

**2021  
VCE  
Mathematical  
Methods  
Trial Examination 1  
Detailed Answers**



**Kilbaha Education**

Quality educational content

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**Question 1**

a.  $y = \log_e(\tan(3x))$  Chain rule

$$y = \log_e(u) \quad , \quad u = \tan(3x)$$

$$\frac{dy}{du} = \frac{1}{u} \quad , \quad \frac{du}{dx} = \frac{3}{\cos^2(3x)}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{3}{\cos^2(3x)} = \frac{1}{\tan(3x)} \cdot \frac{3}{\cos^2(3x)} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{\cos(3x)}{\sin(3x)} \cdot \frac{3}{\cos^2(3x)}$$

$$\frac{dy}{dx} = \frac{3}{\sin(3x)\cos(3x)} \quad \text{A1}$$

b.  $\int \frac{1}{\sin(3x)\cos(3x)} dx = \frac{1}{3} \log_e(\tan(3x)) + c \quad \text{A1}$

**Question 2**

$$X = N(80, 36) \quad , \quad \mu = 80, \sigma = 6, Z = N(0, 1)$$

$$\Pr(Z < -2.5) = \Pr(Z > 2.5) = p \quad \text{and} \quad \Pr(-2.5 < Z < -1.5) = \Pr(1.5 < Z < 2.5) = q,$$

$$\text{Consider } \Pr(0 < Z < 1.5) + \Pr(1.5 < Z < 2.5) + \Pr(Z > 2.5) = 0.5$$

$$\Pr(0 < Z < 1.5) + q + p = 0.5 \quad , \quad \Pr(0 < Z < 1.5) = 0.5 - (p + q) \quad \text{M1}$$

$$\Pr(X > 71 | X < 95) = \Pr\left(Z > \frac{71-80}{6} \mid Z < \frac{95-80}{6}\right)$$

$$\Pr(X > 71 | X < 95) = \Pr(Z > -1.5 | Z < 2.5) = \frac{\Pr(-1.5 < Z < 2.5)}{\Pr(Z < 2.5)}$$

$$\Pr(X > 71 | X < 95) = \frac{\Pr(-1.5 < Z < 0) + \Pr(0 < Z < 1.5) + \Pr(1.5 < Z < 2.5)}{1 - \Pr(Z > 2.5)} \quad \text{M1}$$

$$\Pr(X > 71 | X < 95) = \frac{2\Pr(0 < Z < 1.5) + \Pr(1.5 < Z < 2.5)}{1 - \Pr(Z > 2.5)} = \frac{2(0.5 - (p + q)) + q}{1 - p}$$

$$\Pr(X > 71 | X < 95) = \frac{1 - 2p - q}{1 - p} \quad \text{A1}$$

**Question 3**

$$f(x) = \log_3(x+a) + b$$

$$f(0) = 8 \Rightarrow (1) \quad 8 = \log_3(a) + b$$

$$f(2) = 9 \Rightarrow (2) \quad 9 = \log_3(a+2) + b$$

$$(2) - (1) \quad 1 = \log_3(a+2) - \log_3(a) = \log_3\left(\frac{a+2}{a}\right) = \log_3(3) \quad \text{M1}$$

$$\frac{a+2}{a} = 3, \quad a+2 = 3a, \quad a = 1, \quad (1) \quad b = 8 - \log_3(1) = 8 \quad \text{A1}$$

$$g(x) = p \log_5(q-x) \quad \text{M1}$$

$$g(6) = 0 \Rightarrow 0 = p \log_5(q-6), \quad q-6 = 1, \quad q = 7$$

$$g(2) = 9 \Rightarrow 9 = p \log_5(5), \quad p = 9 \quad \text{A1}$$

**Question 4**

$$f(x) = \log_e(kx) - \frac{kx}{2x+3} \quad \text{differentiating using the quotient rule in the second term}$$

$$f'(x) = \frac{1}{x} - \left[ \frac{k(2x+3) - 2kx}{(2x+3)^2} \right] \quad \text{M1}$$

$$f'(x) = \frac{1}{x} - \left[ \frac{2kx + 3k - 2kx}{(2x+3)^2} \right]$$

$$f'(x) = \frac{1}{x} - \frac{3k}{(2x+3)^2} = \frac{(2x+3)^2 - 3kx}{x(2x+3)^2} = \frac{4x^2 + 12x + 9 - 3kx}{(2x+3)^2}$$

$$f'(x) = \frac{4x^2 + (12-3k)x + 9}{(2x+3)^2} = 0$$

$$4x^2 + (12-3k)x + 9 = 0 \quad \text{for stationary points} \quad \text{A1}$$

$$\Delta = (12-3k)^2 - 4 \times 4 \times 9$$

$$\Delta = (3(4-k))^2 - 16 \times 9 \quad \text{M1}$$

$$\Delta = 9(16-8k+k^2-16) = 9k(k-8)$$

$$\text{for no stationary points } \Delta < 0 \Rightarrow 0 < k < 8 \quad \text{A1}$$

**Question 5**

**a.i.** Since there can be 0, 1, 2 or 3 females, out of 3, the proportion

$$\hat{P} = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\} \quad \text{A1}$$

**ii.**  $\Pr\left(\hat{P} \geq \frac{1}{2}\right) = \Pr\left(\hat{P} = \frac{2}{3}\right) + \Pr(\hat{P} = 1) = \Pr(1M, 2F) + \Pr(3F)$  M1

$$\Pr\left(\hat{P} \geq \frac{1}{2}\right) = 3 \times \frac{6}{9} \times \frac{3}{8} \times \frac{2}{7} + \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{3}{14} + \frac{1}{84} = \frac{18+1}{84}$$

$$\Pr\left(\hat{P} \geq \frac{1}{2}\right) = \frac{19}{84} \quad \text{A1}$$

**b.**  $\left(p - z\sqrt{\frac{p(1-p)}{n}}, p + z\sqrt{\frac{p(1-p)}{n}}\right) = \left(\frac{316}{625}, \frac{484}{625}\right)$

adding  $2p = \frac{316}{625} + \frac{484}{625} = \frac{800}{625}$ ,  $p = \frac{400}{625} = \frac{16}{25}$

subtracting  $2z\sqrt{\frac{p(1-p)}{n}} = 2 \times \frac{49}{25} \times \sqrt{\frac{\frac{16}{25} \times \left(1 - \frac{16}{25}\right)}{n}} = \frac{484}{625} - \frac{316}{625}$  M1

$$= 2 \times \frac{49}{25} \times \sqrt{\frac{\frac{16}{25} \times \frac{9}{25}}{n}} = \frac{168}{625}$$

$$\frac{49}{25} \times \frac{4 \times 3}{25\sqrt{n}} = \frac{84}{625}$$

$$\sqrt{n} = \frac{49 \times 12}{84} = 7$$

$n = 49$  A1

**Question 6**

$$f(x) = \sqrt{9-3x}$$

a.  $9-3x \geq 0, x \leq 3$

$$D = (-\infty, 3] = \text{dom } f = \text{ran } f^{-1}$$

A1

b.  $f^{-1}: x = \sqrt{9-3y}, x^2 = 9-3y$

M1

$$3y = 9 - x^2, y = f^{-1}(x) = \frac{1}{3}(9 - x^2)$$

A1

$$f^{-1}: [0, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = \frac{1}{3}(9 - x^2)$$

c. solving  $f(x) = f^{-1}(x)$

$$\sqrt{9-3x} = \frac{1}{3}(x^2 - 9)$$

$$9-3x = \frac{1}{9}(x^4 - 18x^2 + 81)$$

$$81 - 27x = x^4 - 18x^2 + 81$$

$$x^4 - 18x^2 + 27x = 0$$

$$x(x^3 - 18x + 27) = 0$$

$$x(x-3)(x^2 + 3x - 9) = 0$$

$$x = 0, 3, \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm \sqrt{45}}{2} = \frac{3}{2}(-1 \pm \sqrt{5})$$

M1

but  $x \in [0, 3] x = 0, 3, \frac{3}{2}(\sqrt{5}-1)$

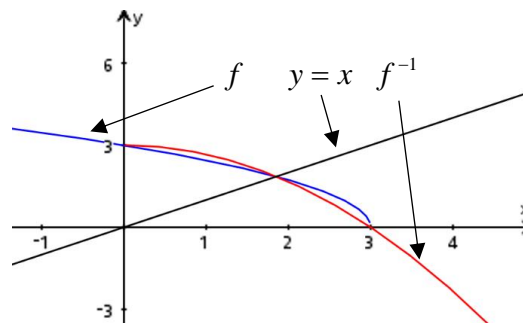
A1

$$(0, 3), (3, 0), \left( \frac{3}{2}(\sqrt{5}-1), \frac{3}{2}(\sqrt{5}-1) \right)$$

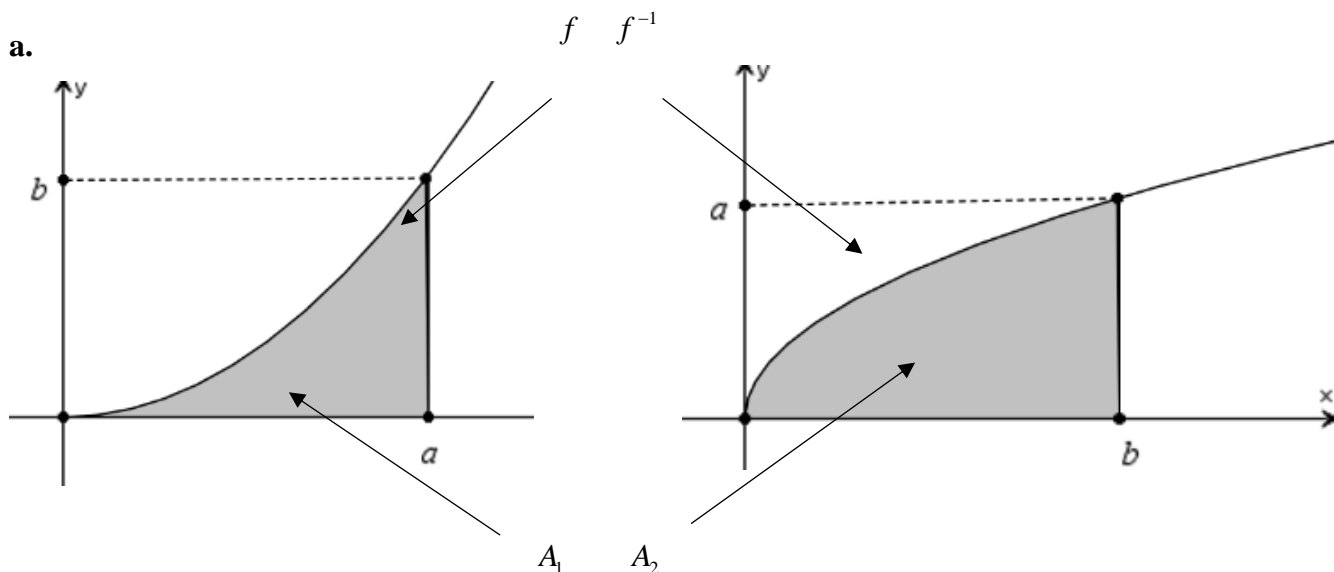
There are three points of intersection between  $f(x) = f^{-1}(x)$  only one of which lies on the line  $y = x$ .

Note that solving  $f(x) = x$  or  $f^{-1}(x) = x$  gives  $x^2 + 3x - 9 = 0$  which gives only

the one point of intersection on the line  $y = x$ , that is the point  $\left( \frac{3}{2}(\sqrt{5}-1), \frac{3}{2}(\sqrt{5}-1) \right)$



**Question 7**



$A_1 = \int_0^a f(x) dx$  ,  $A_2 = \int_0^b f^{-1}(x) dx$  but  $A_1 + A_2 = ab$  the area of the rectangle, hence

$$\int_0^a f(x) dx = ab - \int_0^b f^{-1}(x) dx \quad \text{A1}$$

b. Let  $A_1 = \int_0^3 \log_e(1+3x) dx$  ,  $f(x) = \log_e(1+3x)$  ,  $a = 3$  ,  $b = f(3) = \log_e(10)$

$$f: y = \log_e(1+3x)$$

$$f^{-1}: x = \log_e(1+3y) , 1+3y = e^x , y = \frac{1}{3}(e^x - 1) \quad \text{M1}$$

$$f^{-1}(x) = \frac{1}{3}(e^x - 1)$$

$$A_2 = \int_0^{\log_e(10)} \frac{1}{3}(e^x - 1) dx = \frac{1}{3} [e^x - x]_0^{\log_e(10)} = \frac{1}{3} [(e^{\log_e(10)} - \log_e(10)) - (e^0)] \quad \text{A1}$$

$$A_2 = \frac{1}{3} [10 - \log_e(10) - 1] = \frac{1}{3} (9 - \log_e(10)) = 3 - \frac{1}{3} \log_e(10)$$

$$A_1 = ab - A_2 \quad \text{using a.}$$

$$\int_0^3 \log_e(1+3x) dx = 3 \log_e(10) - \left[ 3 - \frac{1}{3} \log_e(10) \right] = \left( 3 + \frac{1}{3} \right) \log_e(10) - 3$$

$$\int_0^3 \log_e(1+3x) dx = \frac{10}{3} \log_e(10) - 3 , p = 10 , q = 3 \quad \text{A1}$$

**Question 8**

$$\int_0^5 \frac{k}{\sqrt{16-3x}} dx = 1 \quad \text{since it is a valid pdf}$$

$$\int_0^5 \frac{k}{\sqrt{16-3x}} dx = k \int_0^5 (16-3x)^{-\frac{1}{2}} dx = 1 \quad \text{A1}$$

$$k \left[ \frac{1}{-3 \times \frac{1}{2}} (16-3x)^{\frac{1}{2}} \right]_0^5 = - \left[ \frac{2k}{3} \sqrt{16-3x} \right]_0^5 = 1 \quad \text{A1}$$

$$\left( -\frac{2k}{3} \sqrt{16-15} \right) - \left( -\frac{2k}{3} \sqrt{16} \right) = -\frac{2k}{3} (1-4) = 1$$

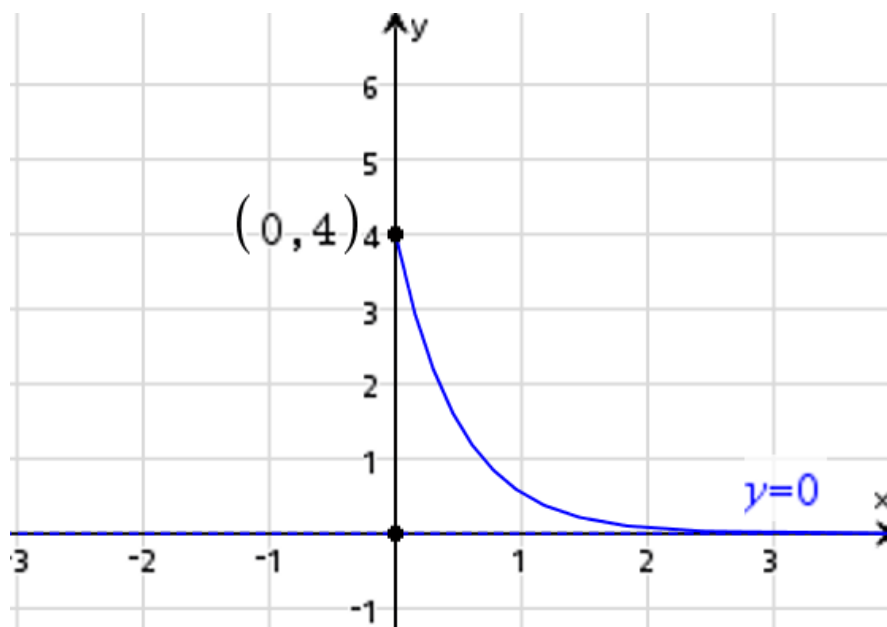
$$2k = 1$$

$$k = \frac{1}{2} \quad \text{A1}$$

**Question 9**

- a) The graph of  $f : [0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = 4e^{-2x}$  crosses the y-axis at  $(0, 4)$  and has the x-axis  $y = 0$  as a horizontal asymptote.

A1





b. 
$$\bar{y} = \frac{1}{3-1} \int_1^3 4e^{-2x} dx = 2 \int_1^3 e^{-2x} dx$$
  

$$\bar{y} = -\left[ e^{-2x} \right]_1^3 = -e^{-6} + e^{-2} = e^{-2} - e^{-6}$$
 A1

c. 
$$P(u, 4e^{-2u}), \frac{dy}{dx} = f'(x) = -8e^{-2x}, \quad f'(u) = -8e^{-2u}$$
 M1

$$T: y - 4e^{-2u} = -8e^{-2u}(x - u)$$
  

$$y = -8e^{-2u}x + 4e^{-2u}(2u + 1)$$
 A1

d. at R,  $x = 0, y_R = 4(2u + 1)e^{-2u}$   $R(0, 4e^{-2u}(2u + 1))$   
 at Q,  $y = 0$  solving  $-8e^{-2u}x + 4e^{-2u}(2u + 1) = 0$   

$$x_Q = \frac{4e^{-2u}(2u + 1)}{8e^{-2u}} = \frac{1}{2}(2u + 1)$$
  

$$Q\left(\frac{1}{2}(2u + 1), 0\right), \quad R(0, 4e^{-2u}(2u + 1))$$
 A1

e. area OQR,  $A(u) = \frac{1}{2} \times \frac{1}{2}(2u + 1) \times 4(2u + 1)e^{-2u} = (2u + 1)^2 e^{-2u}$  A1

f. differentiating using the product rule

$$\frac{dA}{du} = e^{-2u} \frac{d}{du}((2u + 1)^2) + (2u + 1)^2 \frac{d}{du}(e^{-2u})$$
  

$$\frac{dA}{du} = 4e^{-2u}(2u + 1) - 2(2u + 1)^2 e^{-2u}$$
 M1

$$\frac{dA}{du} = 2e^{-2u}(2u + 1)(2 - (2u + 1))$$

$$\frac{dA}{du} = 2e^{-2u}(2u + 1)(1 - 2u) = 0$$
 for a maximum or minimum area

$$u = \frac{1}{2}$$
 since  $u \in [0, \infty)$

$$A\left(\frac{1}{2}\right) = 4e^{-1} = \frac{4}{e}$$
 is the maximum area A1

now as  $u \rightarrow \infty$   $A(u) \rightarrow 0$  the minimum area is 0 A1

**End of detailed answers for the  
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