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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2021

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on pages 13 and 14 of this exam.

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Question 1 (3 marks)

a. Let $y = \log_e(x^2 + 1)$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $g(x) = \frac{\tan(x)}{x^2 + x}$.

Evaluate $g'(\pi)$. 2 marks

Question 2 (3 marks)

Let $f : R \setminus \{-2\} \to R, \quad f(x) = \dfrac{1}{x+2}$.

a. Find the rule for f^{-1}, the inverse function of *f*. 1 mark

b. State the domain of f^{-1}. 1 mark

c. Show that $f^{-1}(f(x)) = x$. 1 mark

Question 3 (3 marks)

Julian randomly selects a die and throws it.
The two dice he selects from, are an unbiased six-sided black die and a biased six sided red die.
With the red die, the probability of throwing a six is $\frac{3}{8}$ and there is equal probability of throwing a 1, 2, 3, 4 or 5.

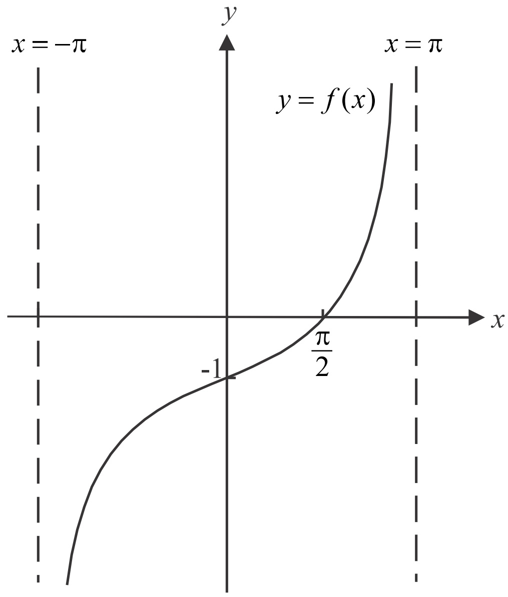
a. What is the probability that Julian throws a six? 2 marks

b. What is the probability that Julian selected the black die given that he threw a six? 1 mark

Question 4 (4 marks)

Let $f:(-\pi, \pi) \to R,\ f(x) = \tan\left(\frac{x}{2}\right) - 1$.

Part of the graph of *f* is shown below.



a. Find the average rate of change of *f* between $x = \frac{\pi}{2}$ and $x = \frac{2\pi}{3}$.

Give your answer in the form $\frac{a\left(\sqrt{b} - c\right)}{d}$ where *a*, *b*, *c* and *d* are constants. 2 marks

b. The graph of *f* undergoes a transformation defined by

$$T: R^2 \to R^2,\ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix},$$

to become the graph of *h*.

On the set of axes above, sketch the graph of *h*.
Indicate clearly on the graph the coordinates of any axis intercepts. 2 marks

Question 5 (5 marks)

Voters in a national referendum can cast a "for" vote or an "against" vote.
In a random sample taken of 100 voters, ten cast an "against" vote.

a. What is the proportion of "against" voters in this sample? 1 mark

b. Thirty of the voters in this sample are randomly selected to complete an additional survey.
The probability that at least two of them are "against" voters is equal to $a-b\,(c)^{n}$, where a and n are positive integers and b and c are positive rational numbers.
Find a, b, c and n. 2 marks

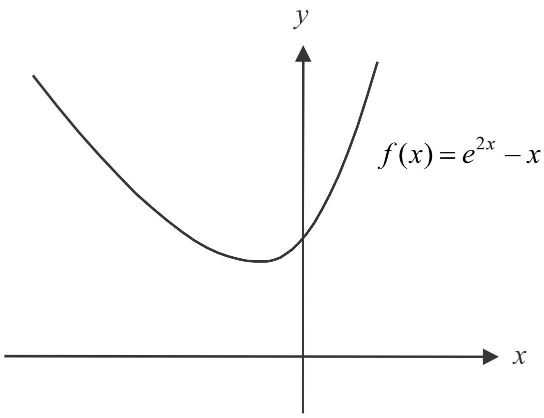
c. Based on the original sample of 100 voters, find the approximate 95% confidence interval for the proportion of "against" voters in the referendum.
Use the z value $\frac{49}{25}$. 2 marks

Question 6 (3 marks)

The area of the region bounded by the curve with equation $y=(x-k)^2$, where k is a positive number, and the x and y axes is $\frac{8}{3}$ square units. Find the value of k.

Question 7 (4 marks)

Part of the graph of $f : R \to R,\ \ f(x) = e^{2x} - x$ is shown below.

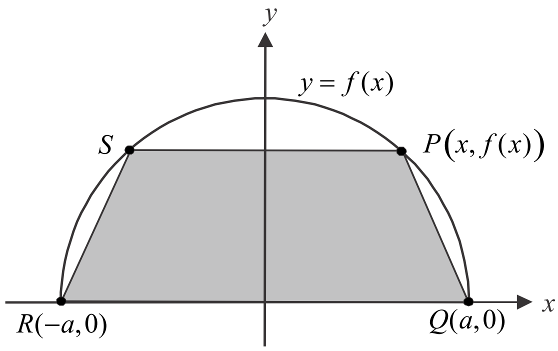


a. Find the minimum value of f. 2 marks

b. Find the average value of f between $x = 0$ and $x = 1$. 2 marks

Question 8 (7 marks)

The points P, Q, R and S lie on the graph of the function $f:[-a, a] \to R,\ f(x) = \sqrt{a^2 - x^2}$.
These four points form a regular trapezium which is shaded below.



a. Show that the area A, in square units, of the trapezium $PQRS$ is given by
$A = (a + x)\sqrt{a^2 - x^2}$. 1 mark

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b. Find the coordinates of point P in terms of a, when A is a maximum. 3 marks

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c. Find the angle PQR when A is a maximum. 1 mark

d. Find the equation of the tangent to the graph of f that is parallel to the line segment PQ when A is a maximum. 2 marks

Question 9 (8 marks)

Let $g:\left[0,\frac{\pi}{2}\right]\to R,\ \ g(x)=\sin^2(x)\cos(x)$.

a. Show that the point $\left(\frac{\pi}{6},\frac{\sqrt{3}}{8}\right)$ lies on the graph of *g*. 1 mark

b. Find $\frac{d}{dx}(\sin^3(x))$. 1 mark

Give your answer in terms of $g(x)$.

c. Hence find the area enclosed by *g*, the line $y=\frac{\sqrt{3}}{8}$ and the *y*-axis. 3 marks

The graph of g is dilated by a factor of a, where $a > 0$, from the y-axis and then reflected in the y-axis to become the graph of the function h.

d. Write down, in terms of a,

i. the rule for h. 1 mark

ii. the domain of h. 1 mark

e. Let p be the x-coordinate of the left endpoint of the graph of h.

Given that $p > -\frac{\pi}{4}$, find the possible values of a. 1 mark

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}\,dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$