

The Mathematical Association of Victoria

Trial Examination 2021

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	D	11	D
2	B	12	E
3	A	13	E
4	E	14	B
5	E	15	C
6	C	16	B
7	A	17	C
8	C	18	B
9	D	19	A
10	A	20	C

Question 1 **Answer D**

$$y = -2 \cos\left(\frac{x}{3}\right) - \frac{1}{2}$$

The amplitude is 2.

$$\text{The period is } \frac{2\pi}{\frac{1}{3}} = 6\pi.$$

Question 2 **Answer B**

$$g : D \rightarrow R, g(x) = \frac{1}{1+3x} \text{ has range } [-0.5, -0.2).$$

$$\text{Solve } \frac{1}{1+3x} = -0.5 \text{ and } \frac{1}{1+3x} = -0.2 \text{ for } x.$$

$$x = -1 \text{ and } x = -2$$

The coordinates of the endpoints are $(-2, -0.2)$ and $(-1, -0.5)$.

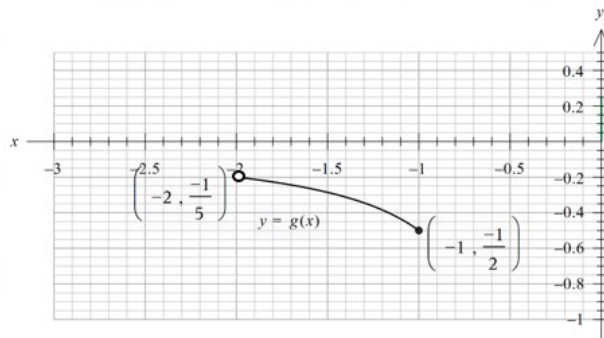
g is a decreasing function.

The domain is $(-2, -1]$

1.1 *MAVMC2021 RAD

solve $\left(\frac{1}{1+3 \cdot x} = -0.5, x\right)$ $x = -1.$

solve $\left(\frac{1}{1+3 \cdot x} = -0.2, x\right)$ $x = -2.$

**Question 3** **Answer A**

$$f(x) = x^5 + mx^3 - nx^2 - 1$$

Solve $f(-2) = 0$ and $f(1) = 5$.

1.1 1.2 *MAVMC2021 RAD

Define $f(x) = x^5 + m \cdot x^3 - n \cdot x^2 - 1$ Done

solve $(f(-2) = 0 \text{ and } f(1) = 5, m, n)$

$m = \frac{-13}{12}$ and $n = \frac{-73}{12}$

Question 4 **Answer E**

$$nx - 2y = m$$

$$n^2x + 6y = m + 1$$

The gradients need to be the same for no solutions.

Using ratios

$$n = -\frac{2}{6}n^2$$

$$-n^2 = 3n$$

$$-n(n+3) = 0$$

$$n = 0 \text{ or } n = -3$$

$$-\frac{1}{3} = \frac{m}{m+1} \text{ for infinite number of solutions}$$

$$m+1 = -3m$$

$$m = -\frac{1}{4}$$

So, for no solutions

$$n = 0, m \in R \setminus \left\{ -\frac{1}{4} \right\} \text{ or } n = -3, m \in R \setminus \left\{ -\frac{1}{4} \right\}$$

OR

The gradients need to be the same for no solutions and the y -intercepts need to be different.

$$nx - 2y = m$$

$$y = \frac{nx}{2} - \frac{m}{2}$$

$$n^2x + 6y = m + 1$$

$$y = \frac{-n^2x}{6} + \frac{m+1}{6}$$

The gradients need to be the same for no solutions.

$$\frac{n}{2} = -\frac{n^2}{6}$$

$$-n^2 = 3n$$

$$-n(n+3) = 0$$

$$n = 0 \text{ or } n = -3$$

$$-\frac{m}{2} = \frac{m+1}{6} \text{ for infinite number of solutions}$$

$$-3m = m+1$$

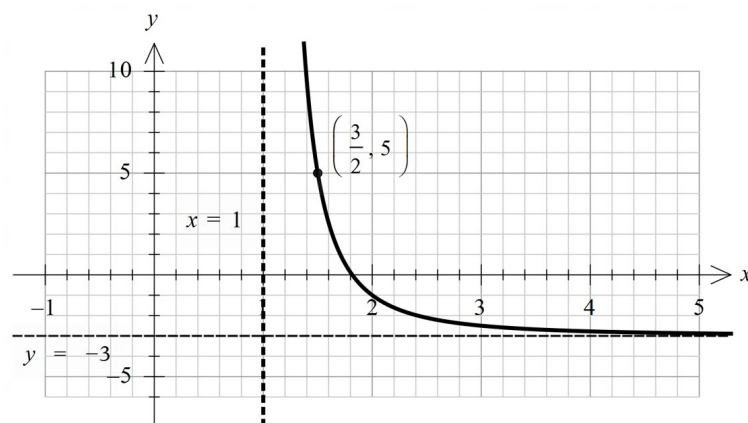
$$m = -\frac{1}{4}$$

So for no solutions

$$n = 0, m \in R \setminus \left\{ -\frac{1}{4} \right\} \text{ or } n = -3, m \in R \setminus \left\{ -\frac{1}{4} \right\}$$

Question 5

Answer E



$$y = \frac{a}{(x-1)^2} - 3$$

$$\text{Solve } 5 = \frac{a}{(1.5-1)^2} - 3 \text{ for } a.$$

$$a = 2$$

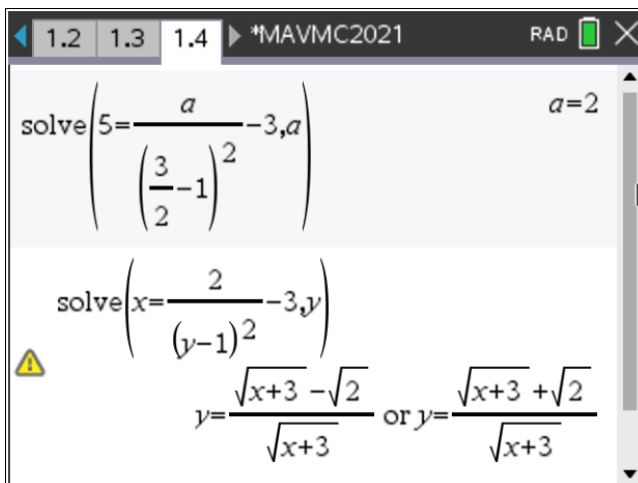
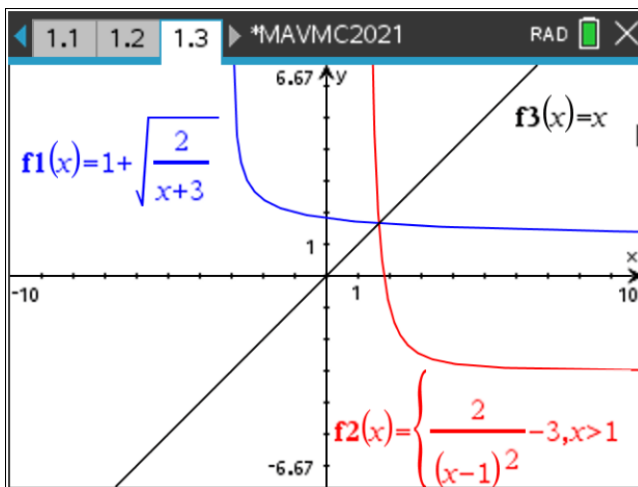
$$y = \frac{2}{(x-1)^2} - 3$$

Inverse swap x and y and solve for y .

$$x = \frac{2}{(y-1)^2} - 3$$

$$y = 1 \pm \sqrt{\frac{2}{x+3}}$$

$$f^{-1}(x) = 1 + \sqrt{\frac{2}{x+3}}$$



Question 6 **Answer C**

$$f: \left[-\frac{\pi}{3}, \pi\right] \rightarrow \mathbb{R}, f(x) = -2\sin(3x) + \sqrt{3}.$$

Maximum rate of change is when the gradient is at its maximum.

One method is to look where the 2nd derivative $f''(x) = 0$, which gives the maximum and minimum gradient points. Remember that the gradient does not exist at an end-point.

Define $f(x) = -2 \cdot \sin(3 \cdot x) + \sqrt{3}$

$\frac{d}{dx}(f(x))$

$-6 \cdot \cos(3 \cdot x)$

solve $\left\{ \frac{d^2}{dx^2}(f(x)) = 0 \mid -\frac{\pi}{3} < x < \pi, \right\}$

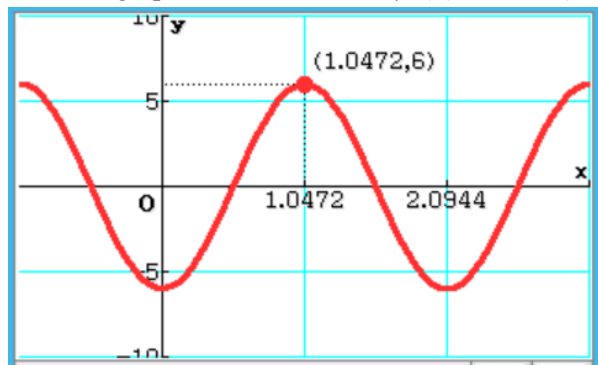
$\left\{ x=0, x=\frac{\pi}{3}, x=\frac{2 \cdot \pi}{3} \right\}$

Note $x=0, x=\frac{2\pi}{3}$ are minimums

The maximum is at $x = \frac{\pi}{3}$.

OR

Sketch a graph of the derivative, $f'(x) = -6\cos(3x)$



This shows that the maximum rate of change is at $x = \frac{\pi}{3}$ only.

Question 7 **Answer A**

$$f: \left[0, \frac{\pi}{2}\right) \rightarrow \mathbb{R}, f(x) = \tan(x) \text{ and } g(x) = \sqrt{2x+1} \text{ over its maximal domain.}$$

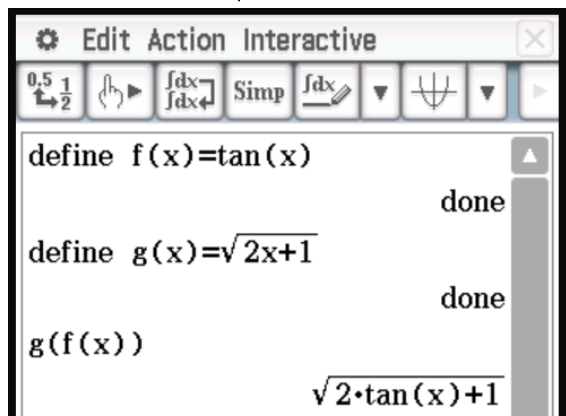
For the composite function $h(x) = g(f(x))$ to exist the range of f must be a subset or equal to the domain of g .

Is the range of $f \subseteq$ domain of g ?

Is $[0, \infty) \subset \left[-\frac{1}{2}, \infty\right)$? Yes true.

So, the domain of $g(f(x))$ is the same as the domain of f which is $\left[0, \frac{\pi}{2}\right)$.

Rule of $g(f(x)) = \sqrt{2 \tan(x) + 1}$



Answer: $h(x) = \sqrt{2 \tan(x) + 1}$ and $x \in \left[0, \frac{\pi}{2}\right)$

Question 8 Answer C

Function $y = -\sqrt{x}$ transformed to the image rule $y = -\sqrt{3x+1} - \frac{1}{4}$

Transformations, in an appropriate order are

- a translation of 1 unit left, giving $y_1 = -\sqrt{x+1}$
- a dilation by a factor of $\frac{1}{3}$ from the y -axis, giving $y_2 = -\sqrt{3x+1}$
- a translation of $\frac{1}{4}$ of a unit down, giving $y_3 = -\sqrt{3x+1} - \frac{1}{4}$

Question 9 Answer D

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is applied to $y = -x^4$

Expanding matrices gives $\begin{bmatrix} 3x+1 \\ -2y \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

$$x = \frac{x' - 1}{3} \quad \text{and} \quad y = \frac{y'}{-2}$$

Substitute in the equation $y = -x^4$

$$\frac{y'}{-2} = -\left(\frac{x' - 1}{3}\right)^4$$

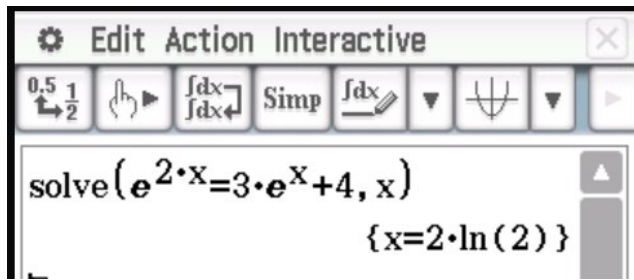
$$y' = 2\left(\frac{x' - 1}{3}\right)^4$$

Matching Answer D: $y = \frac{2(x-1)^4}{81}$

Question 10 Answer A

Area enclosed equals upper $y = g(x) = 3e^x + 4$ minus lower $y = f(x) = e^{2x}$

From $x = 0$ to point of intersection at $x = 2\ln(2) = \ln(4)$



$$\text{Area} = \int_0^{2\log_e(2)} (g(x) - f(x)) dx$$

$$\text{Swapping limits gives } \int_{\log_e(4)}^0 (f(x) - g(x)) dx$$

Question 11

Answer D

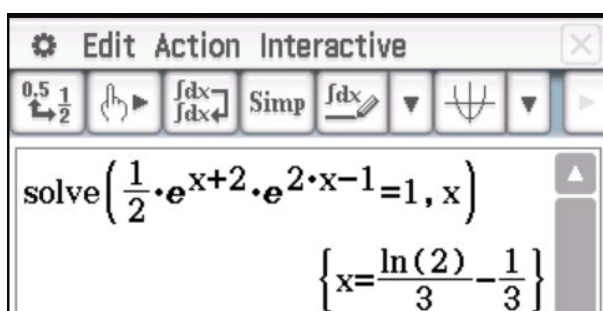
$$g(x) = \frac{1}{2^x} = 2^{-x}$$

$$g(1-x) = \frac{1}{2^{1-x}} = \frac{1}{2} \times 2^x = \frac{1}{2} g(-x)$$

Question 12

Answer E

$$\frac{1}{2} e^{x+2} \times e^{2x-1} = 1$$



$$\text{Using Change of base answer equals } x = \frac{\log_2(2)}{3\log_2(e)} - \frac{1}{3} = \frac{1}{3\log_2(e)} - \frac{1}{3}$$

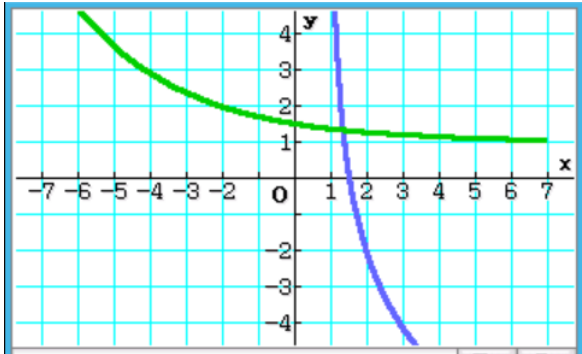
Question 13 **Answer E**

$f(x) = -3 \log_e(2x-2)$ has vertical asymptote at $x = 1$

Inverse f^{-1} is an exponential function with a horizontal asymptote at $y = 1$

Possible answers are D or E

y -intercept of f^{-1} sits between $y = 1$ and $y = 2$



Matching Answer E

Question 14 **Answer B**

The length of the pieces of wire are $20 - 4x$ cm and $4x$ cm.

Rectangle

Perimeter is $20 - 4x$ cm

$6w = 20 - 4x$, where w is the width of the rectangle

$$w = \frac{10 - 2x}{3}, \quad l = \frac{20 - 4x}{3}$$

$$A_{\text{rectangle}} = \left(\frac{10 - 2x}{3}\right)\left(\frac{20 - 4x}{3}\right)$$

$$A_{\text{rectangle}} + A_{\text{square}} = \left(\frac{10 - 2x}{3}\right)\left(\frac{20 - 4x}{3}\right) + x^2$$

For the minimum area $x = \frac{40}{7}$.

Minimum area is $\frac{200}{17}$ cm²

TI-84 Plus calculator screen showing the minimum of a function. The function is $f(x) = \frac{20-4x}{3} \cdot \frac{10-2x}{3} + x^2$ for $0 < x < 20$. The calculator displays the minimum value $x = \frac{40}{17}$ and the corresponding function value $\frac{200}{17}$.

$$f\text{Min}\left(\frac{20-4 \cdot x}{3} \cdot \frac{10-2 \cdot x}{3} + x^2, x\right) | 0 < x < 20$$

$$x = \frac{40}{17}$$

$$\frac{20-4 \cdot x}{3} \cdot \frac{10-2 \cdot x}{3} + x^2 \Big|_{x = \frac{40}{17}} = \frac{200}{17}$$

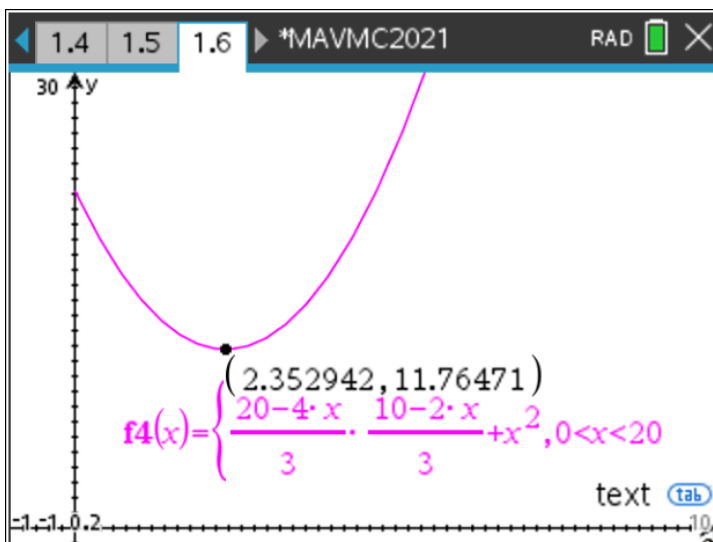
OR

TI-84 Plus calculator screen showing the derivative of the function. The function is $f(x) = \frac{20-4x}{3} \cdot \frac{10-2x}{3} + x^2$ for $0 < x < 20$. The calculator displays the derivative $\frac{d}{dx}\left(\frac{20-4x}{3} \cdot \frac{10-2x}{3} + x^2\right) = 0$ and the solution $x = \frac{40}{17}$. The corresponding function value is $\frac{200}{17}$.

$$\text{solve}\left(\frac{d}{dx}\left(\frac{20-4 \cdot x}{3} \cdot \frac{10-2 \cdot x}{3} + x^2\right) = 0, x\right) | 0 < x < 20$$

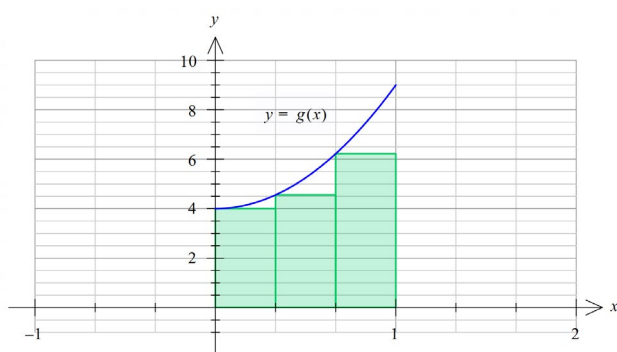
$$x = \frac{40}{17}$$

$$\frac{20-4 \cdot x}{3} \cdot \frac{10-2 \cdot x}{3} + x^2 \Big|_{x = \frac{40}{17}} = \frac{200}{17}$$



Question 15

Answer C



$$y = g(x) = 5x^2 + 4$$

Left-endpoint rectangles

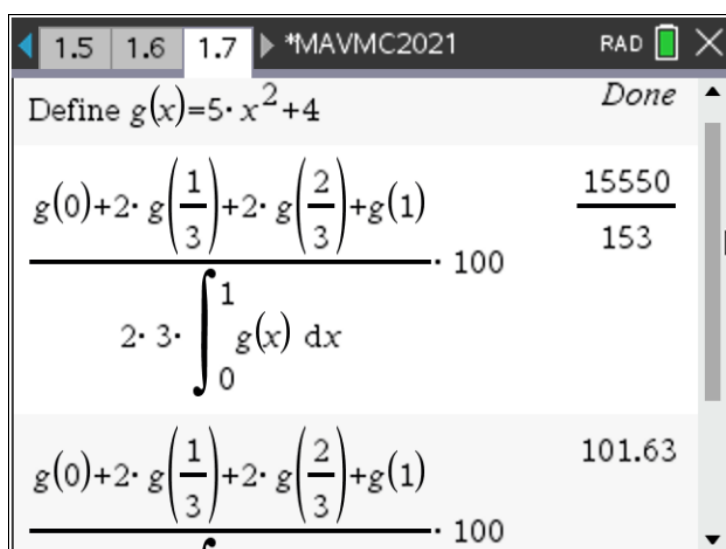
$$A_L = \frac{1}{3} \left(g(0) + g\left(\frac{1}{3}\right) + g\left(\frac{2}{3}\right) \right)$$

Right-endpoint rectangles

$$A_R = \frac{1}{3} \left(g\left(\frac{1}{3}\right) + g\left(\frac{2}{3}\right) + g(1) \right)$$

$$\text{Average} = \frac{g(0) + 2g\left(\frac{1}{3}\right) + 2g\left(\frac{2}{3}\right) + g(1)}{3 \times 2}$$

$$\text{Percentage of exact area} = \frac{g(0) + 2g\left(\frac{1}{3}\right) + 2g\left(\frac{2}{3}\right) + g(1)}{3 \times 2 \int_0^1 (g(x)) dx} \times 100\% = 101.6\%$$

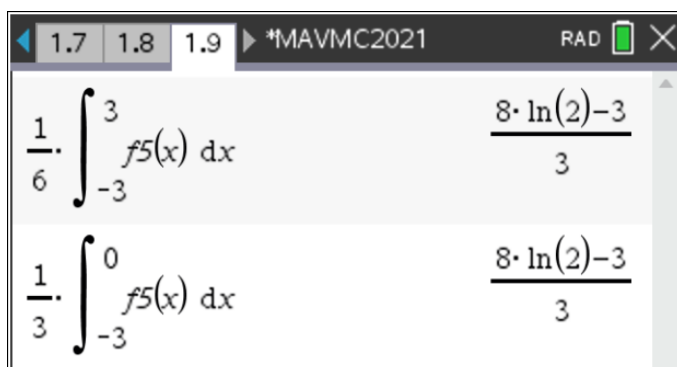
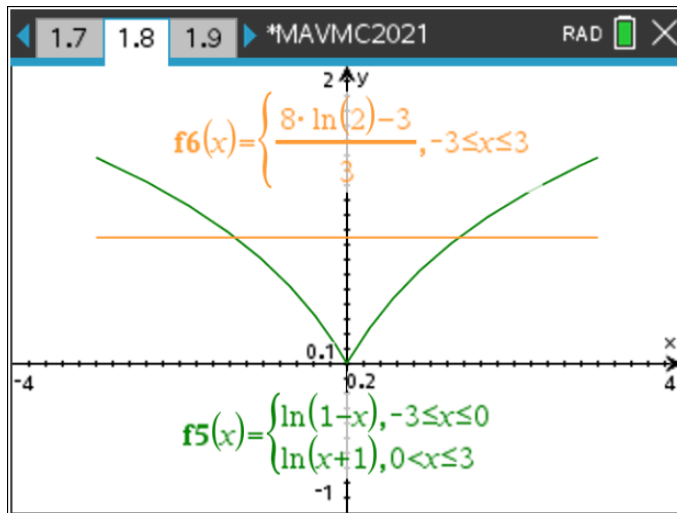


Question 16 **Answer B**

$$\text{Average value} = \frac{1}{6} \int_{-3}^3 (h(x)) dx$$

The branches are symmetrical.

$$\text{Average value} = \frac{2}{6} \int_{-3}^0 (h(x)) dx = \frac{1}{3} \int_{-3}^0 (h(x)) dx$$

**Question 17** **Answer C**

$$\Pr(X = x) = \frac{20}{859} x^2 - \frac{20}{859x}$$

$$\Pr(X = -1) + \Pr(X = 4) + \Pr(X = -5)$$

$$= \frac{40}{859} + \frac{315}{859} + \frac{504}{859} = 1$$

$$\frac{20}{289} \cdot x^2 - \frac{20}{859 \cdot x} | x = \{-1, 4, -5\}$$

$$\left\{ \frac{22960}{248251}, \frac{273435}{248251}, \frac{430656}{248251} \right\}$$

$$\frac{40}{859} + \frac{315}{859} + \frac{504}{859} = 1$$

Question 18**Answer B**

(0.162, 0.238)

zInterval_1Prop 60,300,0.9: *stat.results*

"Title"	"1-Prop z Interval"
"CLower"	0.162014
"CUpper"	0.237986
"p"	0.2
"ME"	0.037986
"n"	300.

C-Level: 0.9
x: 0.2 (300)
n: 300

Lower: 0.1620137
Upper: 0.2379863
p-hat: 0.2
n: 300

Question 19**Answer A**

Solve $\frac{2 - \mu}{\sigma} = -2.527\dots$ and $\frac{4.8 - \mu}{\sigma} = 2.563\dots$

$$\mu = 3.39, \sigma = 0.55$$

$$f(x) = \frac{1}{0.55\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-3.39}{0.55}\right)^2}$$

Giving $z = -2.527\dots$ and $z = 2.563\dots$

$$\begin{cases} -2.527 = \frac{2-m}{s} \\ 2.563 = \frac{4.8-m}{s} \end{cases} \quad m, s$$

{m=3.390098232, s=0.5500982318}

Question 20 Answer C

$$p \sim \text{Bi}\left(n, \frac{2}{5}\right), f \sim \text{Bi}\left(n, \frac{3}{5}\right)$$

Examples

$$f(n) = \binom{n}{n} \left(\frac{3}{5}\right)^n \left(\frac{2}{5}\right)^0 = p(0) = \binom{n}{0} \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^n$$

$$f(n-1) = \binom{n}{n-1} \left(\frac{3}{5}\right)^{n-1} \left(\frac{2}{5}\right) = p(1) = \binom{n}{1} \left(\frac{2}{5}\right) \left(\frac{3}{5}\right)^{n-1}$$

In general

$$f(m) = \binom{n}{m} \left(\frac{3}{5}\right)^m \left(\frac{2}{5}\right)^{n-m} = p(n-m) = \binom{n}{n-m} \left(\frac{2}{5}\right)^{n-m} \left(\frac{3}{5}\right)^m$$

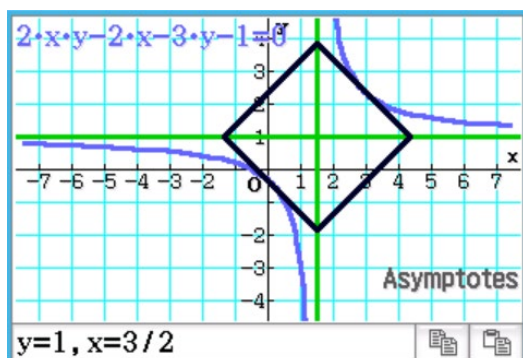
$$f(m) = p(n-m)$$

SECTION B**Question 1**

$$\begin{aligned}
 \text{a. } & \frac{2x+1}{2x-3} \\
 &= \frac{2x-3+3+1}{2x-3} \\
 &= \frac{2x-3+4}{2x-3} \\
 &= \frac{2x-3}{2x-3} + \frac{4}{2x-3} \\
 &= 1 + \frac{4}{2x-3}
 \end{aligned}$$

giving $a=1$ and $b=4$ **1M Show that**

$$f(x) = \frac{2x+1}{2x-3} = 1 + \frac{4}{2x-3}$$

b.i. By inspection the asymptotes are $y=1$ and $x=\frac{3}{2}$.**1A****ii.** domain $R \setminus \left\{ \frac{3}{2} \right\}$, range $R \setminus \{1\}$ **2A**

$$f_1: \left[-\frac{1}{2}, \frac{1}{6} \right] \rightarrow R, f_1(x) = \frac{2x+1}{2x-3}, y = f_1(f_1(x)) = \frac{1-6x}{2x-11}$$

c. For $f_1(f_1(x))$ to exist test range $f_1 \subseteq \text{domain } f_1$.

$$\text{range} \left[-\frac{1}{2}, 0 \right] \subset \left[-\frac{1}{2}, \frac{1}{6} \right] \quad \mathbf{1A}$$

d.i. $\left(0, -\frac{1}{11} \right)$ and $\left(\frac{1}{6}, 0 \right)$ **2A**

```

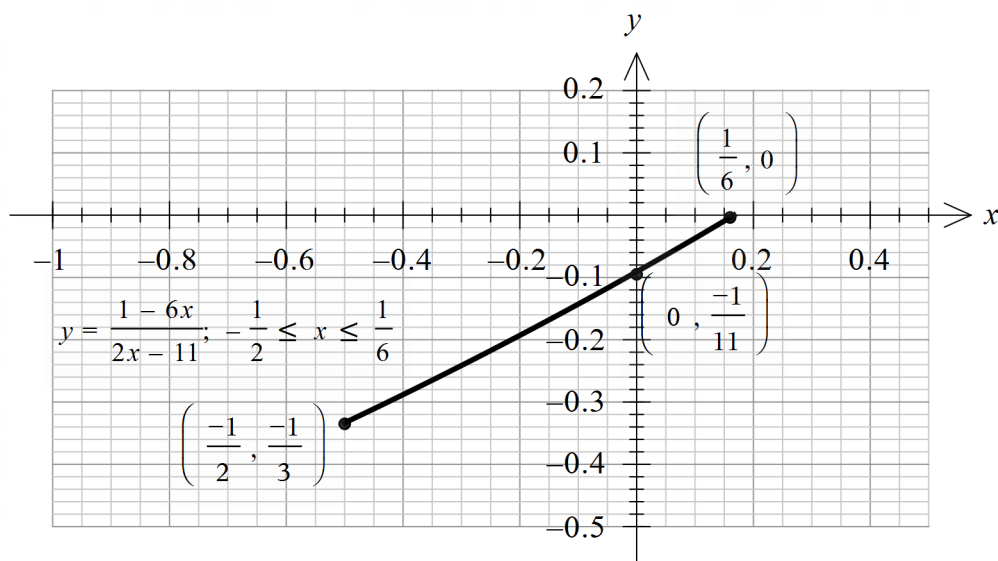
Edit Action Interactive
0.5 1/2  f/dx  f/dx  Simp  f/dx  f/dx
define f(x) = 4/(2*x-3) + 1
done
simplify(f(f(x)))
-(6*x-1)/(2*x-11)
define F(x) = -(6*x-1)/(2*x-11)
done
F(0)
-1/11
solve(F(x)=0, x)
{x=1/6}

```

ii. shape

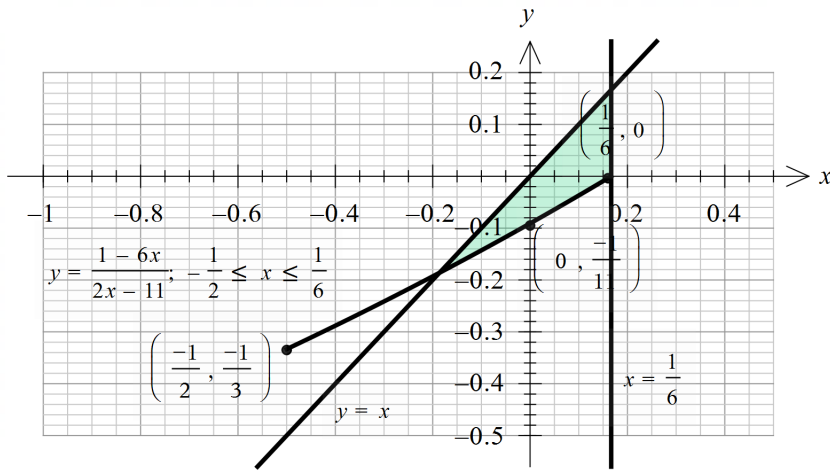
1A

axial intercepts and endpoints $\left(-\frac{1}{2}, -\frac{1}{3}\right)$, $\left(0, -\frac{1}{11}\right)$ and $\left(\frac{1}{6}, 0\right)$ **1A**

e.i. Solve $g(x) = x$

$$x = \frac{5 - \sqrt{33}}{4} \quad \mathbf{1A}$$

$$\text{Area} = \int_{\frac{5-\sqrt{33}}{4}}^{\frac{1}{6}} (x - g(x)) dx = \int_{\frac{5-\sqrt{33}}{4}}^{\frac{1}{6}} \left(x - \frac{1-6x}{2x-11}\right) dx = \int_{\frac{5-\sqrt{33}}{4}}^{\frac{1}{6}} \left(x + \frac{6x-1}{2x-11}\right) dx \quad \mathbf{1A (any)}$$

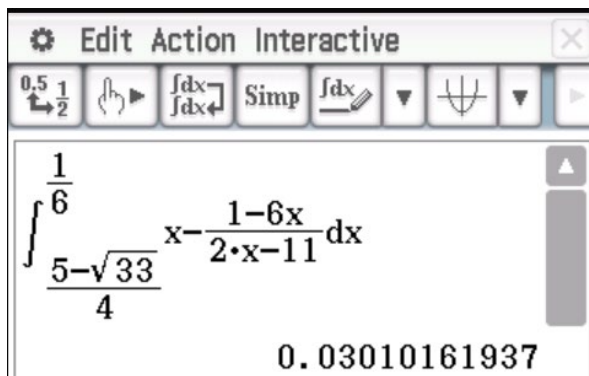


solve $(F(x) = x, x)$

$$\left\{ x = \frac{-\sqrt{33} + 5}{4}, x = \frac{\sqrt{33} + 5}{4} \right\}$$

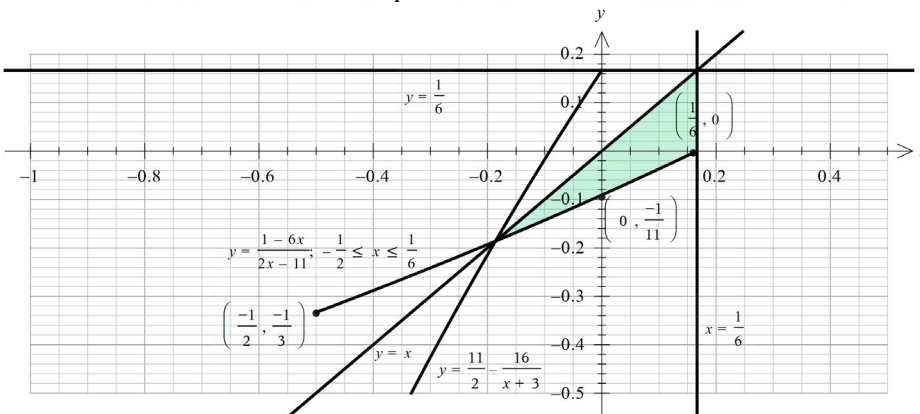
ii. 0.030 correct to three decimal places

1A



iii. 0.030 correct to three decimal places

1H



Question 2

$$d(t) = e^{-kt} \sin(kt)$$

a. $d'(t) = ke^{-kt}(\cos(kt) - \sin(kt))$ **1A**

The screenshot shows a CAS interface with a toolbar containing icons for exponentiation, integration, simplification, and differentiation. The main window contains the following text:

```

define d(t)=e-ktsin(kt)
done
simplify(d/dt(d(t)))
k*(cos(k*t)-sin(k*t))*e-k*t

```

b.i. Solve $d'(t) = ke^{-kt}(\cos(kt) - \sin(kt)) = 0$

gives $ke^{-kt} = 0$ no solution. **1M**

and $(\cos(kt) - \sin(kt)) = 0$

$$\cos(kt) = \sin(kt)$$

$$\frac{\cos(kt)}{\cos(kt)} = \frac{\sin(kt)}{\cos(kt)}$$

giving $\tan(kt) = 1$

1M Show that

ii. $d'(t) = 0$, $\tan(kt) = 1$

giving general solution $t = \frac{\pi}{4k} + \frac{\pi}{k}n$ where $k \in \mathbb{R}^+$, $n \in \{0\} \cup \mathbb{Z}^+$ **1A**

The screenshot shows a CAS interface with the following text:

```

solve(tan(k*t)=1, t)
{t = pi*constn(1)/k + pi/(4*k)}

```

iii. $t = \frac{\pi}{4k} + \frac{\pi}{k}n$

For $d: \left[0, \frac{2}{k}\right] \rightarrow \mathbb{R}$, $d(t) = e^{-kt} \sin(kt)$

Letting $n = 0$ gives the local maximum stationary point at $t = \frac{\pi}{4k}$ **1A**

Let $d_j: [0, 10] \rightarrow \mathbb{R}$, $d_j(t) = 10e^{-0.2t} \sin(0.2t)$

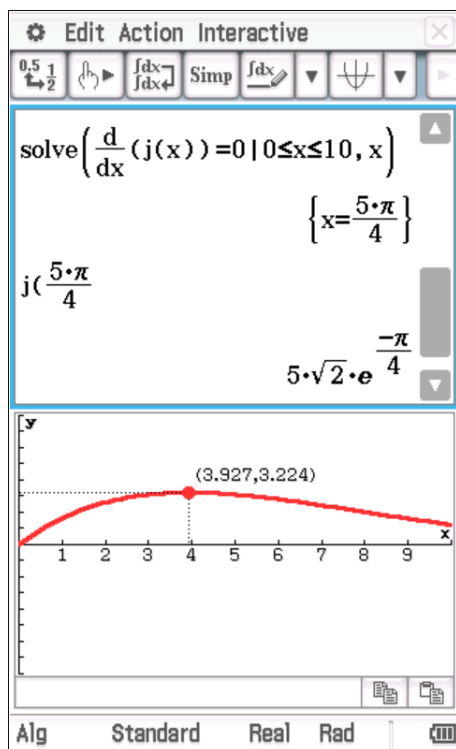
c. Solve $d_j'(t) = 0$

At $t = \frac{5\pi}{4}$ minutes

1A

maximum amount = $5\sqrt{2}e^{-\frac{\pi}{4}}$ mg/litre

1A



d. Average rate of change, for the interval $[0, 10]$

$$\frac{d_j(10) - d_j(0)}{10 - 0} \quad \mathbf{1M}$$

= 0.123 mg/L/min correct to three decimal places 1A

The screenshot shows a TI-84 Plus calculator interface. The top part displays the expression $\frac{j(10) - j(0)}{10 - 0}$ and the result 0.1230600248 .

e. Solve $10e^{-0.2t} \sin(0.2t) = 10e^{-0.2t}$ for $t \in [0, 10]$

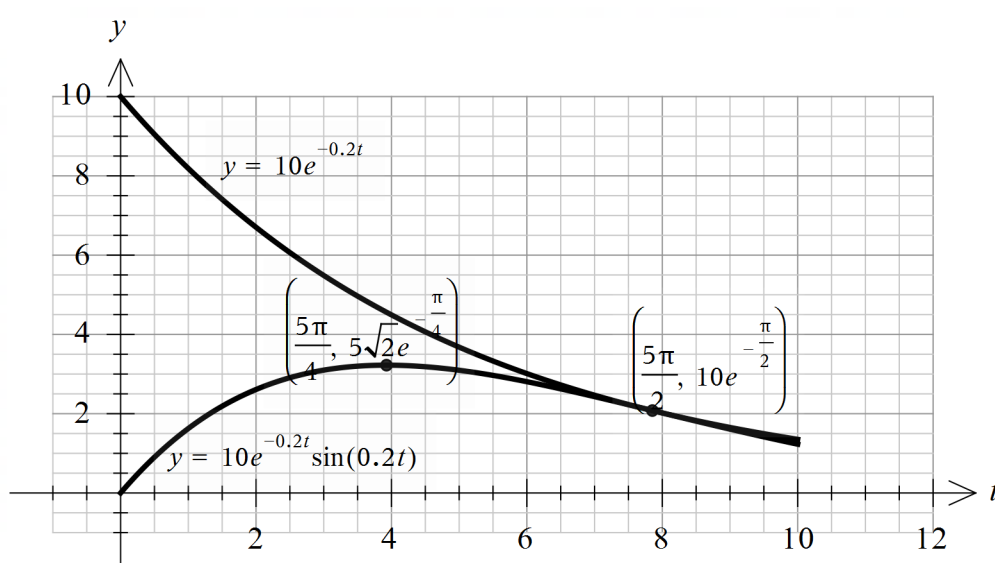
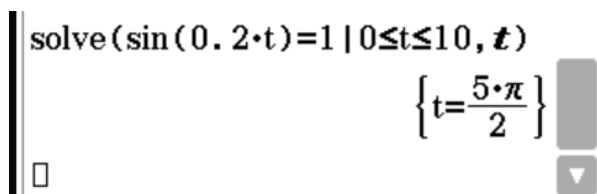
$$10e^{-0.2t} \sin(0.2t) - 10e^{-0.2t} = 0$$

$$10e^{-0.2t} (\sin(0.2t) - 1) = 0$$

$$10e^{-0.2t} \neq 0$$

$$\sin(0.2t) = 1, t \in [0, 10] \quad \mathbf{1M}$$

$$\text{Giving one solution } t = \frac{5\pi}{2} \neq \frac{5\pi}{4} \quad \mathbf{1A}$$



$$\mathbf{f.} \quad d: \left[0, \frac{2}{k}\right] \rightarrow \mathbb{R}, d(t) = e^{-kt} \sin(kt)$$

$$\mathbf{(i)} \quad \text{For domain } \left[0, \frac{2}{0.2}\right] = [0, 10]$$

$$k = 0.2 \quad \text{horizontal distance } t = \frac{5\pi}{2} - \frac{5\pi}{4} = \frac{5\pi}{4} \quad \mathbf{1A}$$

$$\mathbf{(ii)} \quad \text{For domain } \left[0, \frac{2}{0.02}\right] = [0, 100]$$

$$\text{stationary point where } k = 0.02 \text{ at } t = \frac{\pi}{4 \times 0.02} = \frac{25\pi}{2}$$

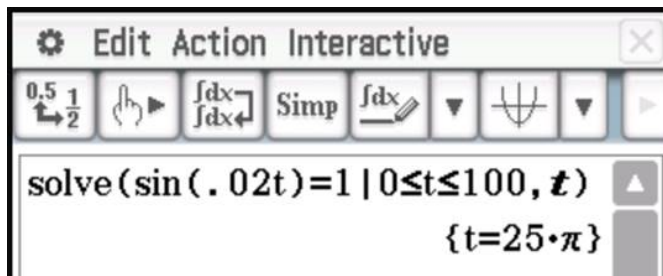
$$k = 0.02$$

$$\text{Solve } 10e^{-0.02t} \sin(0.02t) = 10e^{-0.02t} \text{ for } t \in [0, 100]$$

$$\text{Gives } \sin(0.02t) = 1, t \in [0, 100]$$

$$\text{Giving one solution } t = 25\pi$$

Difference in time $t = 25\pi - \frac{25\pi}{2} = \frac{25\pi}{2}$

1A

g. For domain $\left[0, \frac{2}{k}\right]$

stationary point occurs at $t = \frac{\pi}{4k}$

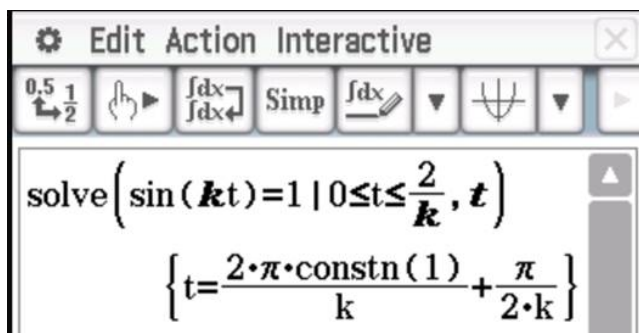
Point of intersection

Gives $\sin(kt) = 1, t \in \left[0, \frac{2}{k}\right]$

Giving one solution $t = \frac{\pi}{2k}$

1M

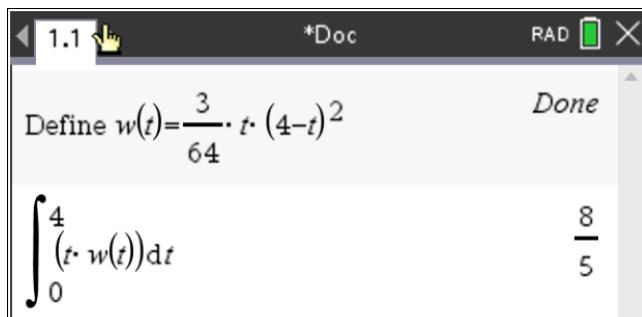
Difference in time $t = \frac{\pi}{2k} - \frac{\pi}{4k} = \frac{\pi}{4k}$

1A

Question 3

$$w(t) = \begin{cases} \frac{3}{64}t(4-t)^2 & 0 \leq t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

a.i. $E(T) = \int_0^4 (t \times w(t)) dt = \frac{8}{5}$ **1A**



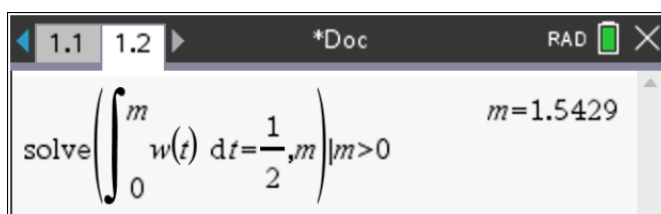
Define $w(t) = \frac{3}{64} \cdot t \cdot (4-t)^2$ Done

$$\int_0^4 (t \cdot w(t)) dt = \frac{8}{5}$$

ii. Let m be the median.

Solve $\int_0^m (w(t)) dt = \frac{1}{2}$ for m .

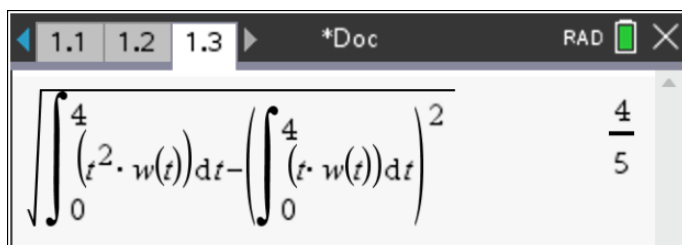
$m = 1.54$ minutes **1A**



solve $\left(\int_0^m w(t) dt = \frac{1}{2}, m \right) | m > 0$ $m = 1.5429$

iii. $sd(T) = \sqrt{\int_0^4 (t^2 \times w(t)) - \left(\int_0^4 (t \times w(t)) dt \right)^2}$ **1M**

$= \frac{4}{5}$ **1A**



$$\sqrt{\int_0^4 (t^2 \cdot w(t)) dt - \left(\int_0^4 (t \cdot w(t)) dt \right)^2} = \frac{4}{5}$$

$$\text{b.i. } w_1(t) = \begin{cases} at(b-t)^2 & 0 \leq t \leq b \\ 0 & \text{elsewhere} \end{cases}$$

Solve $\int_0^b (w_1(t) dt) = 1$ and $\int_0^b (t \times w_1(t)) dt = \frac{4}{5}$ for a and b . **1M**

$$a = \frac{3}{4}, b = 2 \quad \text{1A}$$

Define $w1(t) = a \cdot t \cdot (b-t)^2$ Done

solve $\left(\int_0^b w1(t) dt = 1 \text{ and } \int_0^b (t \cdot w1(t)) dt = \frac{4}{5}, a, b \right)$

$a = \frac{3}{4}$ and $b = 2$

ii. 50% **1A**

Define $w1(t) = a \cdot t \cdot (b-t)^2 | a = \frac{3}{4}$ and $b = 2$ Done

$\sqrt{\int_0^2 (t^2 \cdot w1(t)) dt - \left(\int_0^2 (t \cdot w1(t)) dt \right)^2}$ $\frac{2}{5}$

c.i. (0.846, 0.888) **1A**

zInterval_1Prop 867,1000,0.95: stat.results

"Title"	"1-Prop z Interval"
"CLower"	0.845953
"CUpper"	0.888047
"p"	0.867
"ME"	0.021047
"n"	1000.

ii. No the factory is not misleading their Office Supplies as 80% is below the confidence interval. The factory should be claiming at least 85%. **1A**

d.i. 2923.3 pages

1A

1.4 1.5 1.6 ▶ *EA2202... son RAD	
invNorm(0.05,3005.5,50)	2923.3
2923.2573187047	2923.3

ii. $X \sim \text{Bi}(5, 0.3120\dots)$ $\Pr(X \geq 4) = 0.0356$ correct to four decimal places

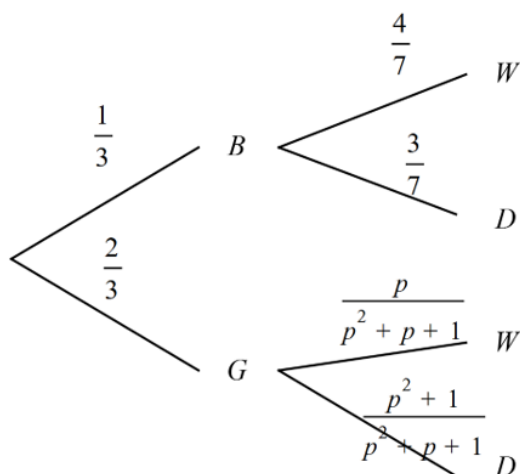
1.5 1.6 1.7 ▶ *EA2202... son RAD	
normCdf(3030,∞,3005.5,50)	0.31207
binomCdf(5,0.31206694831984,4,5)	0.03558

e.i. Let G represent the green box, B the blue box, W a white chocolate and D a dark chocolate.

$$\Pr(G|W) = \frac{\Pr(G \cap W)}{\Pr(W)} \quad \mathbf{1M}$$

$$= \frac{\frac{2}{3} \times \frac{p}{p^2 + p + 1}}{\frac{2}{3} \times \frac{p}{p^2 + p + 1} + \frac{1}{3} \times \frac{20}{35}}$$

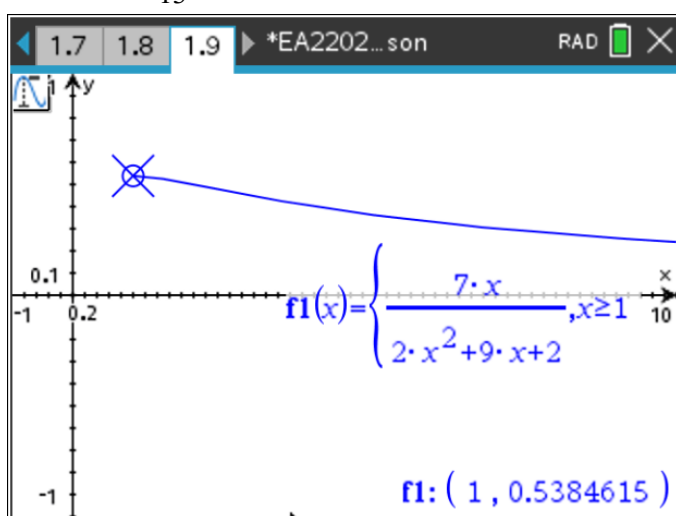
$$= \frac{7p}{2p^2 + 9p + 2} \quad \mathbf{1A}$$



Calculator screen showing the expression $\frac{2}{3} \cdot \frac{p}{p^2+p+1}$ and $\frac{7 \cdot p}{2 \cdot p^2+9 \cdot p+2}$. Below it, the expression $\frac{2}{3} \cdot \frac{p}{p^2+p+1} + \frac{1}{3} \cdot \frac{20}{35}$ is shown.

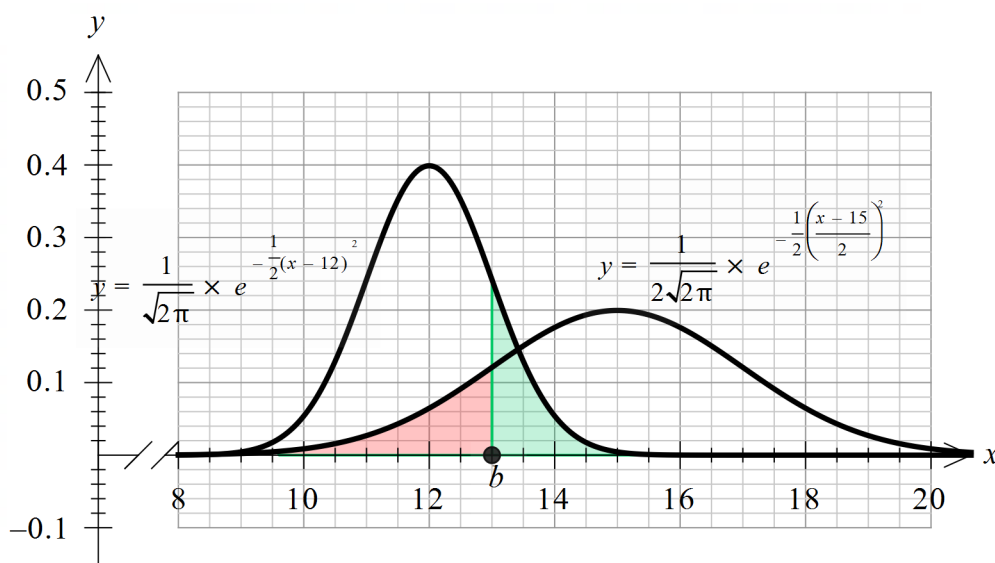
ii. The maximum value occurs when $p = 1$.

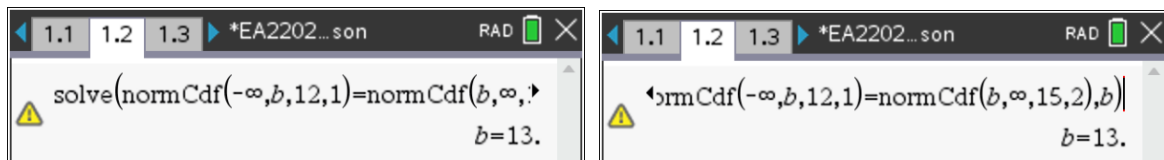
$\Pr(G|W) = \frac{7}{13}$ **1A**



f. Solve $\int_b^\infty \left(\frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}(b-12)^2} \right) = \int_{-\infty}^b \left(\frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{b-15}{2}\right)^2} \right)$ for b . **1M**

$b = 13$ g **1A**



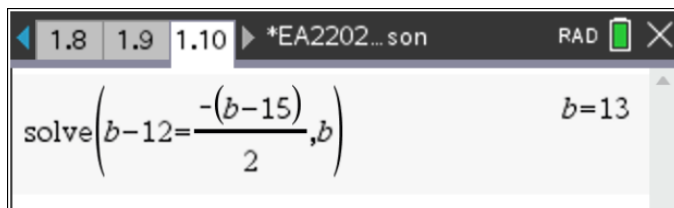
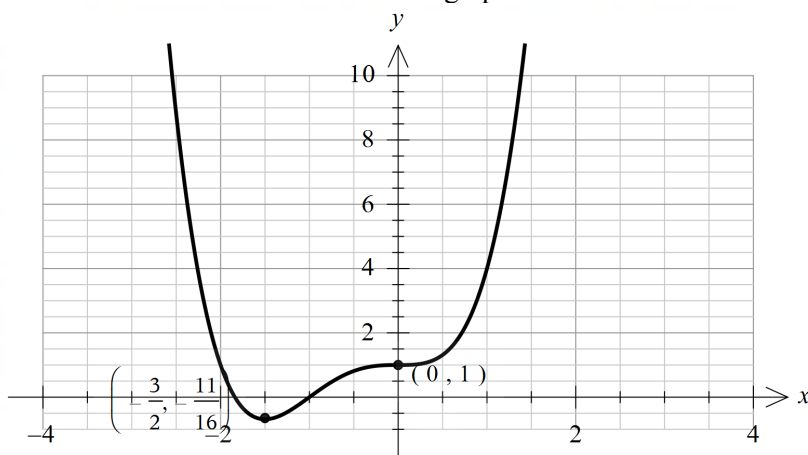


OR

Using z values

$$\text{Solve } b-12 = -\frac{b-15}{2} \quad \mathbf{1M}$$

$$b = 13 \text{ g} \quad \mathbf{1A}$$

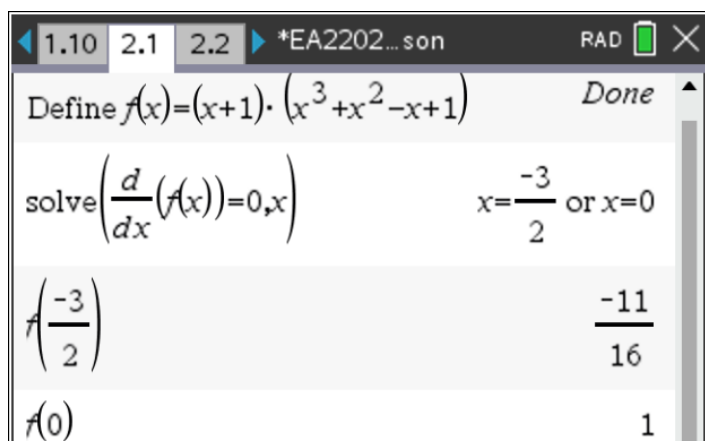
**Question 4**a. Correct coordinates labelled on the graph. **1A**

$$y = f(x) = (x+1)(x^3 + x^2 - x + 1)$$

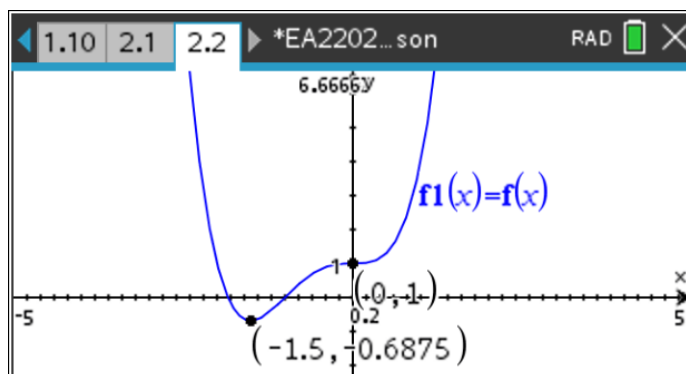
Solve $f'(x) = 0$ for x .

$$x = -\frac{3}{2} \text{ or } x = 0$$

$$f\left(-\frac{3}{2}\right) = -\frac{11}{16} = -0.6875 \text{ and } f(0) = 1$$



OR
Graphically



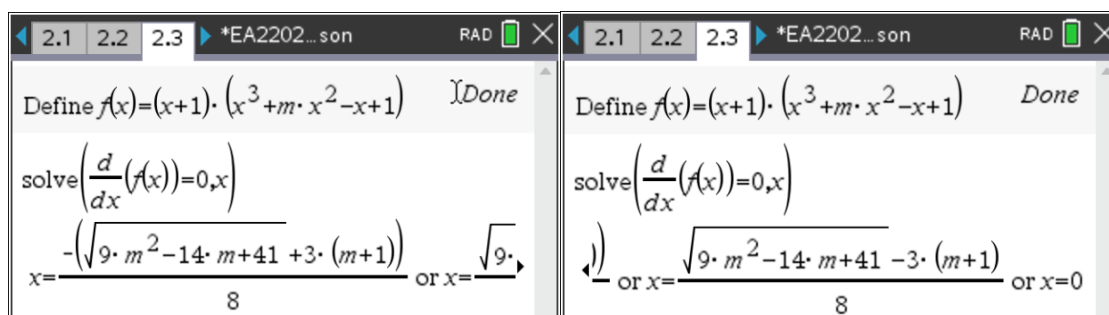
b. $y = f(x) = (x+1)(x^3 + mx^2 - x + 1)$

Solve $f'(x) = 0$ for x .

1M

$$x = \frac{-3(m+1) \pm \sqrt{9m^2 - 14m + 41}}{8} \text{ or } x = 0$$

1A



c. $9m^2 - 14m + 41 > 0$ for $m \in \mathbb{R}$ hence three solutions

$$x = \frac{-3(m+1) \pm \sqrt{9m^2 - 14m + 41}}{8} \text{ or } x = 0 \text{ for } m \in \mathbb{R} \setminus \{1\}.$$

1A

When $m = 1$, $x = \frac{-3(m+1) + \sqrt{9m^2 - 14m + 41}}{8} = 0$. So only two solution.

1A

A screenshot of a calculator window with tabs 2.2, 2.3, and 2.4. The display shows the equation $x = \frac{\sqrt{9 \cdot m^2 - 14 \cdot m + 41} - 3 \cdot (m+1)}{8} \mid m=1$ and $x=0$ to the right.

d. $m > 1$

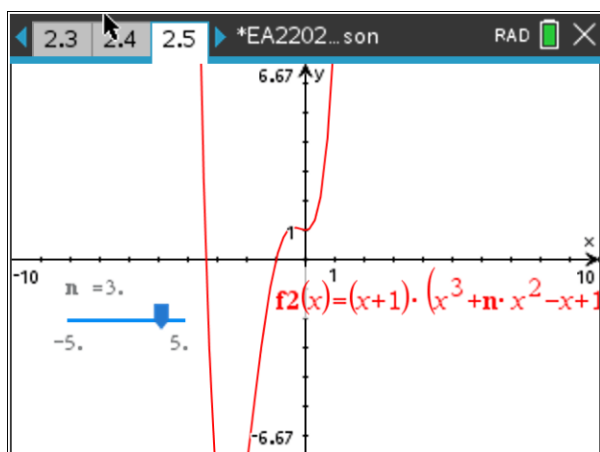
1A

Solve $\frac{-3(m+1) + \sqrt{9m^2 - 14m + 41}}{8} < 0$ for m .

A screenshot of a calculator window with tabs 2.4, 2.5, and 2.6. The display shows the command $\text{solve}(\sqrt{9 \cdot m^2 - 14 \cdot m + 41} - 3 \cdot (m+1) < 0, m)$ and the result $m > 1$.

OR

Graphically



e. 1 correct 1A

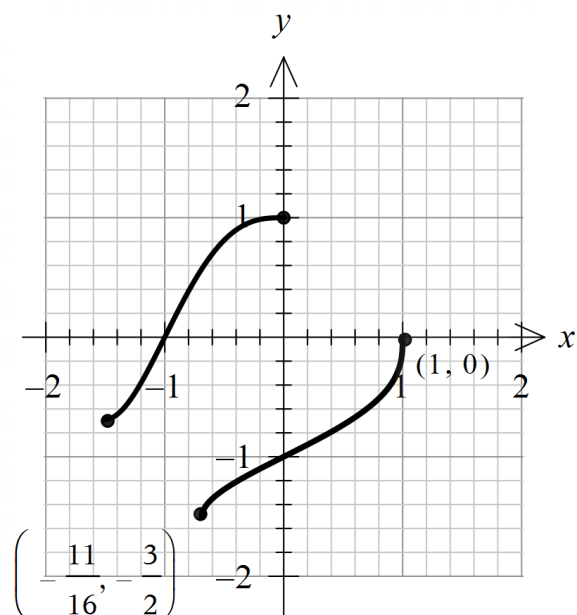
All correct 2A

 $m = 1$, stationary point of inflection $m < 1$, local maximum $m > 1$, local minimum

f. $g: \left[-\frac{3}{2}, 0\right] \rightarrow R, g(x) = (x+1)(x^3 + x^2 - x + 1)$

Shape and endpoints

1A



g. The graphs of g and g^{-1} are symmetrical about the line $y = x + 1$.
The equation of the inverse is too difficult to find.

$$\begin{aligned} \text{So Area} &= 2 \int_{-1}^0 (g(x) - (x+1)) dx && \mathbf{1M} \\ &= \frac{2}{5} && \mathbf{1A} \end{aligned}$$

Define $g(x) = (x+1) \cdot (x^3 + x^2 - x + 1) \mid \frac{-3}{2} \leq x \leq 0$

Done

$$2 \cdot \int_{-1}^0 (g(x) - (x+1)) dx = \frac{2}{5}$$

h. Solve $\frac{d}{dx} \left(\frac{d}{dx} (g(x)) \right) = 0$ for x . **1M**

$$x = -1$$

The equation of the tangent is $y = 2x + 2$. **1A**

solve $\left(\frac{d}{dx} \left(\frac{d}{dx} (g(x)) \right) = 0, x \right)$ $x = -1$

tangentLine $(g(x), x, -1)$ $2 \cdot x + 2$

i. endpoints $\left(-\frac{3}{2}, -\frac{11}{16}\right)$ and $(0, 1)$

$$m = \frac{1 + \frac{11}{16}}{\frac{3}{2}} = \frac{9}{8}$$

The equation of the line passing through the endpoints is $y = \frac{9}{8}x + 1$. **1A**

Dilate by a factor of $\frac{16}{9}$ from the y -axis.

Translate 1 unit down. **1A**

The order does not matter in this case.

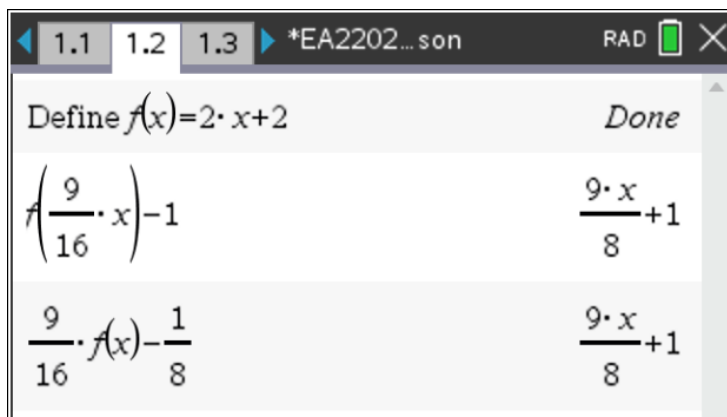
OR

Dilate by a factor of $\frac{9}{16}$ from the x -axis.

Translate $\frac{1}{8}$ th of a unit down. **1A**

Order matters in this case.

There are other possibilities.



END OF SOLUTIONS