2021 VCE Mathematical Methods Trial Examination 1



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Kilbaha Education (Est. 1978) (ABN 47 065 111 373) | T

PO Box 2227 Kew Vic 3101 Australia Tel: +613 9018 5376

Email: kilbaha@gmail.com
Web: https://kilbaha.com.au

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Victorian Certificate of Education 2021

STUDENT NUMBER

						Letter
Figures						
Words						

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (2 + 1 = 3 marks)

a.	For $y = \log_e$	$(\tan(3x))$, find	$\frac{dy}{dx}$ in	n simplified form
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2 marks

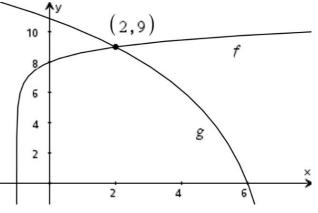
b.	Hence find an antiderivative of	$\frac{1}{\sin(3x)\cos(3x)}.$	1 mark
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Ouestion	2	(3 marks
Question	4	(3 marks

<i>X</i> is a normally distributed random variable, with mean 80 and variance 36, and <i>Z</i> is the standard normal random variable. If $Pr(Z < -2.5) = p$ and $Pr(-2.5 < Z < -1.5) = q$,				
express $Pr(X > 71 X < 95)$ in terms of p and q.				

Question 3 (4 marks)

Part of the graphs of $f(x) = \log_3(x+a) + b$ and $g(x) = p \log_5(q-x)$ are shown. The graph of y = f(x) crosses the y-axis at y = 8, and the graph of y = g(x) crosses the x-axis at x = 6. The two curves intersect at the point (2,9). Determine the values of a, b, p and q.



l	l	2	4	6/

Ouestion	4	(4	marks)
Oucsuon	-	(→	marks

Consider the function $f(x) = \log_e(kx) - \frac{kx}{2x+3}$ where $k \in R \setminus \{0\}$.				
Determine a range of values of k for which the graph of the function has no stationary points.				

Question 5 (5 marks)

- **a.** A veterinarian clinic employs six male and three female veterinarians. On a particular day, three veterinarians are working. Let \hat{P} represent the sample proportion of female veterinarians working on a particular day.
- i. What values can \hat{P} take?

1 mark

ii. Find $\Pr\left(\hat{P} \ge \frac{1}{2}\right)$.

2 marks

b. The veterinarian clinic has a large number of registered animals coming to the surgery. A random sample of animals from the surgery is selected, and an approximate 95% confidence interval for p, the proportion of animals which are dogs that attend the surgery was determined to be $\left(\frac{316}{625}, \frac{484}{625}\right)$. Using $z = 1.96 = \frac{49}{25}$, find the sample size from which this approximate 95% confidence interval was obtained.

2 marks

Question 6 (5 marks)

Consider the function $f: D \to R$, $f(x) = \sqrt{9-3x}$

a. Find the maximal domain of f, that is find D.

1 mark

b. Find the inverse function, f^{-1} .

2 marks

c. Find the coordinates of **all** the points of intersection between the graphs of y = f(x) and $y = f^{-1}(x)$.

2 marks

Question 7 (4 marks)

a.	Let $a > 0$, $b > 0$ and let $f(x)$ be an increasing function, such that $f(0) = 0$
	and $f(a) = b$. Explain why $\int_0^a f(x) dx = ab - \int_0^b f^{-1}(x) dx$.

1	mark

b.	Hence, find the values of p and q , where	$p, q \in Z^+$, if $\int_0^3 \log_e (1+3z)^2$	$x\big)dx = \frac{p}{q}\log_e(p) - q.$
			3 marks

Question 8 (3 marks)

The probability distribution for the continuous random variable *X* is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{16 - 3x}} & 0 \le x \le 5\\ 0 & \text{otherwise} \end{cases}$$

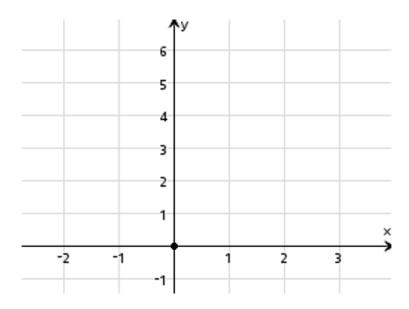
Find the value of k .			

Question 9 (9 marks)

Consider the function $f:[0,\infty) \to R$, $f(x) = 4e^{-2x}$

a. Sketch the graph of the function on the axes below, stating the coordinates of any axial intercepts and equations of any asymptotes.

1 mark



b. Over the interval $1 \le x \le 3$, find the mean value of the function.

1 mark

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c.	Find the equation of the tangent to the graph of the function f at the point P
	where $x = u$. Give your answer in the form $y = mx + c$.

2 marks

d.	This tangent crosses the x-axis at the point Q , and the y-axis at the point R .
	Write down the coordinates of the points Q and R .

1 mark

e. If O is the origin, show that the area A of the triangle OQR is given by $A(u) = (2u+1)^2 e^{-2u}$

		1 mark

f.	Find the maximum value and the minimum value of the area A .	
		3 marks

End of question and answer book for the 2021 Kilbaha VCE Mathematical Methods Trial Examination 1

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EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c , n \neq -1$		
$\frac{d}{dx}\Big(\big(ax+b\big)^n\Big) = na\big(ax+b\big)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, \ n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$		
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$		$\int \cos(ax) dx =$	$=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$				
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$			

Probability

$\Pr(A) = 1 - \Pr(A')$		$Pr(A \cup B) =$	$= \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^{2} = E((X - \mu)^{2}) = E(X^{2}) - \mu^{2}$

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET