= 65°$$ A1

Using a graphics calculator:



e. A translation of $\frac{25}{14}$ in the negative direction of the y-axis moves the graph of $f(x)$ down, so the image is $f(x) = \frac{15}{7}x$. A dilation of factor $\frac{1}{7}$ from the x-axis takes the graph to the image $g(x) = 15x$. A1

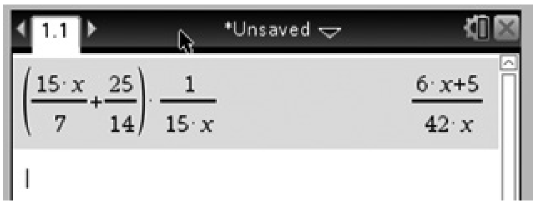
f. **i.** Substituting $f(x)$ and $g(x)$ into $h(x)$ gives:

$$h(x)=f(x)\times\frac{1}{g(x)} \qquad \text{M1}$$

$$=\left(\frac{15}{7}x+\frac{25}{14}\right)\times\frac{1}{(15x)}$$

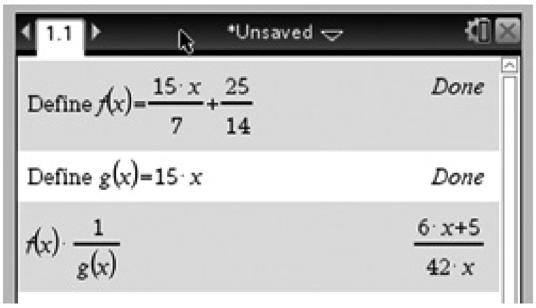
$$=\frac{5}{42x}+\frac{1}{7} \qquad \text{M1}$$

Using a graphics calculator:



Note: Accept answer in either form.

Alternatively, use the graphics calculator to define $f(x)$ and $g(x)$.



ii. $$h(1)=\frac{1}{7}+\frac{5}{42\times 1}$$

$$=\frac{11}{42}$$

$$=0.262 \qquad \text{A1}$$

Using a graphics calculator:

iii.

$$h(x) = 0$$

$$\frac{1}{7} + \frac{5}{42x} = 0$$

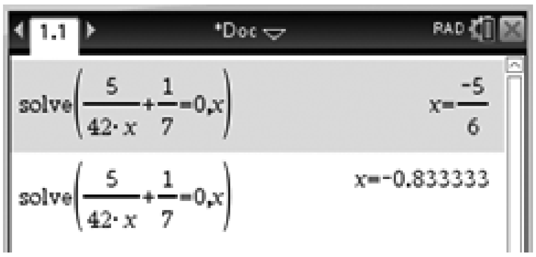
$$42x = -35$$

$$x = -\frac{35}{42}$$

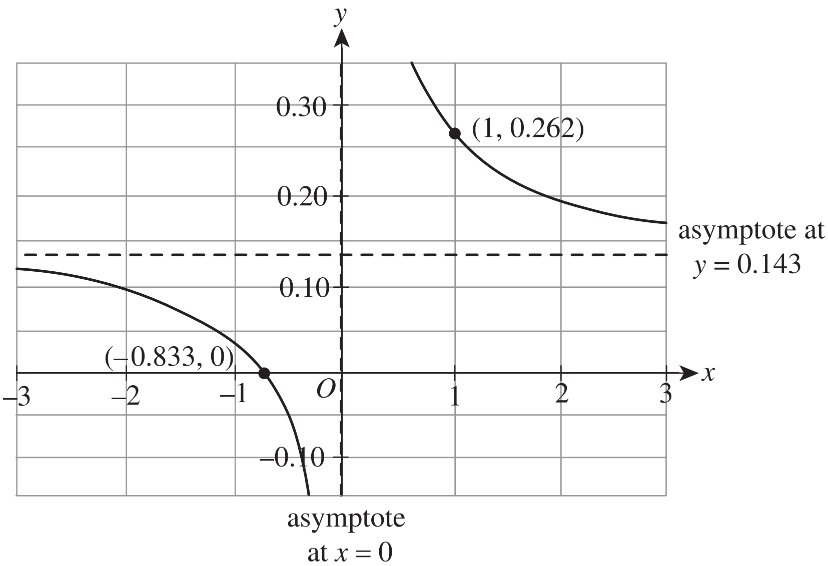
$$= -\frac{5}{6}$$

$$= -0.833$$ A1

Using a graphics calculator, solve for x.



iv.



asymptotes labelled A1

points from Question 1f.ii. and iii. labelled A1

Note: Consequential marks can be awarded for this component only.

correct shape of the graph and position of the two points being relatively accurate A1

Note: Deduct a maximum of 1 mark if decimals are not given to three decimal places.

Question 2 (14 marks)

a. **i.**

	C	C'		
L	**0.05**	**0.43**	**0.48**	A1
L'	**0.30**	**0.22**	**0.52**	A1
	0.35	**0.65**	1	A1

ii. $\Pr(C \cap L) = 0.05$ (or 5%) A1

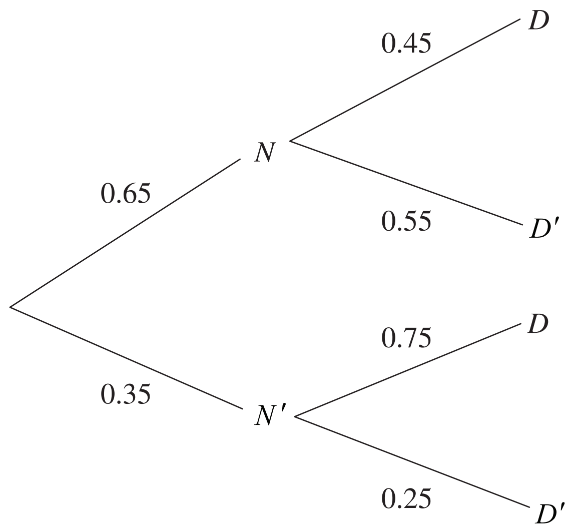
*Note: Consequential on answer to **Question 2a.i.***

iii.
$$\Pr(C \mid L') = \frac{\Pr(C \cap L')}{\Pr(L')}$$
$$= \frac{0.30}{0.52} \quad \text{M1}$$
$$= \frac{15}{26} \quad \text{A1}$$

*Note: Accept 0.58 or 58%. Consequential on answer to **Question 2a.i.***

iv.
$$22\% \text{ of } 267 = \frac{22}{100} \times 267$$
$$= 58.74$$
$$= 58 \text{ students} \quad \text{A1}$$

b. **i.**



correct first stem of the tree diagram A1
correct top half of the second stem of the tree diagram A1
correct lower half of the second stem of the tree diagram A1

ii.
$$\Pr(D) = \Pr(N \cap D) + \Pr(N' \cap D) \quad \text{M1}$$
$$= 0.65 \times 0.45 + 0.35 \times 0.75$$
$$= 0.2925 + 0.2625$$
$$= 0.555$$
$$= 0.56 \quad \text{A1}$$

*Note: Consequential on answer to **Question 2b.i.***

iii. $\Pr(N' \mid D) = \dfrac{\Pr(N' \cap D)}{\Pr(D)}$ M1

$= \dfrac{0.35 \times 0.75}{= 0.65 \times 0.45 + 0.35 \times 0.75}$

$= \dfrac{0.2625}{0.555}$

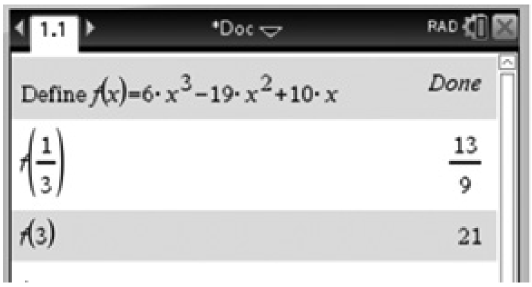
$= 0.47$ A1

Note: Consequential on answer to ***Questions 2b.i.*** *and* ***2b.ii.***

Question 3 (17 marks)

a. **Method 1:**

Defining $f(x)$ on a graphics calculator and finding $f\left(\dfrac{1}{3}\right)$ and $f(3)$:



Hence:

$f\left(\dfrac{1}{3}\right) = \dfrac{13}{9}$ A1

$f(3) = 21$ A1

Method 2:

$f\left(\dfrac{1}{3}\right) = 6\left(\dfrac{1}{3}\right)^3 - 19\left(\dfrac{1}{3}\right)^2 + 10\left(\dfrac{1}{3}\right)$

$= \dfrac{6}{27} - \dfrac{19}{9} + \dfrac{10}{3}$

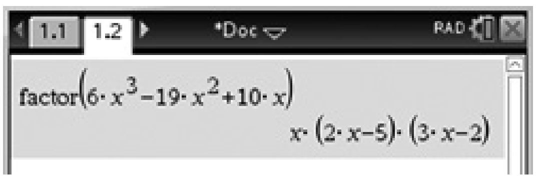
$= \dfrac{13}{9}$ A1

$f(3) = 6(3)^3 - 19(3)^2 + 10(3)$

$= 6 \times 27 - 19 \times 9 + 10 \times 3$

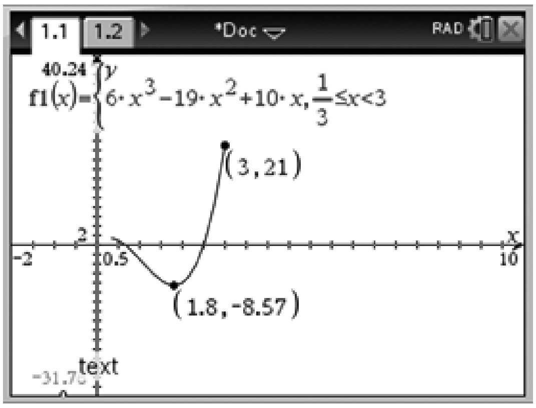
$= 21$ A1

b. The factors of $f(x)$ can be found using a graphics calculator.



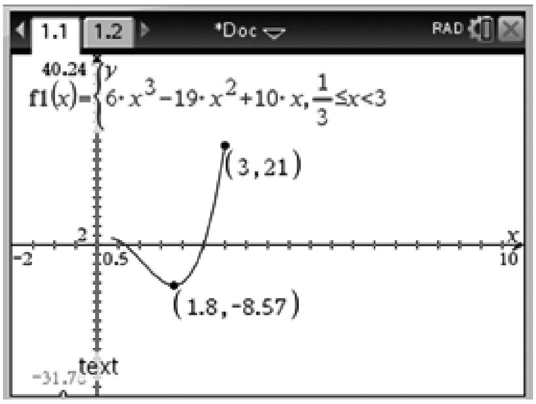
Hence, the factors are x, $2x - 5$ and $3x - 2$. A1

c. The minimum value of $f(x)$ can be found by sketching the graph on a graphics calculator, including the restricted domain and using the minimum functionality to find the minimum point.



Hence, the minimum value is –8.57. A1

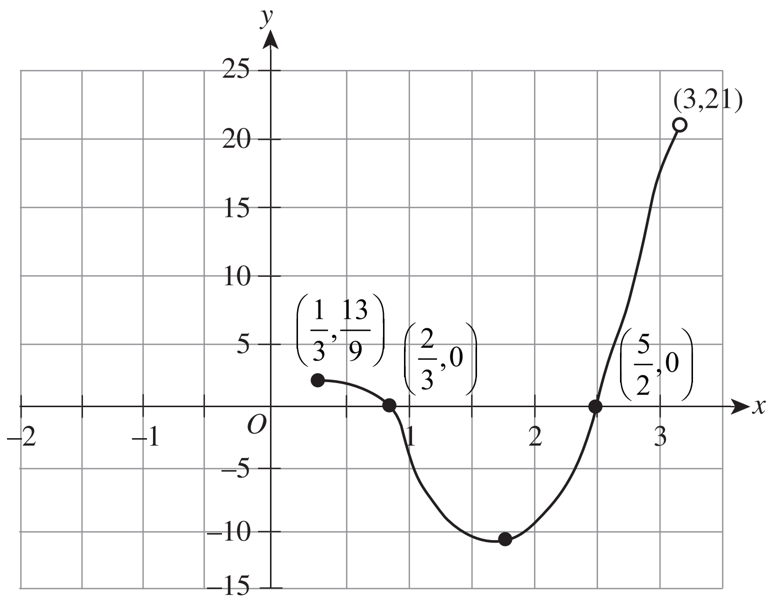
d. The range of $f(x)$ can be found by sketching the graph on a graphics calculator, including the restricted domain and using the maximum functionality to find the maximum point.



From the graph, the maximum point occurs at the coordinate (3, 21). Using the answer from part c., the range is $y \in [-8.57, 21)$. A1

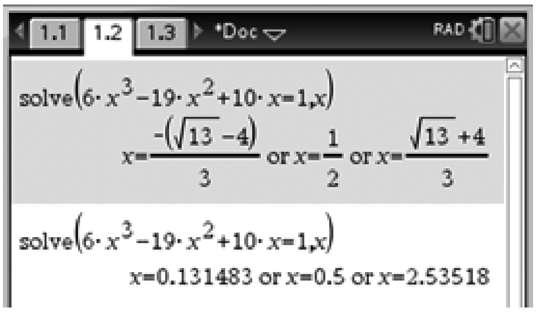
Note: Consequential on answer to ***Question 3c.***

e.



correct endpoints (labels and shading of points) A1
correct x-intercepts A1
correct shape of the graph A1
Note: The point at (1.8, –8.57) does not need to be labelled.

f. Solve $f(x) = 1$ using a graphics calculator:

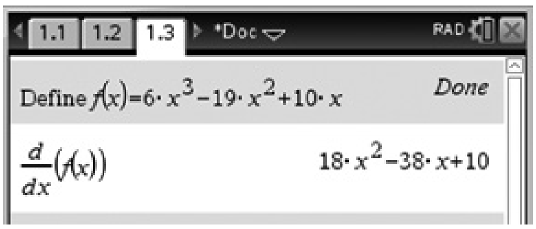


Hence, $x = 0.13148$, 0.5 or 2.53518 A1

However, $x = 0.13148$ is outside of the domain; therefore, it cannot be a solution. M1

By examining the graph and using the solution $x = 0.5$ or $x = 2.535$, $f(x) > 1$ for $x \in [0.33, 0.50] \cup [2.54, 3.00)$. A1

g. **i.**



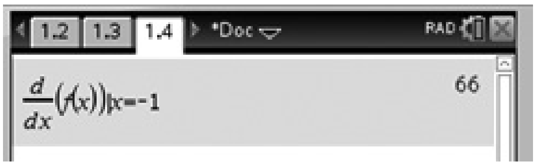
Hence, $f'(x) = 18x^2 - 38x + 10$. A1

ii. **Method 1:**

$$f'(-1) = 18(-1)^2 - 38(-1) + 10$$
$$= 18 + 38 + 10$$
$$= 66$$ A1

Method 2:

Using a graphics calculator gives:



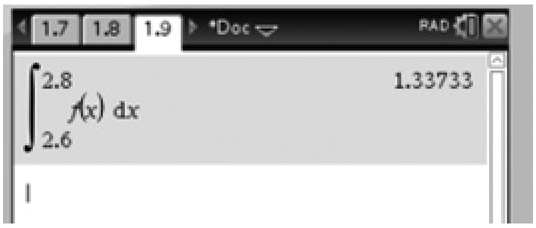
$f'(-1) = 66$ A1

iii. $f'(x) = 18x^2 - 38x + 10$

$f''(x) = 36x - 38$ A1

Note: Consequential on answer to ***Question 3g.ii.***

h. Using a graphics calculator:



Hence, $\int_{2.6}^{2.8} f(x) \cdot dx = 1.34$. A1

i. Examining the components of the hybrid function $h(x)$:

The cubic function sketched in part e. has an endpoint at (3, 21) that is non-inclusive. For $x \geq 3$, the graph is half a parabola. It starts at and includes the turning point of (3, 21). Hence, the hybrid function is continuous at $x = 3$. A1

For $-3 < x < \frac{1}{3}$, the graph is a straight line that starts at (–3, –4) and goes through the point (0, –1) to the point $\left(\frac{1}{3}, -\frac{2}{3}\right)$ (non-inclusive). The cubic function sketched in part e. starts at the point $\left(\frac{1}{3}, \frac{13}{9}\right)$. Hence, the hybrid function has only one discontinuity at $x = \frac{1}{3}$. A1

Question 4 (14 marks)

a. **i.** Since the lowest position above the ground is at 0.5 m (50 cm) and the highest position is at 1.0 m (100 cm), the midpoint of the sine curve is at 75 cm. Hence, $d = 75$. A1

The amplitude will be 25 cm, since the maximum and minimum are 25 cm above and below the midpoint of the sine curve. Hence, $a = 25$. A1

The period is given as 8.4 seconds.

$$P = \frac{2\pi}{n}$$

$$8.4 = \frac{2\pi}{n}$$

$$n = \frac{2\pi}{8.4}$$

$$= \frac{5\pi}{21}$$

Hence, $b = 5$ A1

and $c = 21$. A1

ii. The domain is $t \geq 0$. A1

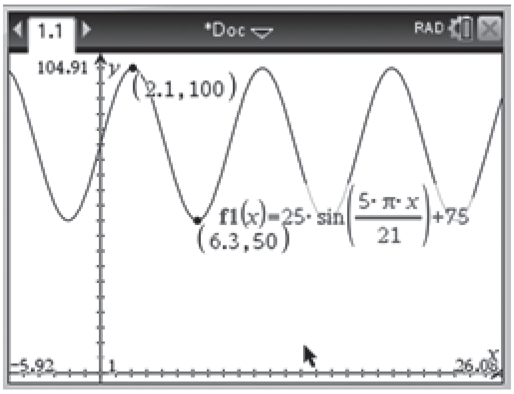
iii. $h(t) = 25\sin\left(\dfrac{5\pi t}{21}\right) + 75$ A1

*Note: Consequential on answer to **Question 4a.i.***

b. When the handle is in the central position, the equation is $h(t) = 75$. A1

Note: Accept $y = 0.75$.
*Consequential on answer to **Question 4a.i.***

c. Using the maximum and minimum functionality of a graphics calculator, the first maximum and minimum points can be found.



The first maximum point is at (2.1, 100). A1

The first minimum point is at (6.3, 50). A1

*Note: Consequential on answer to **Question 4a.i.***

d. $h(85) = 25\sin\left(\dfrac{5\pi \times 85}{21}\right) + 75$

$= 92$ cm A1

Note: Consequential marks can be awarded based on Question 4a.i.
Note: The calculator must be in radian mode, not degrees.
If the answer of 97 cm is obtained, no marks are awarded.

e. 29.4 seconds is $\frac{29.4}{8.4} = 3\frac{1}{2}$ complete cycles M1

$$\begin{aligned} 3\frac{1}{2} \times 1\frac{3}{4} &= \frac{7}{2} \times \frac{7}{4} \\ &= \frac{49}{8} \\ &= 6\frac{1}{8} \\ &\approx 6 \text{ L} \end{aligned}$$

A1

f. **i.** Since the ground level has decreased by 15 cm, this means that the function $g(t)$ has moved up by 15 cm.

Hence, the equation of the transformed function is $g(t) = 25\sin\left(\frac{5\pi t}{21}\right) + 90$. A1

Note: Consequential on answer to ***Question 4a.i.***

ii. 40 + 15 = 55 cm A1