Neap

Trial Examination 2021

VCE Mathematical Methods Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

Question	Answer	Question	Answer
1	B	11	D
2	B	12	D
3	B	13	B
4	D	14	A
5	D	15	E
6	D	16	C
7	E	17	E
8	E	18	C
9	E	19	B
10	C	20	C

Question 1 B

The period of $y = a\tan(nt) + b$ is given by $P = \frac{\pi}{n}$.

For $y = -4\tan\left(\frac{3x}{2}\right) + 2$, $n = \frac{3}{2}$.

$$\begin{aligned} P &= \frac{\pi}{\left(\frac{3}{2}\right)} \\ &= \pi \times \frac{2}{3} \\ &= \frac{2\pi}{3} \end{aligned}$$

Note: A graphics calculator may be used to simplify.

Question 2 B

The midpoint of any two points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$\begin{aligned} \frac{a + 2\sqrt{3}}{2} &= \sqrt{3} \\ a + 2\sqrt{3} &= 2\sqrt{3} \\ a &= 0 \end{aligned} \qquad \begin{aligned} \frac{3\sqrt{5} + b}{2} &= 3\sqrt{5} \\ 3\sqrt{5} + b &= 6\sqrt{5} \\ b &= 3\sqrt{5} \end{aligned}$$

Therefore, a = 0 and $b = 3\sqrt{5}$.

Question 3 B

$$\begin{aligned} \Pr(A' \cap B) &= 0.45 - 0.05 \\ &= 0.40 \end{aligned}$$

$$\begin{aligned} \Pr(A') &= 0.40 + 0.25 \\ &= 0.65 \end{aligned}$$

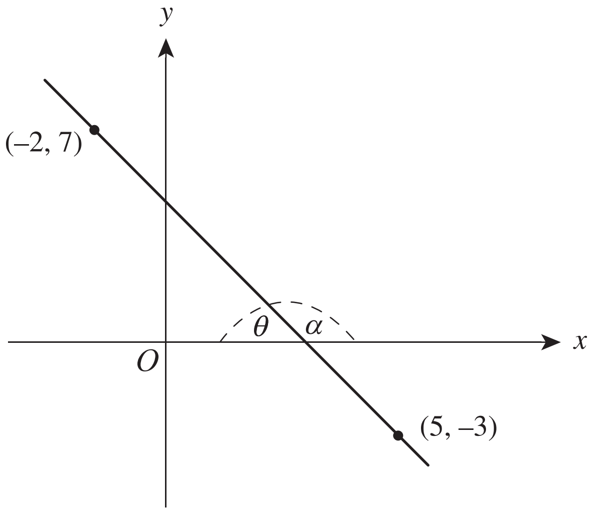
$$\begin{aligned} \Pr(B') &= 1 - 0.45 \\ &= 0.55 \end{aligned}$$

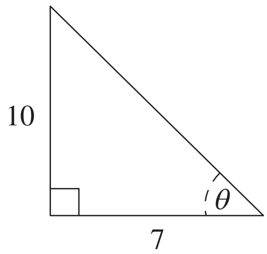
$$\begin{aligned} \Pr(A) &= 1 - 0.65 \\ &= 0.35 \end{aligned}$$

$$\begin{aligned} \Pr(A \cap B') &= 0.35 - 0.05 \\ &= 0.30 \end{aligned}$$

	B	***B′***	**Total**
A	0.05	**0.30**	**0.35**
A′	**0.40**	0.25	**0.65**
Total	0.45	**0.55**	1

Question 4 D



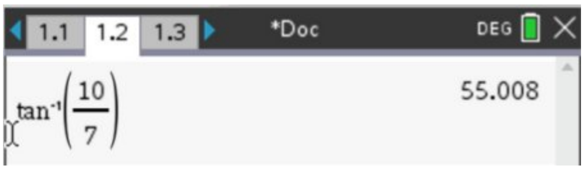


Using CAS calculator:

$$\tan\theta = \frac{10}{7}$$

$$\theta = \tan^{-1}\frac{10}{7}$$

$$= 55.008°$$



$$\alpha = 180° - \theta$$

$$= 180° - 55°$$

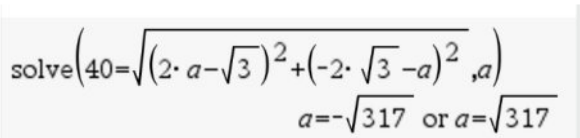
$$= 125°$$

Question 5 D

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$40 = \sqrt{\left(\sqrt{3} - 2a\right)^2 + \left(a - -2\sqrt{3}\right)^2}$$

Solving for a on CAS calculator gives $a = \pm\sqrt{317}$.



Given that a is a positive, real number, $a = \sqrt{317}$.
So, the coordinates are $A\left(\sqrt{3},\ \sqrt{317}\right)$, $B\left(2\sqrt{317},\ -2\sqrt{3}\right)$.

Question 6 D

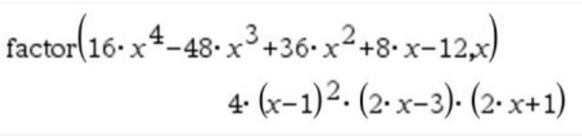
$$\Pr(N|M) = \frac{\Pr(N \cap M)}{\Pr(M)}$$
$$= \frac{\frac{3}{25}}{\frac{13}{25}}$$
$$= \frac{3}{13}$$

Question 7 E

Factorising with a graphics calculator.

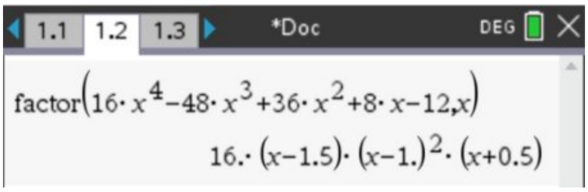
Method 1

Using exact settings gives $4(2x-3)(2x+1)(x-1)^2$.



Method 2

Using approximate settings gives:



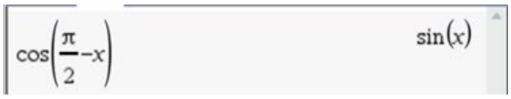
Then, rearrange to represent answer in integers, rather than in decimals.

$$Q(x) = 8(2x-3)(x+0.5)(x-1)^2$$
$$= 4(2x-3)(2x+1)(x-1)^2$$

Question 8 E

Method 1

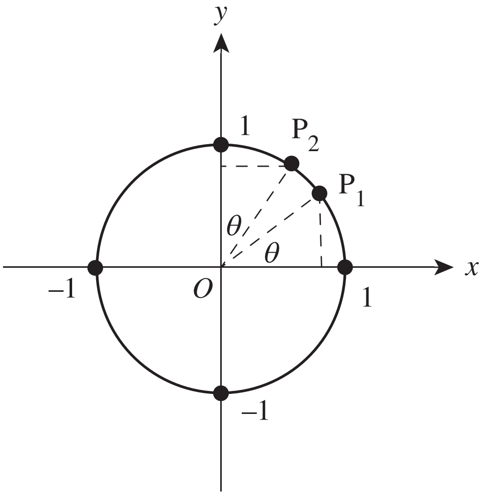
Using a graphics calculator:

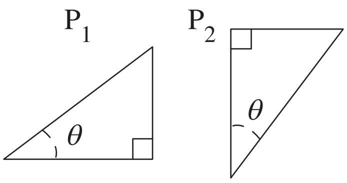


Method 2

Using complimentary angles:

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin(\theta)$$





$$P_1=(x_1,\ x_2)$$
$$=(\cos(\theta),\ \sin(\theta))$$

$$P_2=(x_2,\ y_2)$$
$$=\left(\cos\left(\frac{\pi}{2}-\theta\right),\ \sin\left(\frac{\pi}{2}-\theta\right)\right)$$

The two triangles are similar triangles, therefore $\cos\left(\frac{\pi}{2}-\theta\right)=\sin(\theta)$.

Question 9 E

E is correct. The depth of water is initially increasing linearly from the bottom of the vessel until it reaches the curved part of the bottle. Next, it increases exponentially until the neck of the vessel is reached. The neck of the vessel fills at a much faster rate than the bottom of the vessel and at a constant rate. Therefore, the gradient of the final part of the vessel is much steeper than that of the bottom of the vessel. **A** and **B** are incorrect. These graphs do not show the correct difference in the increase of depth after the water reaches the neck of the vessel. **C** is incorrect. This graph shows a constant increase of water for the bottom and top of the vessel. **D** is incorrect. The gradient of the straight lines of the start of the pouring compared with the end of the pouring are too similar to reflect the shape of the bottle.

Question 10 C

$$\frac{dy}{dx} = x^2 - 3$$

$$x^2 - 3 = -1$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

Using CAS, define the function and solve for *x*.

$f(x) := \frac{x^3}{3} - 3 \cdot x$ Done

$\text{solve}\left(\frac{d}{dx}(f(x)) = -1, x\right)$ $x = -\sqrt{2}$ or $x = \sqrt{2}$

When $x = \sqrt{2}$:

$$y = \frac{(\sqrt{2})^3}{3} - 3 \times \sqrt{2}$$
$$= \frac{(\sqrt{2})^3}{3} - 3\sqrt{2}$$
$$= \frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}$$

Therefore, the first point is $\left(\sqrt{2}, \frac{2^{\frac{3}{2}}}{3} - 3\sqrt{2}\right)$.

When $x = -\sqrt{2}$:

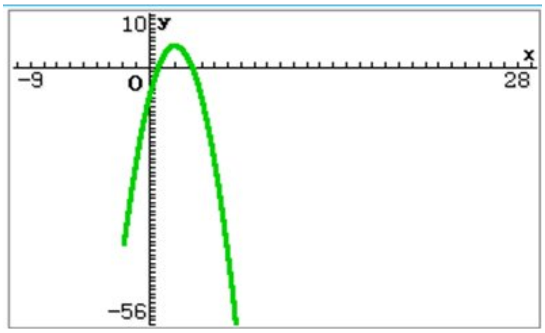
$$y = \frac{(-\sqrt{2})^3}{3} - 3 \times -\sqrt{2}$$
$$= \frac{(-\sqrt{2})^3}{3} + 3\sqrt{2}$$
$$= -\frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}$$

Therefore, the second point is $\left(-\sqrt{2}, -\frac{2^{\frac{3}{2}}}{3} + 3\sqrt{2}\right)$.

 MMU12_SS_Ex2_2021

Question 11 D

D is correct. When sketched on a graphics calculator, this option represents a parabola with a restricted domain. Even with the domain restriction, it is not a one-to-one relationship. Therefore, **D** does not have an inverse function.

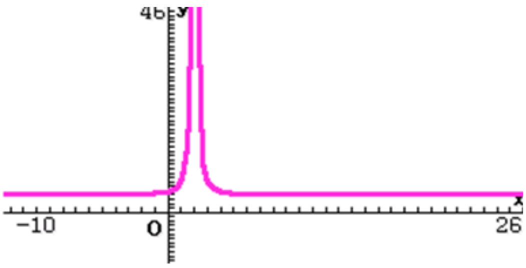


A is incorrect. {(3, 6), (4, 10), (–3, 5), (0, 2)} is a series of points in a one-to-one relationship. Therefore, it does have an inverse function. **B** is incorrect. This option uses the form of $y = mx + c$, which will result in a straight line. A straight line is a one-to-one relationship, so it does have an inverse function. **C** is incorrect. This option is a truncus graph, which is a one-to-one relationship. Therefore, it does have an inverse function. **E** is incorrect. This option is a cubic graph, which is a one-to-one relationship within the domain restriction. Therefore, it does have an inverse function.

Question 12 D

D is correct. This option is a truncus function, which has two asymptotes.

Sketching on a graphics calculator shows:



Asymptote at $x = 2$:

$$(x-2)^2 \neq 0$$
$$x-2 \neq 0$$
$$x \neq 2$$

The second asymptote is at $y = 4$, as the asymptote that is usually at $y = 0$ is translated up 4 units on the y-axis.

Question 13 B

B is correct. The velocity of the ball will decrease with a constant negative gradient until it reaches zero. Then, it will increase at a constant positive gradient until it hits the ground. Therefore, the graph will be made up of two straight lines with gradients of the same magnitude. Of the straight line options remaining, only **B** matches the requirements. **A** is incorrect. The first line increases the velocity with a constant gradient, instead of decreasing. **C** and **D** are incorrect. These graphs use curved lines instead of straight lines. **E** is incorrect. This graph is made of three straight lines, not two.

Question 14 A

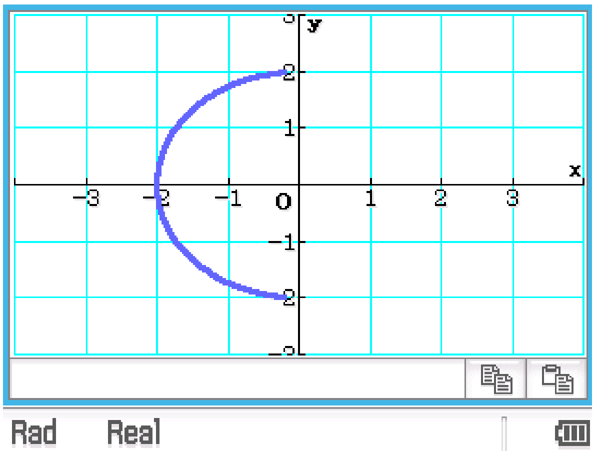
A is correct. Disjoint sets are the same as mutually exclusive sets. Mutually exclusive and disjoint sets have no intersection. **B**, **C**, **D** and **E** are incorrect. These options all show Venn diagrams with intersections.

Question 15 E

$$\begin{aligned}\Pr(2 \le x \le 4) &= 0.05 + 0.20 + 0.15 \\ &= 0.40\end{aligned}$$

Question 16 C

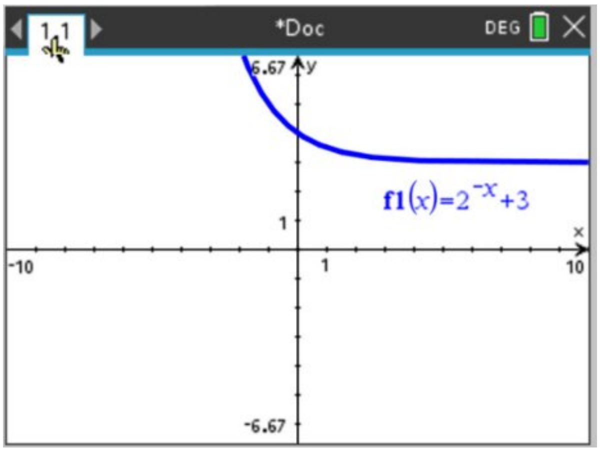
Sketch $x = -\sqrt{4 - y^2}$ on a graphics calculator.



This shows a domain of [–2, 0] and a range of [–2, 2].

Question 17 E

E is correct. Sketch $y = 2^{-x} + 3$ on a graphics calculator.



A is incorrect. This graph represents $y = \log_3(x+3)+3$. **B** is incorrect. This graph represents $y = 2^x + 3$. **C** is incorrect. This graph represents $y = -2^{-x} + 3$. **D** is incorrect. This graph represents $y = -2^x + 3$.

Question 18 C

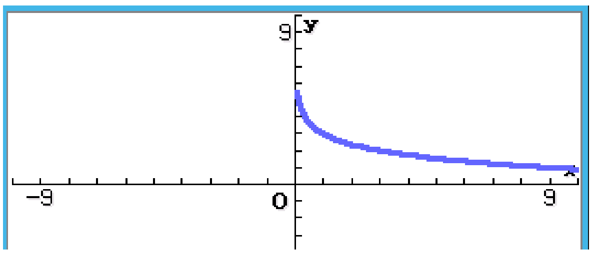
$$\begin{aligned}\Pr\left(X \geq 4 \middle| X > 2\right) &= \frac{0.21+0.25}{0.15+0.21+0.25}\\ &= \frac{0.46}{0.61}\\ &= \frac{46}{61}\end{aligned}$$

Question 19 B

B is correct. Reading from the graph, the range is $2\sqrt{3}$. Therefore, the amplitude of the graph is $\sqrt{3}$. Reading from the graph, one complete cycle is from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$. Therefore, the period is $\frac{5\pi}{4} - \frac{\pi}{4} = \pi$. For sine and cosine graphs, the period is $P = \frac{2\pi}{n}$, where $n = 2$. The graph features a translation of $\frac{\pi}{4}$ in the positive direction of the x-axis. The shape of the graph shows it is a normal sine graph. Therefore, **B** is the only possible option. Alternatively, each response could be graphed to match **B** as the correct answer. **A** and **E** are incorrect. Both equations indicate the graph has been reflected in the x-axis. **C** is incorrect. This option is a cosine graph that has been translated $\frac{\pi}{4}$ in the positive direction of the x-axis. **D** is incorrect. This option represents a cosine graph that has been reflected in the x-axis.

Question 20 C

C is correct. The reflection of the graph of $y = \log_3 x$ in the x-axis results in $y = -\log_3 x$. When this is translated upwards by 3 units, the graph becomes $y = -\log_3 x + 3$. Sketch this on a graphics calculator.



A, **B**, **D** and **E** are incorrect. These options are not the resulting graph.

SECTION B

Question 1 (5 marks)

a. Solving for x using a graphics calculator gives $x = -3$. A1

OR

$$\begin{aligned} x &= \log_{\frac{1}{4}}(64) \\ &= \log_{\frac{1}{4}}(4)^3 \\ &= \log_{\frac{1}{4}}\left(\frac{1}{4}\right)^{-3} \\ &= -3 \end{aligned}$$

A1

b.
$$\begin{aligned} y &= \log_3\left(\sqrt{27}\right) + \log_5(125) + \log_5\left(\frac{1}{25}\right) + \log_3\left(\sqrt{3}\right) \\ &= \log_3(3)^{\frac{3}{2}} + \log_5(5)^3 + \log_5(5)^{-2} + \log_3(3)^{\frac{1}{2}} \end{aligned}$$

M1

$$\begin{aligned} &= \frac{3}{2} + 3 - 2 + \frac{1}{2} \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

A1

c.
$$\begin{aligned} &\left(\log_{\frac{1}{4}}(64)\right)^{\left(\log_3\left(\sqrt{27}\right)+\log_5(125)+\log_5\left(\frac{1}{25}\right)+\log_3\left(\sqrt{3}\right)\right)} \\ &= x^y \\ &= -3^3 \\ &= -27 \end{aligned}$$

A1

d.
$$\begin{aligned} \text{LHS} &= x^{-3}y^{\frac{1}{3}} \times \log_2(16) \\ &= \frac{3^{\frac{1}{3}} \times 4}{(-3)^3} \\ &= \frac{3^{\frac{1}{3}} \times 4}{-(3)^3} \\ &= \frac{-1 \times 4}{3^{\frac{8}{3}}} \\ &= -\frac{4}{3^{\frac{8}{3}}} \end{aligned}$$

A1

Question 2 (8 marks)

a. **i.** $4 \times 5 = 20$

Gem could make 20 outfits. A1

ii. $$\Pr\left(\text{pink T-shirt and pink skirt}\right) = \frac{2}{12} \times \frac{1}{11} = \frac{2}{132} = \frac{1}{66}$$ A1

iii. $$\Pr(\text{at least one pink item}) = 1 - \Pr(\text{no pink items})$$ M1

$$= 1 - \left(\frac{10}{12} \times \frac{9}{11}\right) = 1 - \frac{15}{22} = \frac{7}{22}$$ A1

b. **i.** Total items of clothing: $3 + 4 + 5 = 12$

$12! = 479\,001\,600$ A1

ii. $3! \times 4! \times 5! = 17\,280$ A1

c. $$0.08 + 0.09 + 2k + 0.28 + 3k = 1$$ M1

$$0.45 + 5k = 1$$

$$5k = 0.55$$

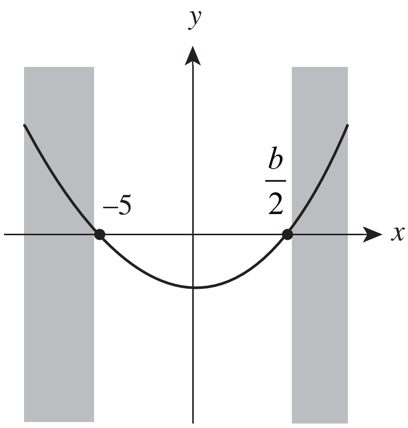
$$k = 0.11$$

$$\Pr\left(\text{pink}\right) = 3k = 0.33$$ A1

Question 3 (7 marks)

a. Solve $(2x-b)(x+5)=0$

$\therefore x=-5$ and $x=-\dfrac{b}{2}$ M1



rough sketch of graph M1

The values of x where the function is greater than or equal to zero are shaded in the diagram above.

Hence, $\left\{x : x \le -5 \cup x \ge \dfrac{b}{2}\right\}$ OR $\{x : x \le -5\} \cup \left\{x : x \ge \dfrac{b}{2}\right\}$. A1

b. Let $y_1=(2x-3)(x+5)$ and $y_2=3kx-15$.

For the tangent line:

$$y_1=y_2$$

$$(2x-3)(x+5)=3kx-15$$

$$2x^2+10x-3x-15=3kx-15 \qquad \text{M1}$$

$$2x^2+7x-15=3kx-15$$

$$2x^2+7x-3kx=0$$

$$2x^2+x(7-3k)=0 \qquad \text{A1}$$

$$b^2-4ac=0$$

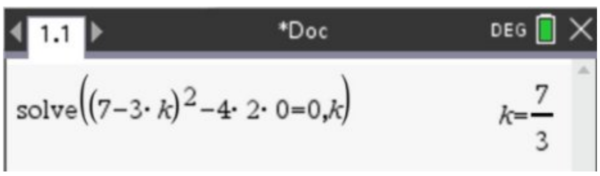
$$(7-3k)^2-4(2)(0)=0 \qquad \text{M1}$$

$$(7-3k)^2=0$$

$$7-3k=0$$

$$k=\frac{7}{3} \qquad \text{A1}$$

Alternatively, solve the second part on a CAS calculator:



 MMU12_SS_Ex2_2021

Question 4 (13 marks)

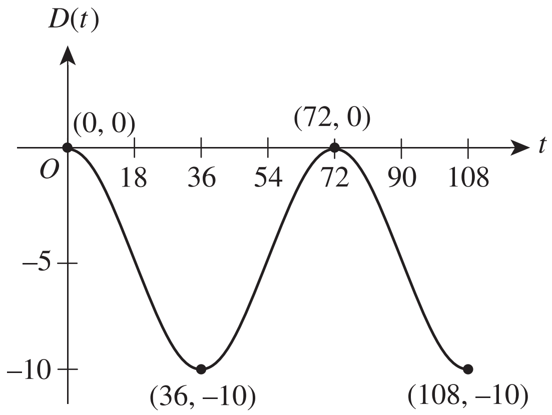
a. The maximum depth is 10 m. A1

Note: Accept –10 m.

b. $P = \frac{2\pi}{n}$, where $n = \frac{\pi}{36}$.

$$P = \frac{2\pi}{\frac{\pi}{36}}$$
$$= 2\pi \times \frac{36}{\pi}$$
$$= 72 \text{ hours}$$ A1

c.



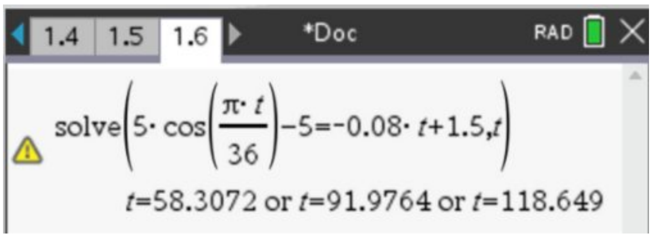
correct shape A1
endpoints labelled (0, 0) and (108, –10) A1
maximum labelled (72, 0) A1
minimum labelled (36, –10) A1

d.
$$5\cos\left(\frac{\pi t}{36}\right) - 5 = -7.5$$
$$5\cos\left(\frac{\pi t}{36}\right) = -2.5$$
$$\cos\left(\frac{\pi t}{36}\right) = -\frac{1}{2}$$ M1
$$\frac{\pi t}{36} = \frac{2\pi}{3},\ \frac{4\pi}{3}$$
$$\frac{t}{36} = \frac{2}{3},\ \frac{4}{3}$$
$$t = \frac{2}{3} \times 36,\ \frac{4}{3} \times 36$$
$$= 24,\ 48,\ (24 + 72)$$
$$= 24,\ 48,\ 96 \text{ hours}$$

correct responses 24 and 48 hours A1
correct response 96 hours A1

e. $5\cos\left(\frac{\pi t}{36}\right)-5=-0.08t+1.5$ M1

Solve the equation using a graphics calculator.



(58.31, –3.16) A1

(91.98, –5.86) A1

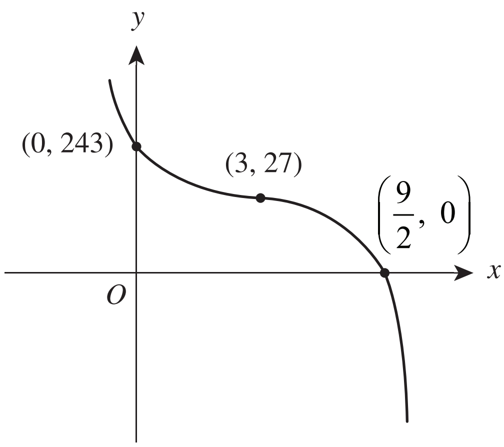
answers correct to two decimal places A1

Note: t = 118.649 is not included as it is outside the domain.

Question 5 (11 marks)

a. **i.** $f'(3)=0$ is a point of inflection. A1

ii.



correct shape A1

both intercepts labelled A1

stationary point of inflection labelled A1

Note: Graph does not need to be to scale.

b. i. $y = ax^3 + bx^2 + cx + d$

d is the y-intercept, so $d = 243$.

Substitute the point (3, 27) to find equation 1:

$27 = a(3)^3 + b(3)^2 + c(3) + 243$

Substitute the point $\left(\frac{9}{2}, 0\right)$ to find equation 2.

$$0 = a\left(\frac{9}{2}\right)^3 + b\left(\frac{9}{2}\right)^2 + c\left(\frac{9}{2}\right) + 243$$

both equations correct M1

For equation 3, find $\frac{dy}{dx}$.

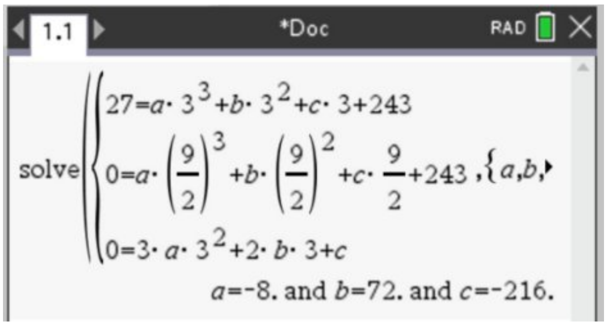
$\frac{dy}{dx} = 3ax^2 + 2bx + c$

Given $\frac{dy}{dx} = 0$ when $x = 3$, substitute $x = 3$ to find equation 3.

$0 = 3a(3)^2 + 2b(3) + c$

derivative and equation correct M1

Solve the three simultaneous equations using a graphics calculator.



$a = -8$, $b = 72$, $c = -216$ A1

$\therefore f(x) = -8x^3 + 72x^2 - 216x + 243$ A1

ii. Let $y = -8(x-3)^3 + 27$.

For inverse, swap x and y.

$$x = -8(y-3)^3 + 27 \qquad \text{M1}$$

$$x - 27 = -8(y-3)^3$$

$$\frac{27-x}{8} = (y-3)^3$$

$$\sqrt[3]{\frac{27-x}{8}} = y - 3$$

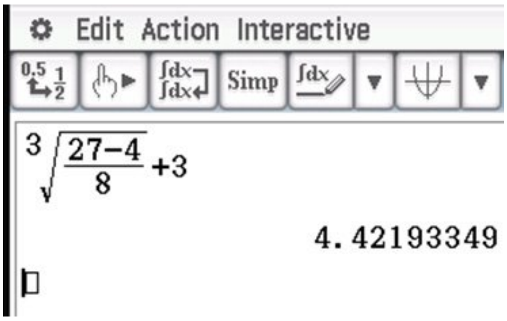
$$\sqrt[3]{\frac{27-x}{8}} + 3 = y$$

$$\therefore f^{-1}(x) = \sqrt[3]{\frac{27-x}{8}} + 3 \qquad \text{A1}$$

Note: Also accept $f^{-1}(x) = \dfrac{\sqrt[3]{27-x}}{2} + 3$.

iii. $f^{-1}(4) = \sqrt[3]{\dfrac{27-4}{3}} + 3$

$= 4.422$ A1



Question 6 (16 marks)

a. **i.** dilation of factor a from the x-axis A1

ii. moves the graph in the negative direction of the x-axis A1

iii. moves the graph in the positive direction of the y-axis A1

b. 1 : 1 function A1

c. $x' = \dfrac{x}{3} - 2$

$\dfrac{x}{3} = x' + 2$

$x = 3x' + 6$ M1

$y' = 2y + 3$

$2y = y' - 3$

$y = \dfrac{y'}{2} - \dfrac{3}{2}$ M1

$\dfrac{y'}{2} - \dfrac{3}{2} = \dfrac{3}{\left(2\left(3x' + 6\right) - 4\right)} + 5$ M1

$\dfrac{y'}{2} - \dfrac{3}{2} = \dfrac{3}{6x' + 12 - 4} + 5$

$\dfrac{y'}{2} = \dfrac{3}{6x' + 12 - 4} + 5 + \dfrac{3}{2}$

$\dfrac{y'}{2} = \dfrac{3}{6x' + 8} + \dfrac{13}{2}$

$\dfrac{y'}{2} = \dfrac{3}{2\left(3x' + 4\right)} + \dfrac{13}{2}$

$y' = \dfrac{3}{3x' + 4} + 13$

The graph of the image is $y' = \dfrac{3}{3x' + 4} + 13$. A1

d. **i.** $x = \dfrac{3}{2y - 4} + 5$ M1

$x - 5 = \dfrac{3}{2y - 4}$

$2y - 4 = \dfrac{3}{x - 5}$ M1

$2y = \dfrac{3}{x - 5} + 4$

$y = \dfrac{3}{2\left(x - 5\right)} + 2$

$f^{-1}(x) = \dfrac{3}{2\left(x - 5\right)} + 2$ A1

As $x - 5 \neq 0$

$x \neq 5$

Domain: $R \setminus \{5\}$

Range: $R \setminus \{2\}$ *domain and range correct* A1

ii. $\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)=\begin{bmatrix} x' \\ y' \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix}=\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x-2 \\ y+1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix}=\begin{bmatrix} 2(y+1) \\ -3(x-2) \end{bmatrix}$$ M1

$x' = 2(y+1)$

$\frac{x'}{2} = y+1$

$y = \frac{x'}{2} - 1$

$y' = -3(x-2)$

$-\frac{y'}{3} = x-2$

$x = \frac{-y'}{3} + 2$

equations $y = \frac{x'}{2} - 1$ *and* $x = \frac{-y'}{3} + 2$ *provided* M1

$f^{-1}(x) = \frac{3}{2(x-5)} + 2$

$\frac{x'}{2} - 1 = \frac{3}{2\left(\frac{-y'}{3} + 2 - 5\right)} + 2$ M1

$x' - 2 = \frac{3}{\left(\frac{-y'}{3} - 3\right)} + 4$

$x' - 6 = \frac{3}{\left(\frac{-y'}{3} - 3\right)}$

$\frac{-y'}{3} - 3 = \frac{3}{(x'-6)}$

$\frac{-y'}{3} = \frac{3}{(x'-6)} + 3$

$-y' = \frac{9}{(x'-6)} + 9$

$y' = -\frac{9}{(x'-6)} - 9$

$= \frac{9}{(6-x')} - 9$

The image is $y = \frac{9}{(6-x)} - 9$. A1

Note: Not all steps are required to be shown.