THE SCHOOL FOR EXCELLENCE (TSFX)

UNIT 3 & 4 MATHEMATICAL METHODS 2020

WRITTEN EXAMINATION 1

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOKLET

Student Name:

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

- Students are **NOT** permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials Supplied

- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the revised 2020 VCE Mathematics Study Design in which significant deletions of content to Mathematical Methods Area of Study 4 (Probability and Statistics) were made by VCAA.

Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than 1 mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 (3 marks)

Let $h:(-\infty,\, a) \to R$, $h(x) = \dfrac{\cos(\pi x)}{\log_e(-3x)}$ and a is a real constant.

a. Find $h'(-1)$.

2 marks

b. Find the largest value of a such that h is defined.

1 mark

Question 2 (4 marks)

Let $f : D \to R$, $f(x) = \dfrac{1}{2} + \dfrac{1}{\sqrt{2-5x}}$ and D is the maximal domain of f.

a. Find $f'(x)$.

1 mark

b. If $g(x)$ is an anti-derivative of $f(x)$ and $g(0) = \sqrt{2}$, find $g(x)$.

3 marks

Let $f : D \to R$, $f(x) = \dfrac{1}{2} + \dfrac{1}{\sqrt{2-5x}}$ and D is the maximal domain of f.

Question 3 (5 marks)

Let $f : \left(-\infty, \dfrac{1}{2}\right) \to R$, $f(x) = x^2 - 2x$.

a. Find the rule of f^{-1}. 2 marks

b. State the domain of f^{-1}. 1 mark

c. Let g be the function obtained by applying the transformation T to the function f where

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix}$$

and $a, b, c \in R$.

Find the values of a, b and c given that $g(x) = -2x^2 + 8x - 1$. 2 marks

Question 4 (3 marks)

Let the functions $f : D \to R$, $f(x) = x^2 - 1$ and $g : (-\infty, 3] \to R$, $g(x) = \log_e(8-x)$.

a. Find the rule for the function h where $h(x) = g(f(x))$.   1 mark

b. Find the maximal domain D for which h exists.   2 marks

Question 5 (3 marks)

Let $f:[-1,\ 1]\to R$, $f(x)=-2x^3+ax^2+bx$ where a and b are real constants.

The average rate of change of f over its domain is -3 and the average value of f is -6.

Find the values of a and b.

Question 6 (2 marks)

Let X be a normally distributed random variable with mean 24 and standard deviation 5.

Find the value of b if $\Pr(X < b) = \Pr\left(Z > \dfrac{b}{4}\right)$ where Z is the standard normal random variable.

Question 7 (4 marks)

The Acme Battery Company makes batteries that are sold in packets of 16. The probability that a battery will last for more than 100 hours is $\dfrac{11}{12}$.

a. Find the probability that at least 15 batteries in a packet will last for more than 100 hours. Express your answer in the form $\dfrac{a}{4}\left(\dfrac{b}{12}\right)^n$ where a, b and n are integers.   2 marks

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James buys a packet of batteries made by the Acme Battery Company. The first battery he uses lasts for more than 100 hours.

b. Find the probability that at least 15 batteries in the packet bought by John last for more than 100 hours. Express your answer in the form $\dfrac{k(11)^p}{12^m - 1}$ where k, p and m are integers.   2 marks

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Question 8 (4 marks)

a. Find the derivative of $x^2 \log_e(x)$. 1 mark

b. Hence find the area bounded by curve $y = x \log_e(x)$ and the x-axis between $x = 1$ and $x = e$. 3 marks

Question 9 (6 marks)

Let $f(\theta) = 2\cos^2(\theta)$ and $g(\theta) = 3\sin(\theta)$.

a. Solve $f(\theta) = g(\theta)$ for θ.   3 marks

b. Find the maximum difference between the values of $f(\theta)$ and $g(\theta)$.   3 marks

Question 10 (6 marks)

a. Solve $3^{2x} < 3^{x+1} + 4$ for x. 3 marks

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b. Solve $\log_{2x}(16) < -2$ for x. 3 marks

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END OF QUESTION AND ANSWER BOOK