Neap

Trial Examination 2021

VCE Mathematical Methods Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 24 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The period of $y = -4\tan\left(\frac{3x}{2}\right) + 2$ is

A. $\frac{3}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{2\pi x}{3}$

D. $\frac{4\pi}{3}$

E. 4

Question 2

The points $P\left(3\sqrt{5},\ a\right)$ and $Q\left(b,\ 2\sqrt{3}\right)$ have a midpoint of $M\left(3\sqrt{5},\ \sqrt{3}\right)$.
The values of a and b are

A. $a = 0,\ b = -3\sqrt{5}$

B. $a = 0,\ b = 3\sqrt{5}$

C. $a = 3\sqrt{5},\ b = 0$

D. $a = 4\sqrt{3},\ b = -3\sqrt{5}$

E. $a = 4\sqrt{3},\ b = 9\sqrt{3}$

Question 3

The Karnaugh map shown below is incomplete.

	B	B'	**Total**
A	0.05		
A'		0.25	
Total	0.45		1

The value of $\Pr\left(A \cap B'\right)$ is

A. 0.25

B. 0.30

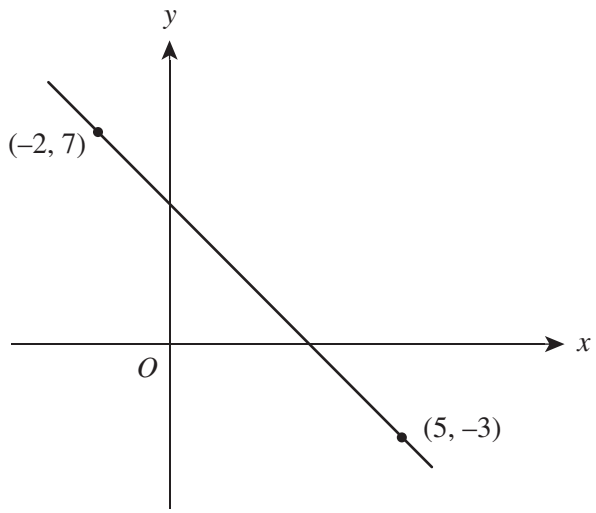
C. 0.35

D. 0.40

E. 0.55

Question 4

Consider the line connecting two points in the graph below.



The angle that the line makes with the positive direction of the x-axis, correct to the nearest degree, is

A. 2°

B. 35°

C. 55°

D. 125°

E. 145°

Question 5

The distance between the two points $A\left(\sqrt{3},\ a\right)$ and $B\left(2a,\ -2\sqrt{3}\right)$ is 40, where $a \in R^{+}$.

The coordinates of A and B are

A. $A\left(\sqrt{3},\ \sqrt{5}\right),\ B\left(2\sqrt{5},\ -2\sqrt{3}\right)$

B. $A\left(\sqrt{3},\ 5\right),\ B\left(10,\ -2\sqrt{3}\right)$

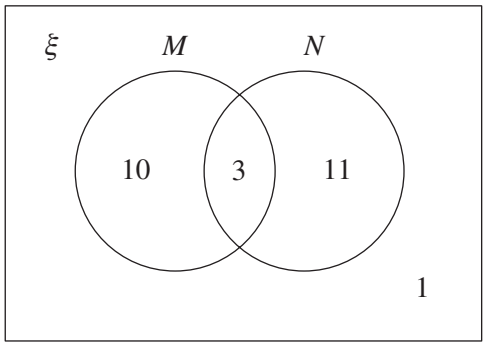
C. $A\left(\sqrt{3},\ \sqrt{29}\right),\ B\left(\sqrt{29},\ -2\sqrt{3}\right)$

D. $A\left(\sqrt{3},\ \sqrt{317}\right),\ B\left(2\sqrt{317},\ -2\sqrt{3}\right)$

E. $A\left(\sqrt{3},\ \sqrt{317}\right)$ and $\left(\sqrt{3},\ -\sqrt{317}\right),\ B\left(2\sqrt{317},\ -2\sqrt{3}\right)$ and $\left(-2\sqrt{317},\ -2\sqrt{3}\right)$

Question 6

Consider the Venn diagram below.



The value of $\Pr(N \mid M)$ is

A. $\frac{1}{8}$

B. $\frac{1}{7}$

C. $\frac{3}{14}$

D. $\frac{3}{13}$

E. $\frac{7}{12}$

Question 7

Let $Q(x) = 16x^4 - 48x^3 + 36x^2 + 8x - 12.$

The factorised form of $Q(x)$ is

A. $(4x^2 - 1)(4x - 3)(x - 1)$

B. $(4x^2 - 6)(4x + 2)(x - 1)$

C. $2(2x - 3)(4x + 2)(x + 1)(x - 1)$

D. $2(2x - 3)(4x + 2)(x - 1)^2$

E. $4(2x - 3)(2x + 1)(x - 1)^2$

Question 8

An equivalent expression for $\cos\left(\frac{\pi}{2} - \theta\right)$ is

A. $-\cos\left(\frac{\pi}{2}\right)$

B. $\sin\left(\frac{\pi}{2}\right)$

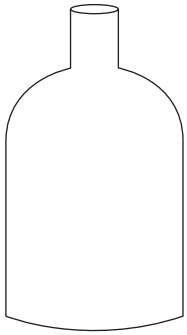
C. $-\sin\left(\frac{\pi}{2}\right)$

D. $\cos(\theta)$

E. $\sin(\theta)$

Question 9

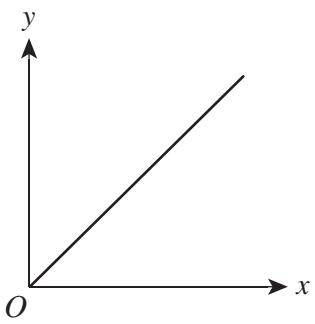
Water is being poured into the following vessel at a constant rate.



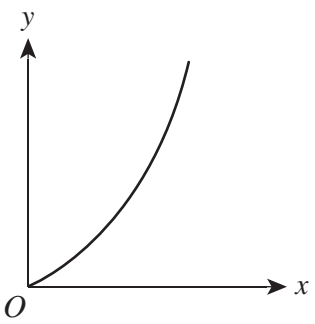
A graph is drawn showing the depth of the water from the bottom of the vessel as a function of time, where x is time, in seconds, and y is the depth of the water, in centimetres.

The graph that best shows this is

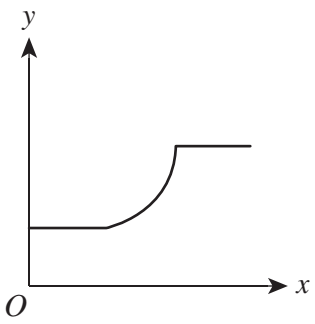
A.



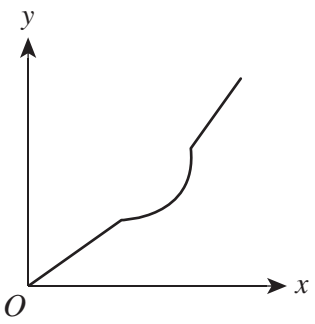
B.



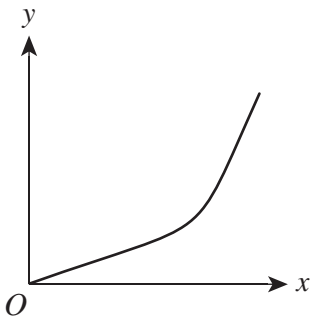
C.



D.



E.



Question 10

The equation of a curve is $y = \frac{x^3}{3} - 3x$.

The coordinates of the points where the gradient of the tangent line is –1 are

A. $\left(-\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}\right), \left(-\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right)$

B. $\left(-\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right), \left(-\sqrt{2}, -\frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right)$

C. $\left(\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}\right), \left(-\sqrt{2}, -\frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right)$

D. $\left(\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right), \left(-\sqrt{2}, -\frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}\right)$

E. $\left(\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}\right), \left(-\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right)$

Question 11

The function that does **not** have an inverse function is

A. {(3, 6), (4, 10), (–3, 5), (0, 2)}

B. $f : [-2, 7) \to R, f(x) = 3x - 1$

C. $f : (3, \infty) \to R, f(x) = \frac{2}{(x-3)^2}$

D. $f : (-2, \infty) \to R, f(x) = -3(x-2)^2 + 5$

E. $f : [1, 0] \to R, f(x) = 3x^3 - 4x^2 + x - 2$

Question 12

The asymptote(s) of the function $y = \frac{3}{(x-2)^2} + 4$ is/are

A. $x = -2$

B. $x = \frac{3}{2}, y = 4$

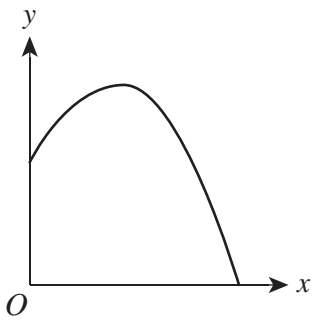
C. $x = 2$

D. $x = 2, y = 4$

E. $x = 4, y = 2$

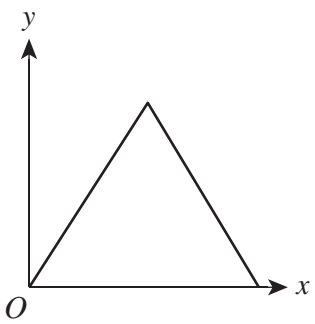
Question 13

A ball is thrown from a point above the origin to the ground. The following graph shows the quadratic relationship, where x is the time, in seconds, from the origin and y is the height of the ball, in metres.

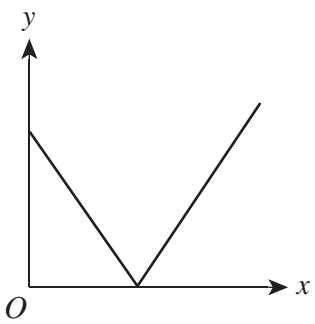


The graph that best describes the velocity of the ball is

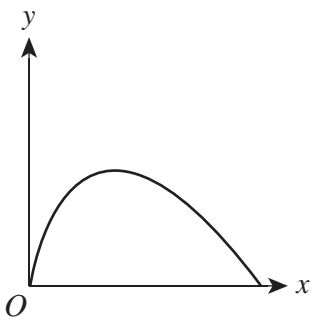
A.



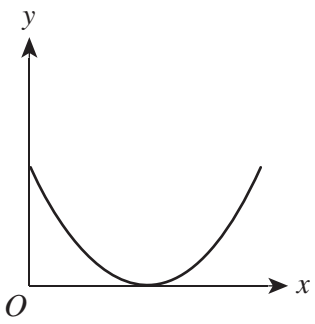
B.



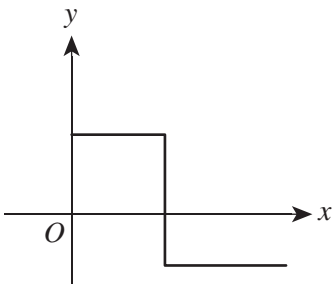
C.



D.



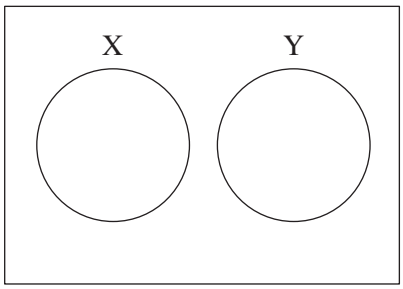
E.



Question 14

The Venn diagram that shows an example of disjoint sets is

A.



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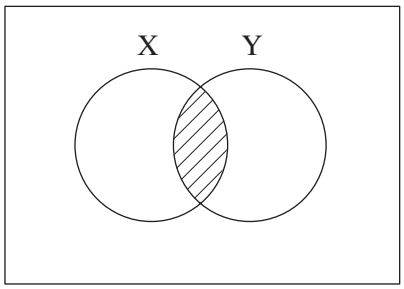
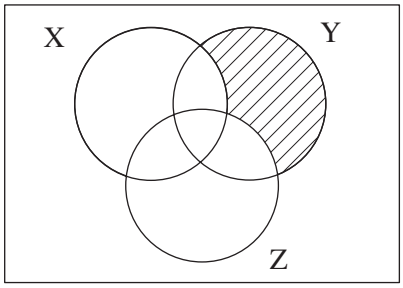
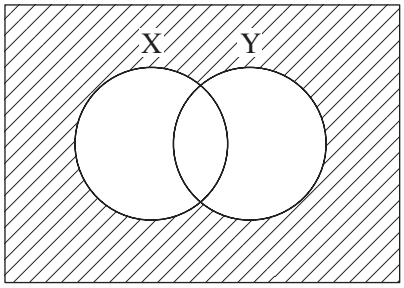


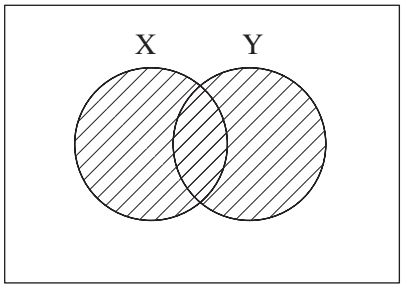
C.



D.



E.



Question 15

Consider the following probability distribution table.

x	1	2	3	4	5
Pr $(X = x)$	0.25	0.05	0.20	0.15	0.35

The value of $\Pr(2 \le x \le 4)$ is

A. 0.20

B. 0.25

C. 0.33

D. 0.35

E. 0.40

Question 16

The implied domain and range of the relation $x = -\sqrt{4 - y^2}$ is

A. domain (–2, 0), range (–2, 0)

B. domain (–2, 0), range (–2, 2)

C. domain [–2, 0], range [–2, 2]

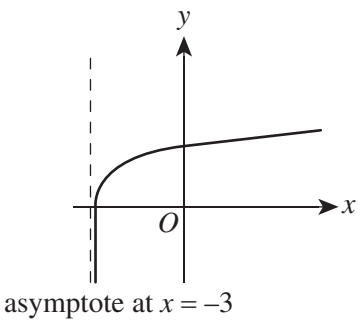
D. domain [–2, 2], range [–2, 0]

E. domain [–2, 2], range [2, 2]

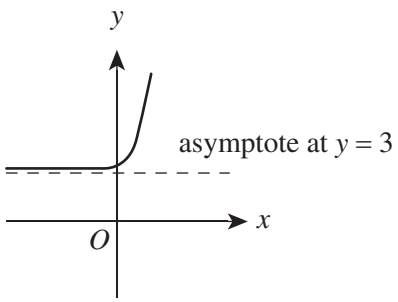
Question 17

The graph of the equation $y = 2^{-x} + 3$ is given by

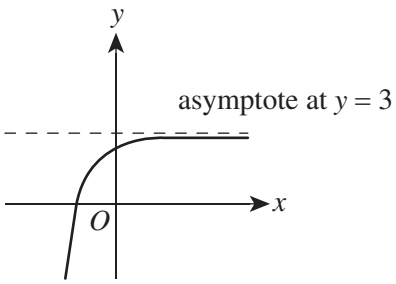
A.



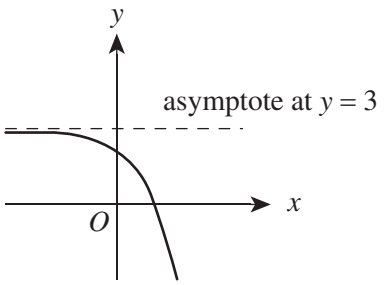
B.



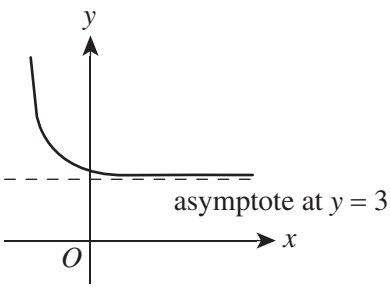
C.



D.



E.



Question 18

The following table shows the probability distribution for a random variable *X*.

x	1	2	3	4	5
$\Pr(X = x)$	0.33	0.06	0.15	0.21	0.25

The value of $\Pr\left(X \geq 4 \middle| X > 2\right)$ is

A. $\frac{25}{61}$

B. $\frac{46}{67}$

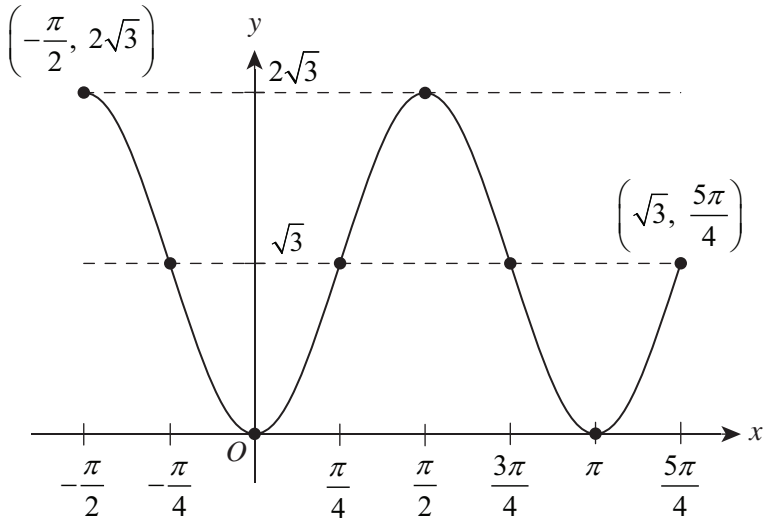
C. $\frac{46}{61}$

D. $\frac{54}{61}$

E. $\frac{46}{39}$

Question 19

Consider the following graph.



A possible equation for this graph is

A. $y = -\sqrt{3}\sin 2\left(x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

B. $y = \sqrt{3}\sin 2\left(x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

C. $y = \sqrt{3}\cos 2\left(x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

D. $y = \sqrt{3}\cos 2\left(x\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

E. $y = -\sqrt{3}\sin 2\left(x - \frac{\pi}{4}\right) + 2\sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{5\pi}{4}$

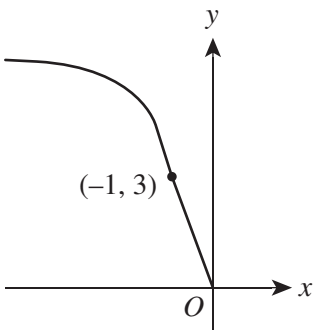
Question 20

The image of $y = \log_3 x$ undergoes the following transformations.

- reflection in the x-axis
- translation of 3 units upwards

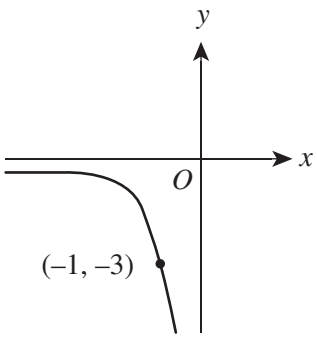
The transformed graph is

A.



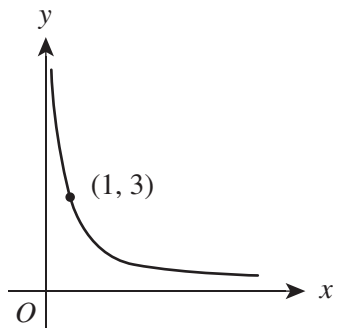
asymptote at $x = 0$

B.



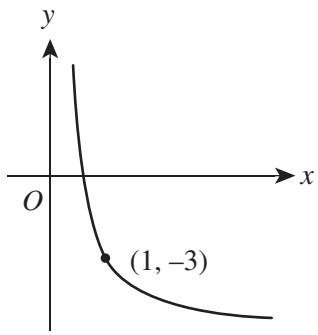
asymptote at $x = 0$

C.



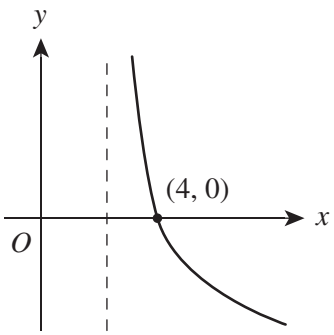
asymptote at $x = 0$

D.



asymptote at $x = 0$

E.



asymptote at $x = 3$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (5 marks)

Let $x = \log_{\frac{1}{4}}(64)$ and $y = \log_3\left(\sqrt{27}\right) + \log_5(125) + \log_5\left(\frac{1}{25}\right) + \log_3\left(\sqrt{3}\right)$.

a. Solve $x = \log_{\frac{1}{4}}(64)$. 1 mark

b. Solve $y = \log_3\left(\sqrt{27}\right) + \log_5(125) + \log_5\left(\frac{1}{25}\right) + \log_3\left(\sqrt{3}\right)$. 2 marks

c. Find $\left(\log_{\frac{1}{4}}(64)\right)^{\left(\log_3\left(\sqrt{27}\right)+\log_5(125)+\log_5\left(\frac{1}{25}\right)+\log_3\left(\sqrt{3}\right)\right)}$. 1 mark

d. Show that $x^{-3}y^{\frac{1}{3}}\log_2(16) = -\frac{4}{3^{\frac{8}{3}}}$. 1 mark

Question 2 (8 marks)

Gem is going on a holiday next week. Before she leaves, she packs her suitcase with three types of clothing: coats, T-shirts and skirts. She packs three coats, four T-shirts and five skirts.

a. **i.** How many different outfits can Gem make if she randomly selects one T-shirt and one skirt? 1 mark

Of the items Gem has packed, one T-shirt is pink and one skirt is pink. She randomly draws two items of clothing from her suitcase.

ii. What is the probability that she chooses both pink items? 1 mark

iii. What is the probability that she chooses at least one pink item? 2 marks

b. Once she arrives in her hotel room, Gem hangs her clothes in the wardrobe.

i. If Gem hangs her clothing in any order, how many different combinations are possible? 1 mark

ii. If Gem organises her clothing by type, how many different combinations are possible? 1 mark

c. Gem ranks her skirts from least favourite (1) to most favourite (5). She chooses a skirt according to the following distribution table.

x	1	2	3	4	5
Colour	grey	green	red	blue	pink
Pr $(X = x)$	0.08	0.09	$2k$	0.28	$3k$

What is the probability that Gem chooses a pink skirt? 2 marks

Question 3 (7 marks)
A relation is defined as $\{x : (2x - b)(x + 5) \geq 0\}$, where $b > 0$.

a. Using a sketch, solve for x. 3 marks

b. If $b = 3$, what is the value of k when the line $y = 3kx - 15$ forms a tangent with $y = (2x - b)(x + 5)$? 4 marks

Question 4 (13 marks)

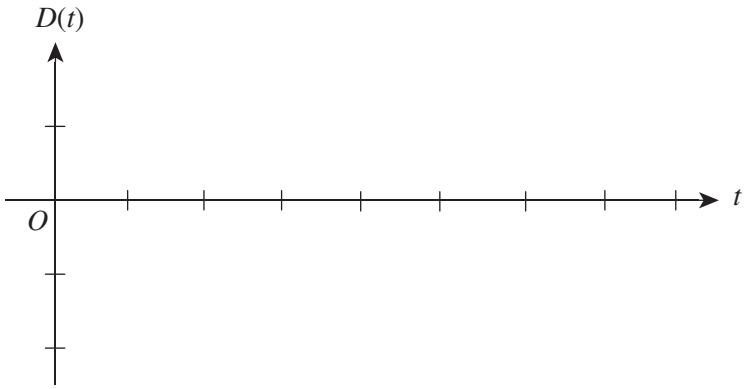
A submarine dives from the surface of the ocean according to the following relationship, where D is depth, in metres, and t is time, in hours.

$$D(t) = 5\cos\left(\frac{\pi t}{36}\right) - 5,\ 0 \le t \le 108$$

a. What is the maximum depth that the submarine reaches? 1 mark

b. How long is the submarine underwater before it resurfaces again? 1 mark

c. On the axes below, sketch the graph of $D(t)$. Label the endpoints and any maximum or minimum points, giving their coordinates. 4 marks



d. When the submarine is 7.5 m below the surface of the water, it is no longer able to be detected by radar.

Find the times when the submarine is exactly 7.5 m below the surface. 3 marks

e. A drone delivers supplies to the submarine by dropping them into the ocean on the path of $D(t) = -0.08x + 1.5$.

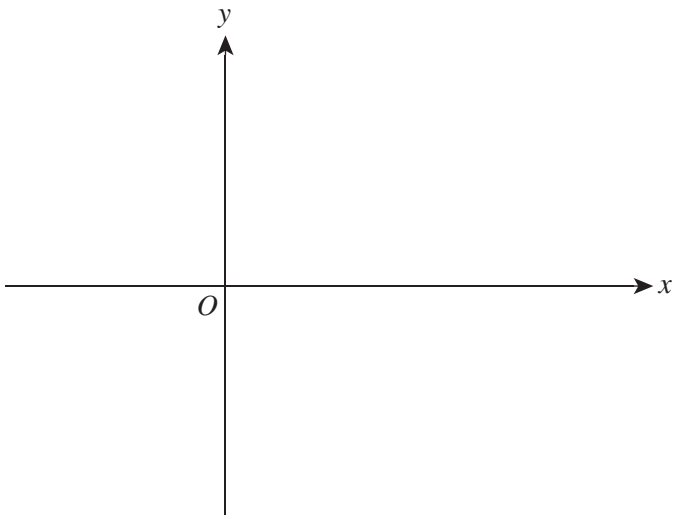
What are the points of intersection between the path of the drone and the path of the submarine, correct to two decimal places? 4 marks

Question 5 (11 marks)

Given $f(0) = 243, f(3) = 27, f'(3) = 0, f\left(\frac{9}{2}\right) = 0$ and $f'(x) \leq 0$ for $x \in R$.

a. **i.** State the nature of $f'(3) = 0$. 1 mark

ii. On the axes below, sketch the graph of $f(x)$. Label the intercepts and the stationary point of inflection, giving their coordinates. 3 marks



b. **i.** Given $f(x)$ is a cubic function, find the equation of $f(x)$. 4 marks

ii. Find the inverse function $f^{-1}(x)$. 2 marks

iii. Find $f^{-1}(4)$ correct to three decimal places. 1 mark

Question 6 (16 marks)

The family of graphs $y = \frac{a}{(bx + c)} + d$ exists, where $bx + c \neq 0$, $a \neq 0$ and $d \neq 0$.

a. **i.** What is the effect on the graph when the value of a is increased? 1 mark

_____

_____

ii. What is the effect on the graph when the value of c is increased? 1 mark

_____

_____

iii. What is the effect on the graph when the value of d is increased? 1 mark

_____

_____

b. What type of relationship does $f(x) = \frac{3}{(2x - 4)} + 5$ for $x \in R \setminus \{2\}$ represent? 1 mark

_____

c. Consider the following transformations.

$$(x, y) \rightarrow \left(\frac{x}{3}, 2y\right) \rightarrow \left(\frac{x}{3} - 2, 2y + 3\right)$$

Find the effect of the transformations on $f(x)$. 4 marks

d. **i.** Find the inverse function of $f(x)$, stating its domain and range. 4 marks

ii. A transformation is described by the equation

$$T\left(X+B\right)=X', \text{ where } T=\begin{bmatrix}0 & 2\\ -3 & 0\end{bmatrix} \text{ and } B=\begin{bmatrix}-2\\ 1\end{bmatrix}.$$

Find the image of $f^{-1}\left(x\right)$ under this transformation. 4 marks

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2021

VCE Mathematical Methods Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: _____

Teacher's Name: _____

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out. Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: [A] B C D E

Use pencil only

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

Neap

Trial Examination 2021

VCE Mathematical Methods Units 1&2

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	

END OF FORMULA SHEET