# 2021
# VCE
# Mathematical Methods
## Trial Examination 1

Kilbaha Education

Quality educational content


| | |
|---|---|
| **Kilbaha Education**<br>**PO Box 2227**<br>**Kew Vic 3101**<br>**Australia** | **Tel: (03) 9018 5376**<br>**kilbaha@gmail.com**<br>**https://kilbaha.com.au** |

*All publications from Kilbaha Education are digital and are supplied to the purchasing school in both WORD and PDF formats with a school site licence to reproduce for students in both print and electronic formats.*

Kilbaha Education

Quality educational content


| | |
|---|---|
| **Kilbaha Education (Est. 1978) (ABN 47 065 111 373)**<br>**PO Box 2227**<br>**Kew Vic 3101**<br>**Australia** | **Tel: +613 9018 5376**<br>**Email: kilbaha@gmail.com**<br>**Web: https://kilbaha.com.au** |

# Victorian Certificate of Education
# 2021

**STUDENT NUMBER**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | Letter |
| Figures | | | | | | | | | | |
| Words | | | | | | | | | | |

# MATHEMATICAL METHODS

# Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 9 | 9 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** ( 2 + 1 = 3 marks )

**a.** For $y = \log_e\left(\tan\left(3x\right)\right)$, find $\frac{dy}{dx}$ in simplified form.

2 marks

**b.** Hence find an antiderivative of $\frac{1}{\sin\left(3x\right)\cos\left(3x\right)}$. 1 mark

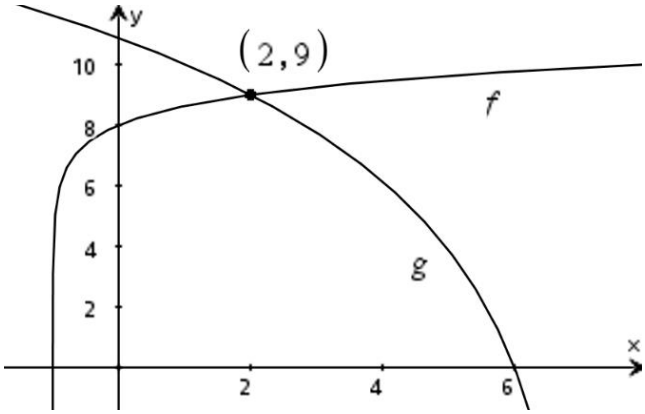
**Question 2** (3 marks)

$X$ is a normally distributed random variable, with mean 80 and variance 36, and $Z$ is the standard normal random variable. If $\Pr(Z < -2.5) = p$ and $\Pr(-2.5 < Z < -1.5) = q$, express $\Pr(X > 71 | X < 95)$ in terms of $p$ and $q$.

**Question 3** (4 marks)

Part of the graphs of $f(x) = \log_3(x+a)+b$ and $g(x) = p\log_5(q-x)$ are shown. The graph of $y = f(x)$ crosses the *y*-axis at $y = 8$, and the graph of $y = g(x)$ crosses the *x*-axis at $x = 6$. The two curves intersect at the point $(2,9)$.

Determine the values of *a* , *b* , *p* and *q*.



**Question 4** (4 marks)

Consider the function $f(x) = \log_e(kx) - \frac{kx}{2x+3}$ where $k \in R \setminus \{0\}$.

Determine a range of values of $k$ for which the graph of the function has no stationary points.

**Question 5** (5 marks)

**a.** A veterinarian clinic employs six male and three female veterinarians. On a particular day, three veterinarians are working. Let $\hat{P}$ represent the sample proportion of female veterinarians working on a particular day.

**i.** What values can $\hat{P}$ take?

1 mark

**ii.** Find $\Pr\left(\hat{P} \geq \frac{1}{2}\right)$.

2 marks

**b.** The veterinarian clinic has a large number of registered animals coming to the surgery. A random sample of animals from the surgery is selected, and an approximate 95% confidence interval for *p*, the proportion of animals which are dogs that attend the surgery was determined to be $\left(\frac{316}{625}, \frac{484}{625}\right)$. Using $z = 1.96 = \frac{49}{25}$, find the sample size from which this approximate 95% confidence interval was obtained.

2 marks

**Question 6** (5 marks)

Consider the function $f : D \to R$, $f(x) = \sqrt{9 - 3x}$

**a.** Find the maximal domain of *f*, that is find *D*.

1 mark

**b.** Find the inverse function, $f^{-1}$.

2 marks

**c.** Find the coordinates of **all** the points of intersection between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

2 marks

**Question 7** (4 marks)

**a.** Let $a > 0$ , $b > 0$ and let $f(x)$ be an increasing function, such that $f(0) = 0$ and $f(a) = b$. Explain why $\int_0^a f(x)\,dx = ab - \int_0^b f^{-1}(x)\,dx$.

1 mark

**b.** Hence, find the values of $p$ and $q$, where $p, q \in Z^+$, if $\int_0^3 \log_e(1+3x)\,dx = \frac{p}{q}\log_e(p) - q$.

3 marks

**Question 8** (3 marks)

The probability distribution for the continuous random variable $X$ is given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{16-3x}} & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

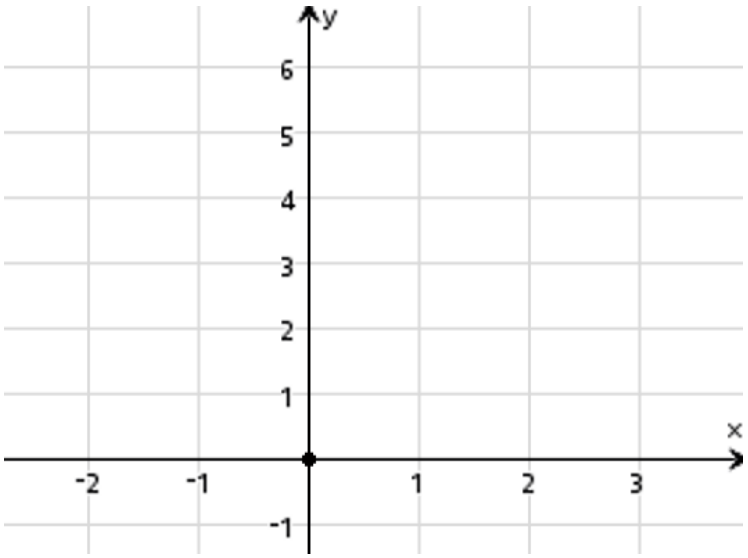
Find the value of $k$.

**Question 9** (9 marks)

Consider the function $f:[0,\infty) \to R$, $f(x) = 4e^{-2x}$

**a.** Sketch the graph of the function on the axes below, stating the coordinates of any axial intercepts and equations of any asymptotes.

1 mark




**b.** Over the interval $1 \leq x \leq 3$, find the mean value of the function.

1 mark

**c.** Find the equation of the tangent to the graph of the function $f$ at the point $P$ where $x = u$. Give your answer in the form $y = mx + c$.

2 marks

**d.** This tangent crosses the $x$-axis at the point $Q$, and the $y$-axis at the point $R$. Write down the coordinates of the points $Q$ and $R$.

1 mark

**e.** If $O$ is the origin, show that the area $A$ of the triangle $OQR$ is given by $A(u) = (2u+1)^2 e^{-2u}$

1 mark

**f.** Find the maximum value and the minimum value of the area $A$.

3 marks

**End of question and answer book for the**
**2021 Kilbaha VCE Mathematical Methods Trial Examination 1**


| | |
|---|---|
| **Kilbaha Education**<br>**PO Box 2227**<br>**Kew Vic 3101**<br>**Australia** | **Tel: (03) 9018 5376**<br>**kilbaha@gmail.com**<br>**https://kilbaha.com.au** |

**EXTRA WORKING PAGE**

# MATHEMATICAL METHODS

## Written examination 1

## FORMULA SHEET

### Directions to students

| |
|---|
| Detach this formula sheet during reading time.<br><br>This formula sheet is provided for your reference. |

# Mathematical Methods formulas

## Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium | $\frac{1}{2}(a+b)h$ | volume of a pyramid | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder | $2\pi rh$ | volume of a sphere | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder | $\pi r^2 h$ | area of triangle | $\frac{1}{2}bc\sin(A)$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ | | |

## Calculus

| | |
|---|---|
| $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ | $\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$ |
| $\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$ | $\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$ |
| $\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$ | $\int e^{ax}dx=\frac{1}{a}e^{ax}+c$ |
| $\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$ | $\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$ |
| $\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$ | $\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$ |
| $\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$ | $\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$ |
| $\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$ | |

| | | | |
|---|---|---|---|
| product rule | $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ | quotient rule | $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ |
| chain rule | $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ | | |

## Probability

| | | | |
|---|---|---|---|
| $\Pr(A) = 1 - \Pr(A')$ | | $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ | |
| $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$ | | | |
| mean | $\mu = E(X)$ | variance | $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$ |

| Probability distribution | | Mean | Variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \sum x\, p(x)$ | $\sigma^2 = \sum (x-\mu)^2\, p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)\,dx$ | $\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$ |

## Sample proportions

| | | | |
|---|---|---|---|
| $\hat{P} = \dfrac{X}{n}$ | | mean | $E(\hat{P}) = p$ |
| standard deviation | $\text{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$ | approximate confidence interval | $\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$ |

**END OF FORMULA SHEET**