GOLD STRIPE

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter

MATHEMATICAL METHODS

Written examination 2

Monday 31 May 2021

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 29 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

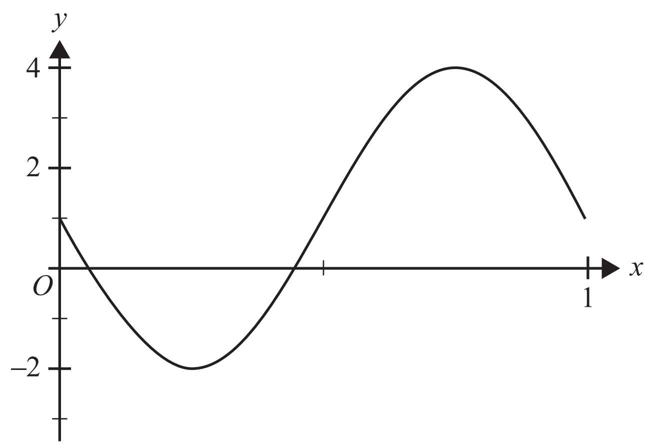
SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The graph below shows one cycle of a circular function.



The rule for the function could be

A. $y = 3\sin(x) + 1$

B. $y = -3\sin\left(\frac{x}{2\pi}\right) + 1$

C. $y = -3\cos(2\pi x) + 1$

D. $y = 3\sin(2\pi x) - 1$

E. $y = -3\sin(2\pi x) + 1$

Question 2

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 3

The function $f: D \to R,\ f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x$ will have an inverse function for

A. $D = R$

B. $D = (-3, 1)$

C. $D = (1, \infty)$

D. $D = (-\infty, 0)$

E. $D = (0, \infty)$

Question 4

The graph of $f: R \to R, f(x) = x^3 + ax^2 + bx + c$ has a turning point at $x = 3$ and a y-intercept at $y = 9$.

The values of a, b and c could be, respectively

A. -5, 3 and 9

B. 7, -15 and -9

C. -2, $-\frac{3}{2}$ and 9

D. 5, -3 and -9

E. -1, -3 and 9

Question 5

The expression $\log_3\left(\frac{\sqrt{5}}{q^2 p}\right)$ is equivalent to

A. $\log_3(5) - \log_3(q) - \log_3(p)$

B. $\frac{1}{2}\log_3(5) - 2\log_3(q) - 2\log_3(p)$

C. $\frac{1}{2}\log_3(5) - 2\log_3(q) - \log_3(p)$

D. $2\log_3(5) - 2\log_3(q) - \log_3(p)$

E. $2\log_3(5) - 2\log_3(q) - 2\log_3(p)$

Question 6

The sum of the first four positive solutions to the equation $\tan(2x) - 1 = 0$ is

A. $\frac{3\pi}{2}$

B. $\frac{5\pi}{2}$

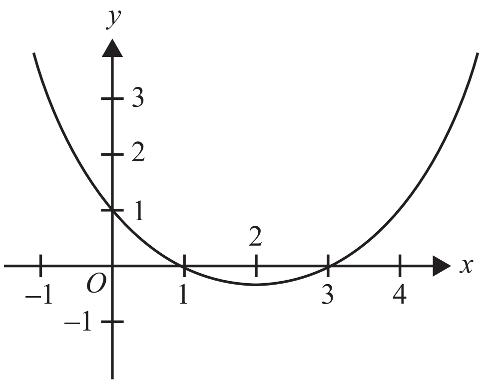
C. 2π

D. $\frac{7\pi}{2}$

E. 4π

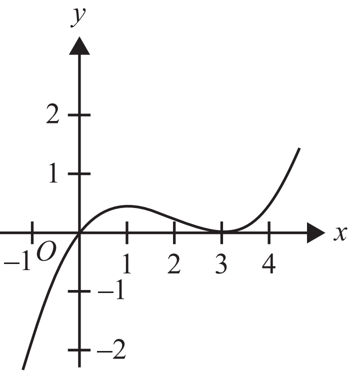
Question 7

Part of the graph of $y = f'(x)$ is shown in the diagram below.

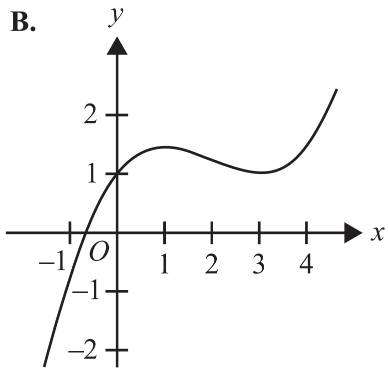


Given that $f(0) = 1$, the corresponding part of the graph of $y = f(x)$ could be

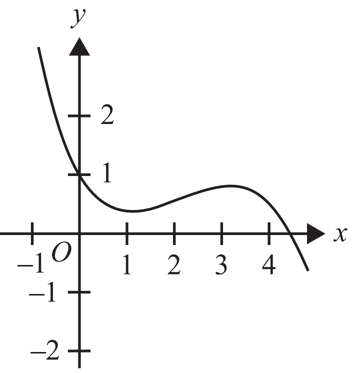
A.



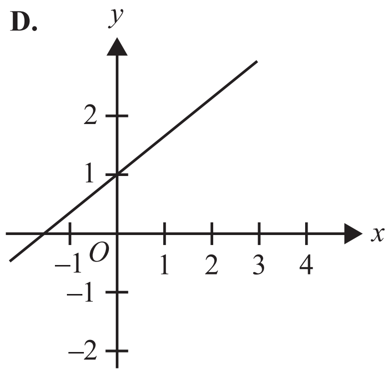
B.



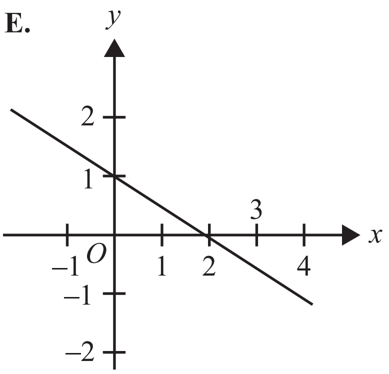
C.



D.

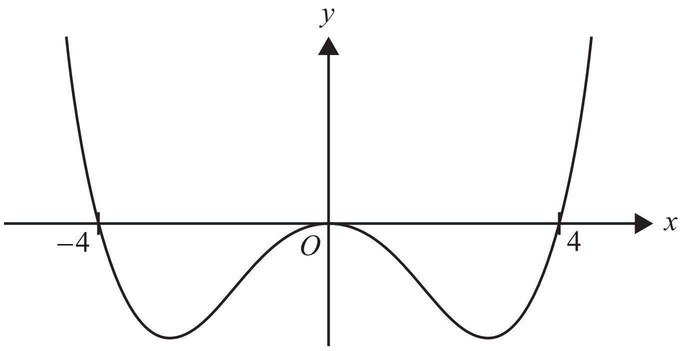


E.



Question 8

Part of the graph of a polynomial function f is shown below. This graph has turning points at $\left(-2\sqrt{2}, -1\right)$ and $\left(2\sqrt{2}, -1\right)$.



$f(x)$ is strictly decreasing for

A. $x \in (-\infty, -4] \cup [4, \infty)$

B. $x \in [-4, 4]$

C. $x \in \left[-2\sqrt{2}, 2\sqrt{2}\right]$

D. $x \in \left(-\infty, -2\sqrt{2}\right] \cup \left[0, 2\sqrt{2}\right]$

E. $x \in \left[-2\sqrt{2}, 0\right] \cup \left[2\sqrt{2}, \infty\right)$

Question 9

The continuous and differentiable function $f: R \to R$ has roots at $x = 1$ and $x = 6$ and a repeated root at $x = 4$.

Given that $\int_1^4 f(x)dx = a$ and $\int_4^6 f(x)dx = b$, where $a, b \in R$, $\int_1^6 \left(f(x)+1\right)dx$ is equal to

A. $a + b + 1$
B. $a - b + 1$
C. $a + b + 5$
D. $a - b - 5$
E. $a - b$

Question 10

Consider the graph of $f: R \to R, f(x) = -x^2 - 4x + 5$.

The tangent to the graph of f is parallel to the line connecting the negative x-intercept and the y-intercept of f when x is equal to

A. -3

B. $-\frac{5}{2}$

C. $-\frac{3}{2}$

D. -1

E. $-\frac{1}{2}$

Question 11

A survey of a large random sample of people found that an approximate 95% confidence interval for the proportion of people who owned a yellow rubber duck was (0.6299, 0.6699).

The number of people in the random sample is closest to

A. 569

B. 1793

C. 2108

D. 2179

E. 2185

Question 12

The transformation $T: R^2 \rightarrow R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x-1)^3 - 2(x-1) + 4$.

The transformation T could be given by

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Question 13

For the function $p(x) = ke^{-kx}$, where $x \geq 0$ and $k > 0$, the value of a for which $p(a) = \frac{1}{2}p(0)$ is

A. $\frac{1}{k}\log_e\left(\frac{1}{2}\right)$

B. $\frac{1}{k}\log_e(2)$

C. $k\log_e(2)$

D. $k\log_e\left(\frac{1}{2}\right)$

E. $k\log_e\left(\frac{1}{2k}\right)$

Question 14

A continuous random variable, X, has the probability density function

$$f(x)=\begin{cases}0.2e^{-0.2x} & x\geq 0\\ 0 & x<0\end{cases}$$

The **variance** of X is

A. 25

B. 12.5

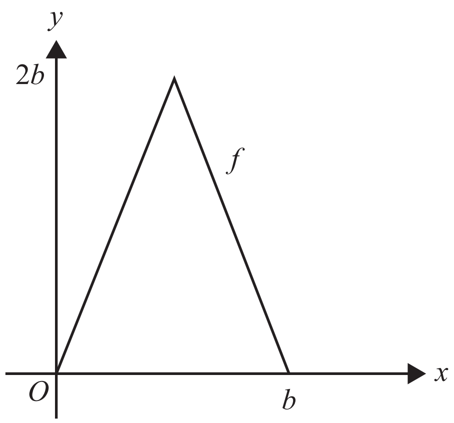
C. 6.25

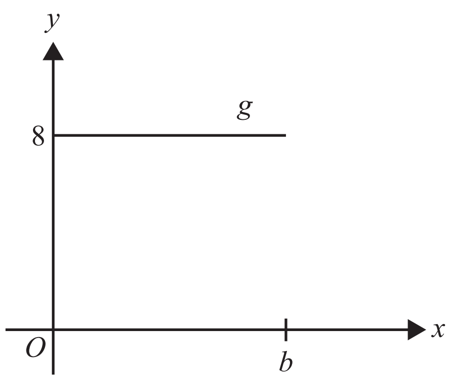
D. 3.125

E. 0

Question 15

The graphs of functions f and g are shown below. Both functions have the same domain of $[0, b]$, where $b > 0$, and the same average value.





The value of b is

A. 1

B. 2

C. 4

D. 8

E. 16

Question 16

In a particular city, it is known that 70% of all adults get their hair cut every month. A random sample of 720 adults from this city is selected.

From this sample, the probability that the proportion of adults who get their hair cut every month is greater than 0.72 is

A. 0.2104

B. 0.1359

C. 0.1187

D. 0.0847

E. 0.0392

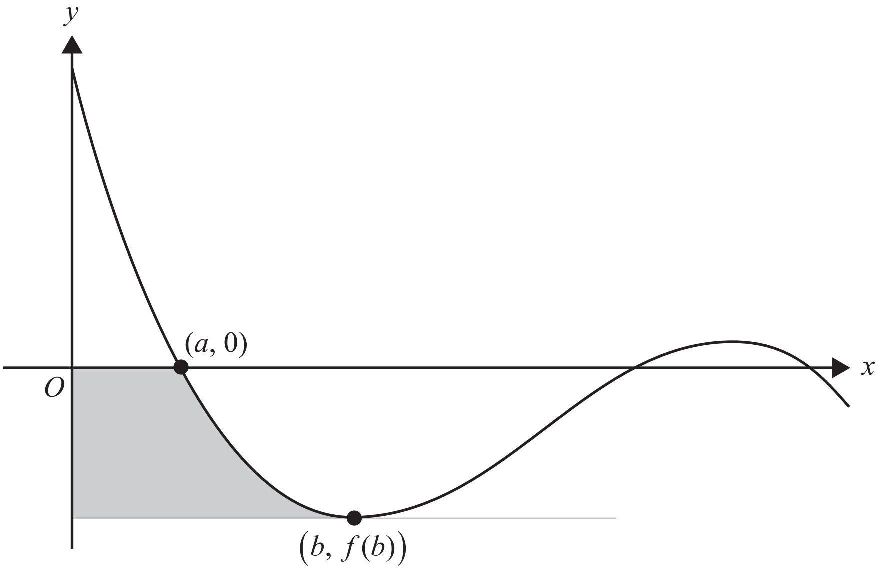
Question 17

Part of the graph of the function f is shown below.

The smallest positive x-intercept of the graph occurs at $x = a$.

The horizontal line is a tangent to f at the local minimum $(b, f(b))$.

The shaded area is the area bounded by the graph of f, the x-axis, the y-axis and the graph of $y = f(b)$.



The area of the shaded region is

A. $af(b) + \int_a^b f(x)dx$

B. $af(b) - \int_a^b f(x)dx$

C. $\int_a^b f(x)dx + bf(b)$

D. $bf(b) - \int_a^b f(x)dx$

E. $\int_a^b f(x)dx - bf(b)$

Question 18

Given that $\frac{d(x\cos(x))}{dx} = \cos(x) - x\sin(x)$, $\int x\sin(x)dx$ is equal to

A. $\cos(x) - x\cos(x)$

B. $\cos(x) + \int x\cos(x)dx$

C. $x\cos(x) - \int \cos(x)dx$

D. $\int \cos(x)dx - x\cos(x)$

E. $\frac{-x\cos(x)}{\cos(x)}$

DO NOT WRITE IN THIS AREA

Question 19

A cubic polynomial function $f: R \to R$ has roots at $x = 1$ and $x = 3$ only and its graph has a y-intercept at $y = 3$.

Which one of the following statements **must** be true about the function g, where $g(x) = \sqrt{f(x)}$?

A. The function g has a local maximum at $x = 2$

B. $g(2) = 1$

C. The domain of g does not include the interval $(1, 3)$

D. The domain of g includes the interval $(1, 3)$

E. The domain of g does not include the interval $(3, \infty)$

Question 20

The probability distribution for the discrete random variable X, where $b \in R$, is shown in the table below.

x	0	1	2	3
$\Pr(X = x)$	$\frac{4}{5}$	$\frac{1}{10} - b^3$	$\frac{15b^2 - 2}{50}$	$\frac{9b + 5}{50}$

The value of b is

A. –0.4

B. –0.3

C. –0.2

D. 0.2

E. 0.5

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

Let $f: R \to R,\ f(x) = -\left(\cos(2x) + \cos(4x)\right)$ and $g: R \to R,\ g(x) = 2\cos(x)$.

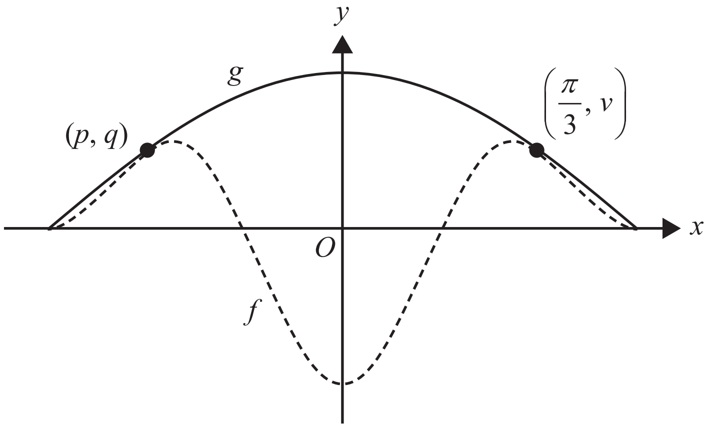
a. State the period and the amplitude of g. 1 mark

b. Find the value of c for which $f(c) = 0$, where $c \in \left(0, \frac{\pi}{2}\right)$. 1 mark

c. Find the minimum value of f. 1 mark

Parts of the graphs of the functions of f and g are shown below.

The graphs of f and g touch, but do not cross, at the points (p, q) and $\left(\frac{\pi}{3}, v\right)$.



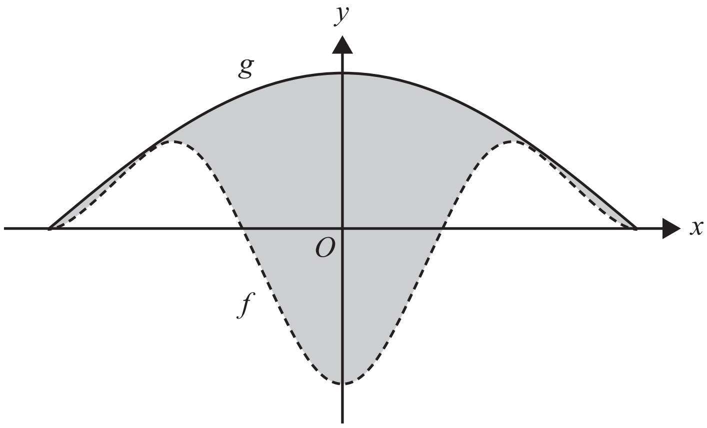
d. Find the values of p and q. 2 marks

e. **i.** Find the value of the derivative of f and the value of the derivative of g at $x = \frac{\pi}{3}$. 2 marks

ii. Find the equation of the tangent to the graphs of f and g at $x = \frac{\pi}{3}$. 1 mark

iii. Find the equation of the line perpendicular to the graphs of f and g at $x = \frac{\pi}{3}$. 1 mark

f. The area bounded by the graphs of f and g is shaded in the diagram below.



Find the area of the shaded region. 2 marks

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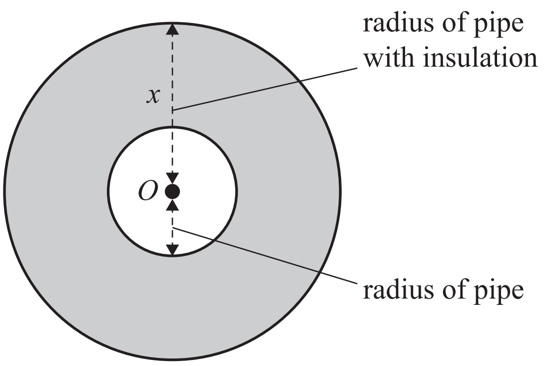
Question 2 (10 marks)

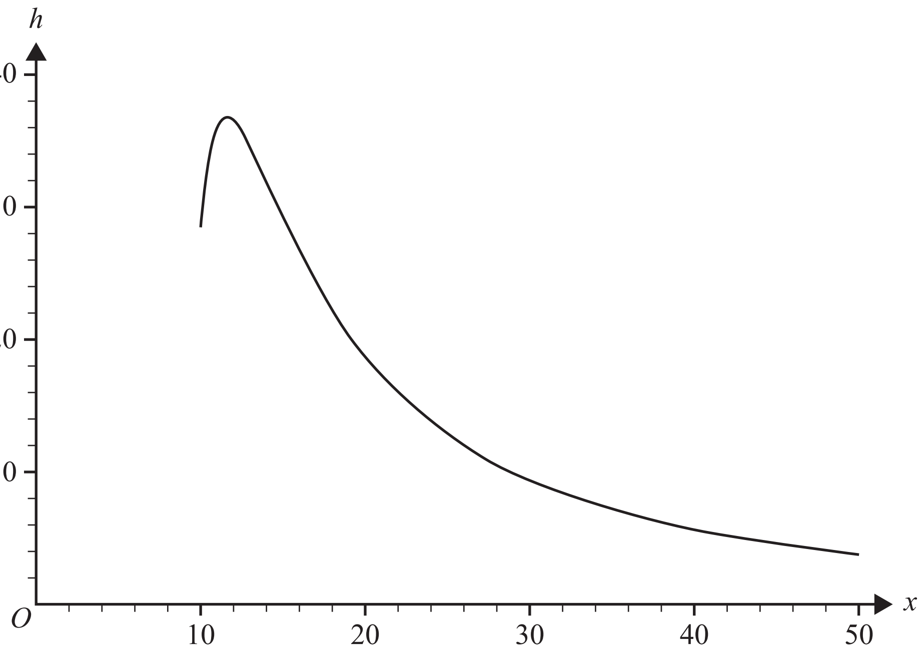
The function $h(x) = \frac{3200}{(x-5)^2} \log_e\left(\frac{x-5}{4}\right)$, where $x \in [10, 50]$, models the rate at which heat is lost from the water in a hot-water pipe with insulation, where $h(x)$ is the rate at which units of heat are lost from the water and x is the radius of the hot-water pipe with its insulation, in millimetres.

The diagram below shows a cross-section of the pipe with its insulation.



The radius of the pipe without its insulation is 10 mm.

The graph of the rate of heat lost from the water over the given domain is shown below.



a. Find the rate at which heat is lost from the water in a pipe with no insulation, correct to three decimal places. 1 mark

b. **i.** State the derivative of $h(x)$. 1 mark

ii. Find the maximum rate at which heat is lost from the water, correct to three decimal places. 1 mark

c. A particular insulated pipe has the same rate of heat lost from the water as a pipe with no insulation.

Find the thickness of insulation for this pipe, in millimetres, correct to three decimal places. 1 mark

d. **i.** If both the radius of the pipe without insulation and the radius of the pipe with insulation, as shown in the diagram on page 14, are doubled, show that the rate of heat lost from the water, h_1, is now given by

$$h_1(x) = \frac{12\,800}{(x-10)^2}\log_e\left(\frac{x-10}{8}\right)$$

and state the domain of h_1. 2 marks

ii. Describe the transformation that maps the graph of h to the graph of h_1. 1 mark

e. **i.** Find the area between the graph of h_1 and the horizontal axis over its domain. Give your answer correct to three decimal places. 2 marks

ii. Let the area found in **part e.i.** be A.

Determine the area between the graph of h and the horizontal axis over its domain, in terms of A. 1 mark

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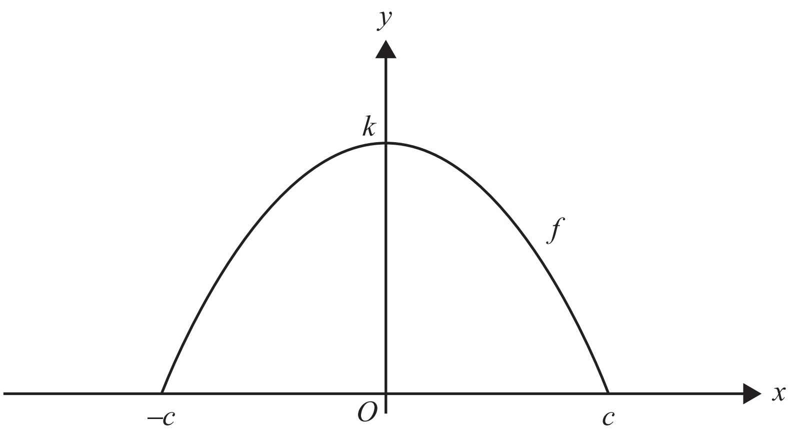
Question 3 (10 marks)

The parabolic arch of a tunnel is modelled by the function $f : [-c, c] \to R$, $f(x) = ax^2 + b$, where $a < 0$, $b \in R$ and $c > 0$.

Let x be the horizontal distance, in metres, from the origin and let y be the vertical distance, in metres, above the base of the arch.

The graph of f is shown below, where the coordinates of the y-intercept are $(0, k)$ and the coordinates of the x-intercepts are $(-c, 0)$ and $(c, 0)$.



a. Express a and b in terms of c and k. 2 marks

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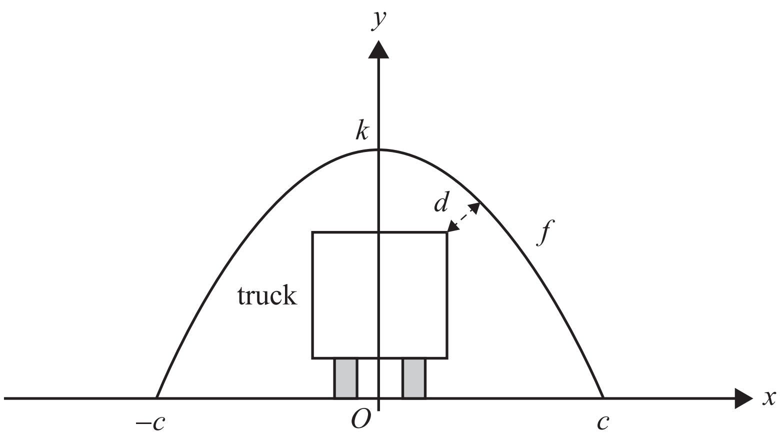
A particular tunnel has an arch modelled by f. It has a height of 6 m at the centre and a width of 8 m at the base.

b. **i.** Find the rule for this arch. 1 mark

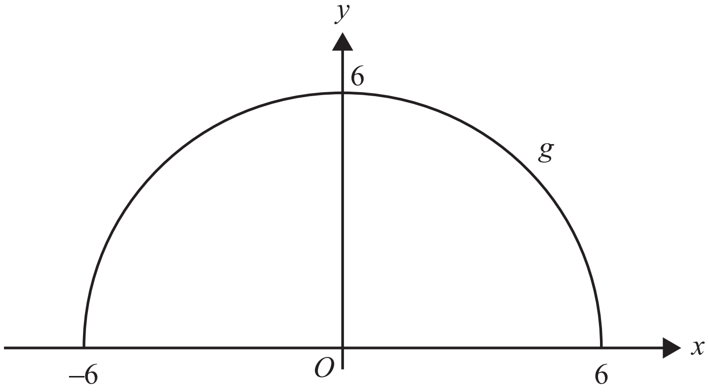
ii. A truck that has a height of 3.7 m and a width of 2.7 m will fit through the arch with the function f found in **part b.i.**



Assuming that the truck drives directly through the middle of the arch, let d be the minimum distance between the arch and the top corner of the truck.

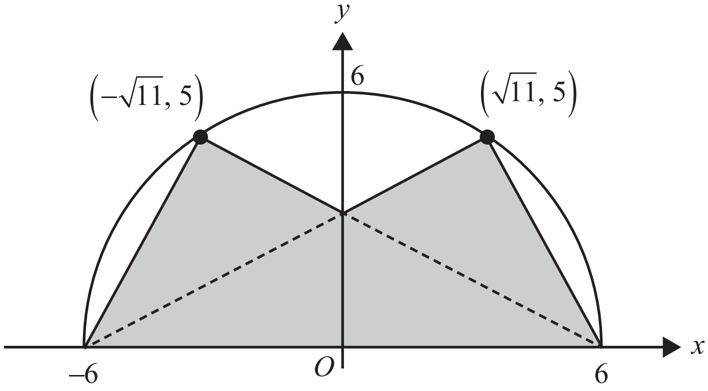
Find d and the value of x for which this occurs, correct to three decimal places. 3 marks

A different tunnel has a semicircular arch. This arch can be modelled by the function
$g:[-6,\ 6] \rightarrow R,\ g(x)=\sqrt{r^2-x^2}$, where $r>0$.
The graph of g is shown below.



c. State the value of r. 1 mark

d. Two lights have been placed on the arch to light the entrance of the tunnel. The positions of the lights are $\left(-\sqrt{11}, 5\right)$ and $\left(\sqrt{11}, 5\right)$. The area that is lit by these lights is shaded in the diagram below.



Determine the proportion of the cross-section of the tunnel entrance that is lit by the lights. Give your answer as a percentage, correct to the nearest integer. 3 marks

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Question 4 (17 marks)

A particular petrol station has two air pumps, *A* and *B*, to inflate tyres. Each inflation of a tyre is independent of any other inflation of a tyre.

When pump *A* is set to 320 kilopascals (kPa), the pressure that the tyres will be inflated to follows a normal distribution with a mean of 320 kPa and a standard deviation of 10 kPa.

a. Find the probability that a tyre will be inflated to a pressure greater than 330 kPa when inflated by pump *A*, correct to four decimal places. 1 mark

b. The probability that a tyre is inflated by pump *A* to a pressure greater than *a* is 0.9

Find the value of *a*, correct to the nearest kilopascal. 2 marks

When pump *B* is set to 320 kPa, the pressure that the tyres will be inflated to is modelled by the following probability density function.

$$b(x) = \begin{cases} \dfrac{3}{40000}(x-310)^2(330-x) & 310 \le x \le 330 \\ 0 & \text{elsewhere} \end{cases}$$

c. Determine the mean tyre pressure for tyres inflated by pump *B*. 2 marks

d. A randomly selected tyre is inflated by pump *B*.

Find the probability that this tyre will be inflated to a pressure greater than the mean tyre pressure of tyres inflated by pump *B*, correct to four decimal places. 2 marks

SECTION B – Question 4 – continued

e. The probability that a tyre is inflated by pump B to a pressure less than k is 0.95

Find the value of k, correct to the nearest kilopascal. 2 marks

f. A motorist is equally likely to use either pump A or pump B to inflate one of their car's tyres.

Find the probability that the motorist has used pump A given that the tyre is inflated to a pressure greater than 325 kPa. Give your answer correct to four decimal places. 2 marks

The company that manufactures the pumps tests all of its pumps and removes those that are defective.

The probability that a randomly selected pump is defective, from all of the pumps tested, is 0.08

g. Find the probability that four pumps are defective from a sample of 25 randomly selected pumps, correct to four decimal places. 2 marks

h. For random samples of 25 pumps, $\hat{P}$ is the random variable that represents the proportion of pumps that are defective.

Find the probability that $\hat{P}$ is greater than 15%, correct to four decimal places. 2 marks

i. For random samples of n pumps, $\hat{P}_n$ is the random variable that represents the proportion of pumps that are defective.

Find the least value of n such that $\Pr\left(\hat{P}_n < \frac{1}{n}\right) < 0.15$ 2 marks

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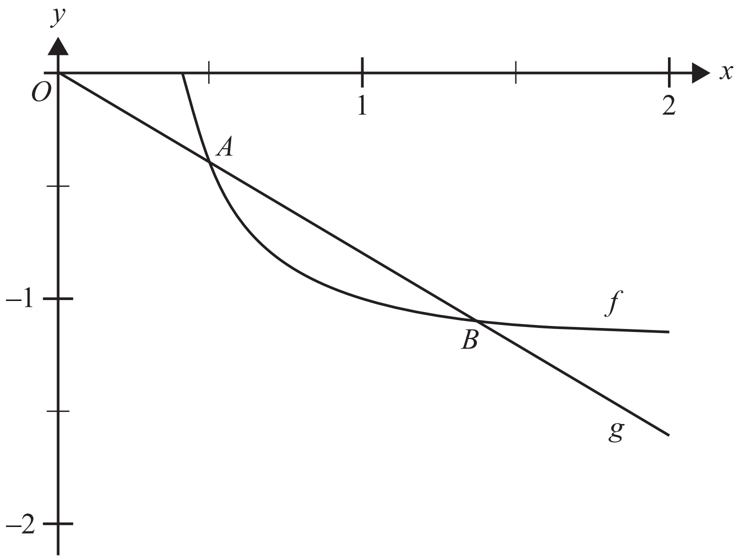
Question 5 (12 marks)

Let $f: R^+ \to R,\ f(x) = \frac{1}{5x^2} - \frac{6}{5}$ and $g: R \to R,\ g(x) = -\frac{4x}{5}$.

Parts of the graphs of f and g are shown below.



a. Find the coordinates of the points of intersection of the graphs of f and g, labelled A and B in the diagram above. 2 marks

b. Determine the area bounded by the graphs of f and g between A and B. Give your answer in the form $\frac{r + s\sqrt{t}}{u}$, where r, s, t and u are integers. 2 marks

Let $p: R^+ \to R,\ p(x) = \frac{1}{ax^2} - \frac{a+1}{a}$ and $q: R \to R,\ q(x) = \frac{(1-a)x}{a}$ for $a > 1$.

c. Find the value of a for which $p(x) = f(x)$ and $q(x) = g(x)$ for all x. 1 mark

d. Find the positive x-intercept of p in terms of a. 1 mark

Point M lies on the graph of $y = p(x)$. The tangent to p at M is parallel to q.

e. Find the x-coordinate of M in terms of a. 2 marks

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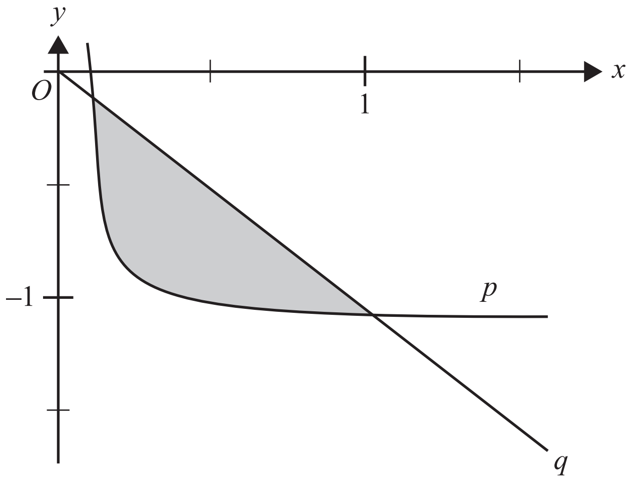
f. **i.** Find the y-intercept of the tangent to p at M in terms of a. 1 mark

ii. Given that $\frac{2x}{3} \geq 2^{\frac{1}{3}}(x-1)^{\frac{2}{3}}$ for $x > 1$, show that the tangent to p parallel to q will have a negative y-intercept for all $a > 1$. 1 mark

iii. The tangent to p parallel to q has a negative y-intercept.

Explain why this implies p and q will always enclose a region bounded by both graphs for all $a > 1$. 1 mark

g. Parts of the graphs of p and q are shown below for when $a = 100$.



The shaded area is bounded by the graphs of p and q.

Find the smallest value, b, such that the shaded area is less than b for all $a \geq 100$. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET