Neap

Trial Examination 2021

VCE Mathematical Methods Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 10 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 VCE Mathematical Methods Units 3&4 Written Examination 1.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = \frac{3}{2}\cos\left(\frac{3x}{2}\right)$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $f(x) = \frac{\log_e(-x)}{e^x}$.

Evaluate $f'(-1)$. 2 marks

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Question 2 (3 marks)

Let $\int_1^5 \frac{1}{1-2x}\,dx = \log_e(b)$.

Find the value of b.

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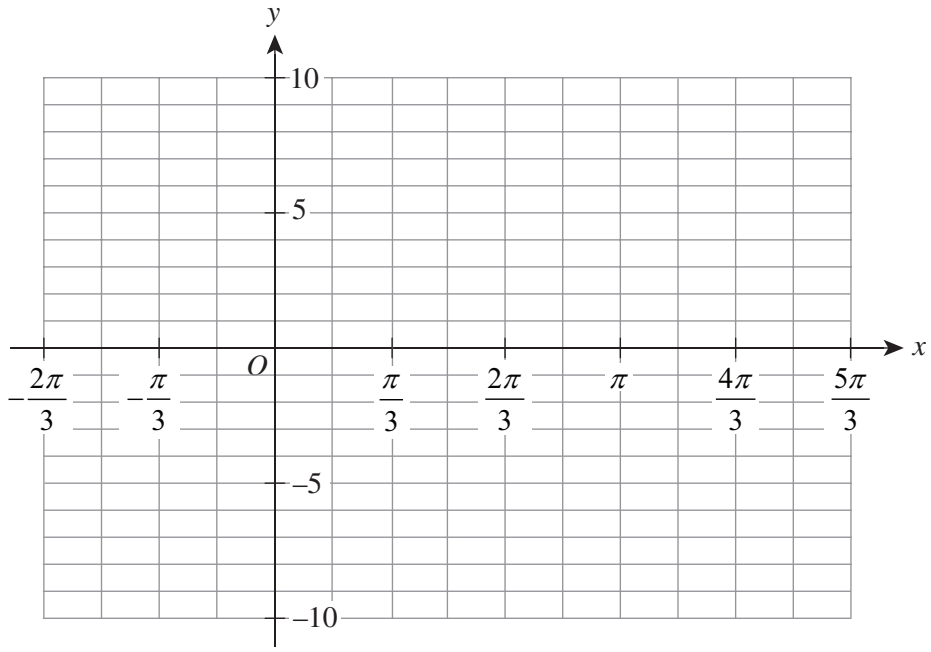
Question 3 (7 marks)

a. Solve $\tan\left(\frac{\pi}{3}-x\right)=0$ for $x\in\left(-\frac{\pi}{6},\frac{4\pi}{3}\right]$. 2 marks

Let $f:\left(-\frac{\pi}{6},\frac{4\pi}{3}\right]\to R, f(x)=\tan\left(\frac{\pi}{3}-x\right)$.

b. Sketch the graph of $y=f(x)$ on the axes below. Label the axes intercepts with their coordinates and give the equations of asymptotes. 3 marks



c. Find the equation of the tangent to the graph of $y=f(x)$ at the point where the graph crosses the *y*-axis. 2 marks

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Question 4 (3 marks)

Solve $\log_2(2x+4) - 2\log_2(x+2) - 1 = 0$.

Question 5 (3 marks)

The results of four independent trials show that $4\Pr(X=1) = \Pr(X=3)$, where the random variable X represents the number of successes. X is binomially distributed.

If p is the probability of success on any trial, find the value(s) of p.

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Question 6 (2 marks)

An animal rescue shelter has a population of rabbits of different breeds. European rabbits form $\frac{3}{5}$ of the population. If a sample of size n is taken from the population, $\hat{P}$ represents the proportion of European rabbits in the sample.

Find the value of n such that the standard deviation of $\hat{P}$ is equal to $\frac{1}{10}$.

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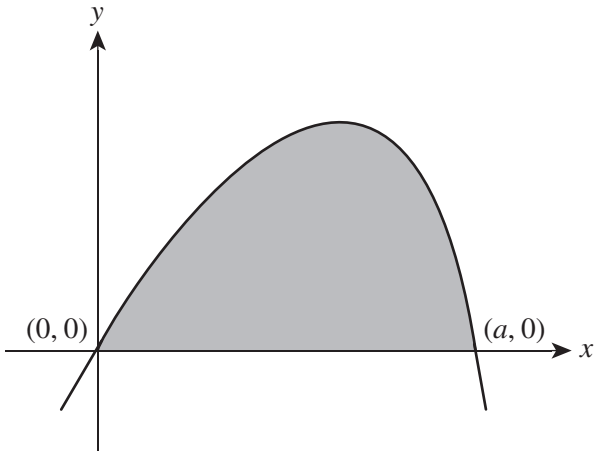
Question 7 (5 marks)

Part of the graph of $f(x) = x\cos\left(x^2\right)$ is shown below. The point $(a, 0)$ is the first positive x-intercept of the graph of $y = f(x)$.



a. Show that $a = \dfrac{\sqrt{2\pi}}{2}$. 1 mark

b. Find $\dfrac{d\left(\sin\left(x^2\right)\right)}{dx}$ and hence find the shaded area of the graph. 4 marks

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Question 8 (6 marks)

The continuous random variable X has a probability density function that is given by

$$f(x) = \begin{cases} a\sqrt{p-x} & 0 \leq x \leq p \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $a = \dfrac{3}{2p^{\frac{3}{2}}}$. 2 marks

b. Let the y-intercept of the function f be given by the coordinate $(0, q)$.
Express q in terms of p. 1 mark

c. The minimum value of $(p+q)$ is equal to $\sqrt{m}$.
Find the value of m. 3 marks

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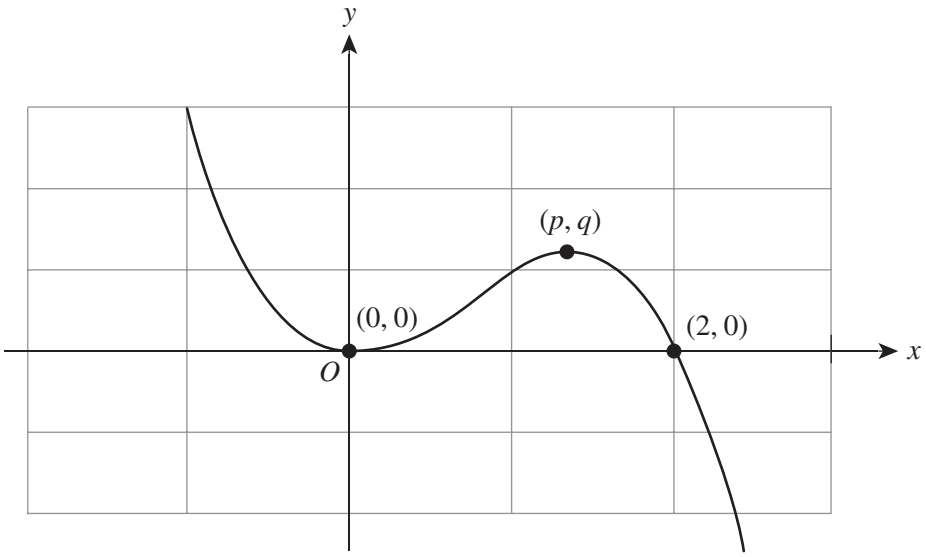
Question 9 (8 marks)

Part of the graph of a cubic polynomial $y = f(x)$ is shown below. The rule of $f(x)$ can be written as $f(x) = ax^2(x-2)$, where $a < 0$. The graph has a local maximum at (p, q).



A transformation is applied to $f(x)$. The image of $f(x)$ under this transformation is defined as $g(x)$. The rule for $g(x)$ can be written as $g(x) = kf(x+1)$, where $k < -\frac{3}{2}$.

a. Sketch the graph of $y = g(x)$ on the axes above. Label the axes intercepts and turning points with their coordinates in terms of p and q where required. 3 marks

b. Show that $p = \frac{4}{3}$. 2 marks

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c. The average value of g over the interval $[-1, 1]$ is equal to -1.
Find q in terms of k. 3 marks

END OF QUESTION AND ANSWER BOOKLET

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VCE Mathematical Methods Units 3&4

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(-ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

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Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$		

Probability distribution		**Mean**	**Variance**
Bernoulli	$P(X = x) = \begin{cases} p & x = 1 \\ 1-p & x = 0 \end{cases}$	$\mu = p$	$\sigma^2 = p(1-p)$
binomial	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
normal	$\Pr(X \le a) = \int_{-\infty}^{a} \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$	μ	σ^2

END OF FORMULA SHEET

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