Neap

Trial Examination 2021

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. $y = \frac{3}{2}\cos\left(\frac{3x}{2}\right)$

$\frac{dy}{dx} = -\frac{3}{2} \times \frac{3}{2}\sin\left(\frac{3x}{2}\right)$

$= -\frac{9}{4}\sin\left(\frac{3x}{2}\right)$ A1

b. $f'(x) = -e^{-x} \times \log_e(-x) + e^{-x} \times -\frac{1}{-x}$ M1

Note: Product or quotient rule should be used.

$= -e^{-x}\log_e(-x) + \frac{1}{xe^x}$

$f'(-1) = -e^{-(-1)}\log_e(-(-1)) + \frac{1}{(-1)e^{(-1)}}$

$= -e$ A1

Question 2 (3 marks)

$\int_1^5 \frac{1}{1-2x}\,dx = -\int_1^5 \frac{1}{2x-1}\,dx$

$= -\frac{1}{2}\left[\log_e(2x-1)\right]_1^5$ M1

$= -\frac{1}{2}\left(\log_e(9) - \log_e(1)\right)$

$= -\log_e(3)$ M1

$= \log_e\left(\frac{1}{3}\right)$

$\therefore b = \frac{1}{3}$ A1

Question 3 (7 marks)

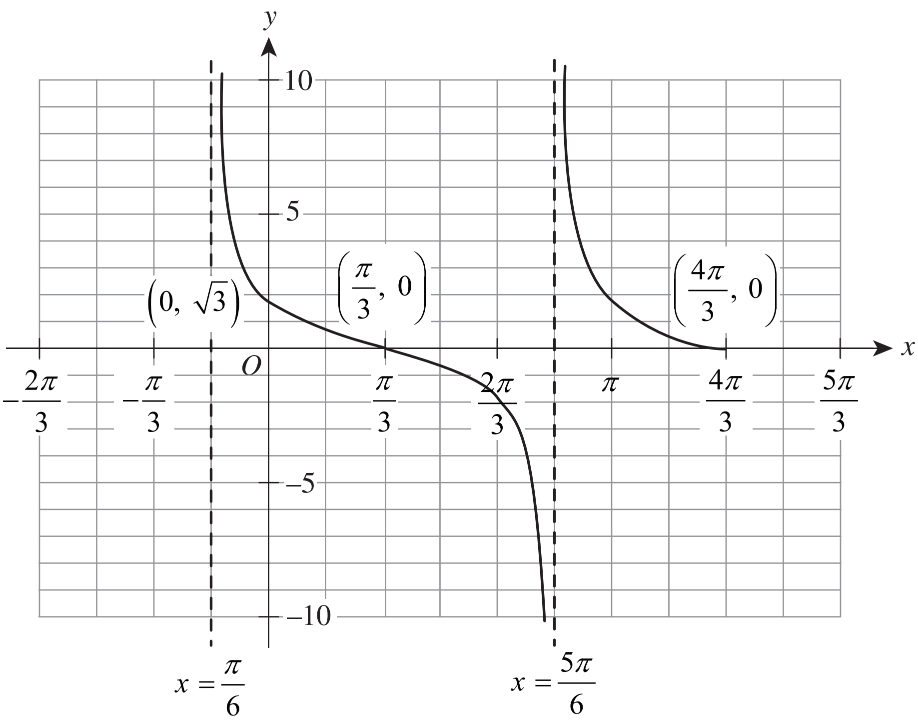
a. $\tan\left(\frac{\pi}{3}-x\right)=0$

$\tan\left(x-\frac{\pi}{3}\right)=0$ M1

$x-\frac{\pi}{3}=0+n\pi,\ n\in Z$

$x=\frac{\pi}{3}$ and $x=\frac{4\pi}{3}$ A1

b.



correct intercepts A1
correct asymptotes A1
correct shape A1

c. $f(x)=\tan\left(\frac{\pi}{3}-x\right)$

$f'(x)=-\sec^2\left(\frac{\pi}{3}-x\right)$ M1

$f'(0)=-\sec^2\left(\frac{\pi}{3}\right)$

$=-\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right)^2$

$=-4$

$m_T=-4$ and point $\left(0,\sqrt{3}\right)$

$y-\sqrt{3}=-4x$

$y=-4x+\sqrt{3}$ A1

Question 4 (3 marks)

$$\log_2(2x+4)-2\log_2(x+2)-1=0$$

$$\log_2(2x+4)-\log_2(x+2)^2=1 \qquad \text{M1}$$

$$\log_2\left(\frac{2x+4}{(x+2)^2}\right)=\log_2(2)$$

$$\frac{2x+4}{(x+2)^2}=2 \qquad \text{M1}$$

$$2x+4=2(x+2)^2$$

$$x+2=(x+2)^2$$

$$(x+2)^2-(x+2)=0$$

$$(x+2)(x+2-1)=0$$

$$(x+2)(x+1)=0$$

$$x=-2 \text{ or } x=-1$$

But $x>-2$ and therefore $x=-1$ only. A1

Question 5 (3 marks)

$X \sim Bi(4, p)$

$$\Pr(X=1)={}^4C_1\,p(1-p)^3$$
$$=4p(1-p)^3$$

$$\Pr(X=3)={}^4C_3\,p^3(1-p) \qquad \text{M1}$$
$$=4p^3(1-p)$$

$$4\Pr(X=1)=\Pr(X=3)$$

$$4\times 4p(1-p)^3=4p^3(1-p)$$

$$4p(1-p)^3=p^3(1-p)$$

$$4p(1-p)^3-p^3(1-p)=0$$

$$p(1-p)(4(1-p)^2-p^2)=0$$

$$p(1-p)(4(1-2p+p^2)-p^2)=0$$

$$p(1-p)(3p^2-8p+4)=0 \qquad \text{M1}$$

$$p(1-p)(p-2)(3p-2)=0$$

$\therefore p=0$ or $p=\dfrac{2}{3}$ or $p=1$ as $0\le p\le 1$ A1

Question 6 (2 marks)

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

Let $\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}$.

$$\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}$$ M1

$$\sqrt{\frac{6}{25n}} = \frac{1}{10}$$

$$\frac{6}{25n} = \frac{1}{100}$$

$$25n = 600$$

$$n = 24$$ A1

Question 7 (5 marks)

a. Let $f(x) = 0$.

$$x\cos\left(x^2\right) = 0$$

$x = 0$ or $\cos(x^2) = 0$

$x^2 = \frac{\pi}{2}$ gives first positive solution.

$$a = \sqrt{\frac{\pi}{2}} \text{ as } a > 0$$

$$= \frac{\sqrt{\pi}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2\pi}}{2}$$ A1

b. $\dfrac{d\left(\sin\left(x^2\right)\right)}{dx} = 2x \times \cos\left(x^2\right)$ A1

$\int 2x\cos\left(x^2\right)dx = \sin\left(x^2\right)$

$\int x\cos\left(x^2\right)dx = \dfrac{1}{2}\sin\left(x^2\right)$ M1

Area $= \int_0^{\frac{\sqrt{2\pi}}{2}} x\cos\left(x^2\right)dx$ M1

$= \left[\dfrac{1}{2}\sin\left(x^2\right)\right]_0^{\frac{\sqrt{2\pi}}{2}}$

$= \left[\dfrac{1}{2}\sin\left(\left(\dfrac{\sqrt{2\pi}}{2}\right)^2\right)\right] - \left[\dfrac{1}{2}\sin\left((0)^2\right)\right]$

$= \left[\dfrac{1}{2}\sin\left(\dfrac{\pi}{2}\right)\right]$

$= \dfrac{1}{2}$ A1

Question 8 (6 marks)

a. $\int_0^p a\sqrt{p-x}\ dx = 1$

$\left[-\frac{2a}{3}(p-x)^{\frac{3}{2}}\right]_0^p = 1$ M1

$(0) - \left(-\frac{2a}{3}p^{\frac{3}{2}}\right) = 1$

$\frac{2a}{3}p^{\frac{3}{2}} = 1$

$a = \frac{3}{2p^{\frac{3}{2}}}$ A1

b. $f(x) = \frac{3}{2p^{\frac{3}{2}}}\sqrt{p-x}$

$q = f(0)$

$= \frac{3}{2p^{\frac{3}{2}}} \times p^{\frac{1}{2}}$

$= \frac{3}{2p}$ A1

c. Let $g(p) = p + q$.

$g(p) = p + \frac{3}{2p}$

$g'(p) = 1 - \frac{3}{2p^2}$

Let $g'(p) = 0$.

$1 - \frac{3}{2p^2} = 0$ M1

$p^2 = \frac{3}{2}$

$p = \sqrt{\frac{3}{2}}$ as $p > 0$ M1

$p + q = \sqrt{\frac{3}{2}} + \frac{3}{2\left(\sqrt{\frac{3}{2}}\right)}$

$= \frac{\sqrt{3}}{\sqrt{2}} + \frac{3\sqrt{2}}{2\sqrt{3}}$

$= \frac{6+6}{2\sqrt{6}}$

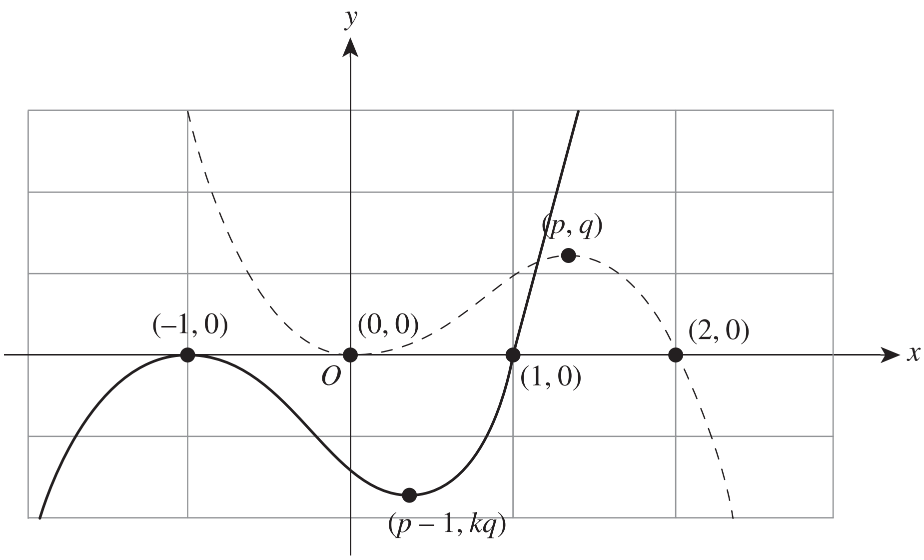
$= \sqrt{6}$

As $(p+q) = \sqrt{m}$, $\sqrt{m} = \sqrt{6}$

$\therefore m = 6$ A1

Question 9 (8 marks)

a.



correct x-intercepts A1
correct turning point A1
correct shape and scale A1

b. $f(x) = ax^2(x-2)$ where $a < 0$

$f(x) = a(x^3 - 2x^2)$

$f'(x) = a(3x^2 - 4x)$

Let $f'(x) = 0$.

$3x^2 - 4x = 0$ M1

$x(3x-4) = 0$

$x = 0$ or $x = \frac{4}{3}$

$\therefore p = \frac{4}{3}$ A1

c. The average value of g(x) over the interval [−1,1] is equal to −1.

Find q in terms of k.

$$\frac{1}{1--1}\int_{-1}^{1} g(x)dx = -1$$ M1

$$\frac{1}{2}\int_{-1}^{1} kf(x+1)dx = -1$$

$$\frac{k}{2}\int_{-1}^{1} f(x+1)dx = -1$$

$$\frac{k}{2}\int_{0}^{2} f(x)dx = -1$$

$$\frac{k}{2}\int_{0}^{2} ax^2(x-2)dx = -1$$

$$\frac{ka}{2}\int_{0}^{2} x^2(x-2)dx = -1$$

$$\frac{ka}{2}\int_{0}^{2} x^3-2x^2dx = -1$$

$$\int_{0}^{2} x^3-2x^2dx = \frac{-2}{ka}$$

$$\left[\frac{2^4}{4}-\frac{2(2)^3}{3}\right] = \frac{-2}{ka}$$

$$\frac{-4}{3} = \frac{-2}{ka}$$

$$a = \frac{3}{2k}$$ M1

$$f(x) = \frac{3}{2k}(x^3-2x^2)$$

$$q = f\left(\frac{4}{3}\right)$$

$$= \frac{3}{2k}\left(\left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2\right)$$

$$= -\frac{16}{9k}$$

$$q = -\frac{16}{9k}$$ A1

