Neap

Trial Examination 2020

VCE Mathematical Methods Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

Students are to write in blue or black pen.

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 10 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = \dfrac{1}{(1-2x)^2}$.

Find $\dfrac{dy}{dx}$. 1 mark

b. Let $f(x) = x^3 \cos(2x)$.

Evaluate $f'\left(\dfrac{\pi}{4}\right)$. 2 marks

Question 2 (2 marks)

Let $f : \left(-\infty, \frac{1}{3}\right) \to R, f(x) = \frac{3}{1-3x}$ and let $\int_{-1}^{0} f(x)dx = \log_e(b)$.

Find the value of b.

Question 3 (6 marks)

Let $f'(x) = 1 + e^{-\frac{x}{2}}$.

a. Evaluate $f'(\log_e(9))$. Give your answer in the form $\frac{m}{n}$ where m and n are integers. 2 marks

b. Given that $f(-2) = -2e$, find the rule for $f(x)$. 2 marks

c. The transformation $T : R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Find the rule of the image of the curve of $f'(x)$ under this transformation. 2 marks

Question 4 (5 marks)

a. Solve $25^m - \frac{1}{5^{1-2m}} = 48$ for $m \in R$. Give your answer in the form $m = \frac{\log_e(a)}{\log_e(b)}$, where $a, b \in Z^+$. 3 marks

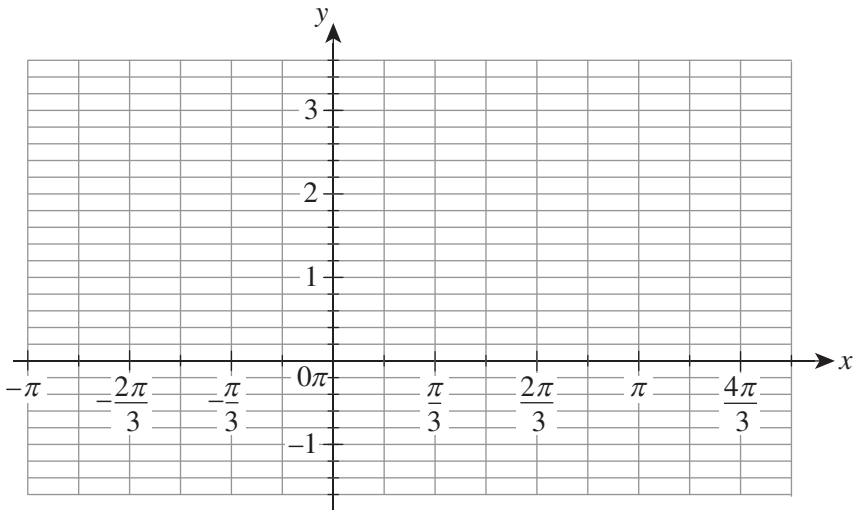
b. Solve $\frac{5}{\log_e(x) + 2} = \log_e(x) - 2$ for $x > 0$. 2 marks

Question 5 (7 marks)

a. Solve $1 - 2\sin(2x) = 0$ for $x \in \left(-\frac{\pi}{2}, \pi\right]$. 2 marks

Let $f : \left(-\frac{\pi}{2}, \pi\right] \to R, f(x) = 1 - 2\sin(2x)$.

b. Sketch the graph of $y = f(x)$ on the axes below. Label the axes intercepts and endpoints with their coordinates as appropriate. 3 marks



c. Find the equation of the tangent to the graph of $y = f(x)$ at the point where the graph crosses the *y*-axis. 2 marks

Question 6 (8 marks)

A phone dealer has eight new J-phones in stock. It is found that three of these phones have faulty batteries.

a. Two of the eight J-phones are selected at random.

i. Find the probability that at least one of the selected phones has a faulty battery. 2 marks

ii. Given that at least one of the selected phones has a faulty battery, find the probability that exactly one phone has a faulty battery. 2 marks

b. It is known that, overall, 20% of J-phones will have a faulty battery. The probability of any given phone having a faulty battery is independent to any other phone. A phone dealer receives a delivery of four new J-phones.

Find the probability that at least two of these phones will have faulty batteries. 2 marks

c. Working batteries have a lifespan, measured in minutes, after which they require recharging.

The lifespan of working batteries is a normally distributed variable, X, with a mean of 540 minutes and a standard deviation of 70 minutes. Let Z be the standard normal variable such that $Z \sim N(0, 1)$.

i. Find b such that $\Pr(X < b) = \Pr(Z > -1.5)$. 1 mark

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ii. Correct to two decimal places, $\Pr(Z < -1) = 0.16$.

Find the probability that a randomly selected phone battery has a lifespan of greater than 470 minutes, given that it has a lifespan of less than 540 minutes. 1 mark

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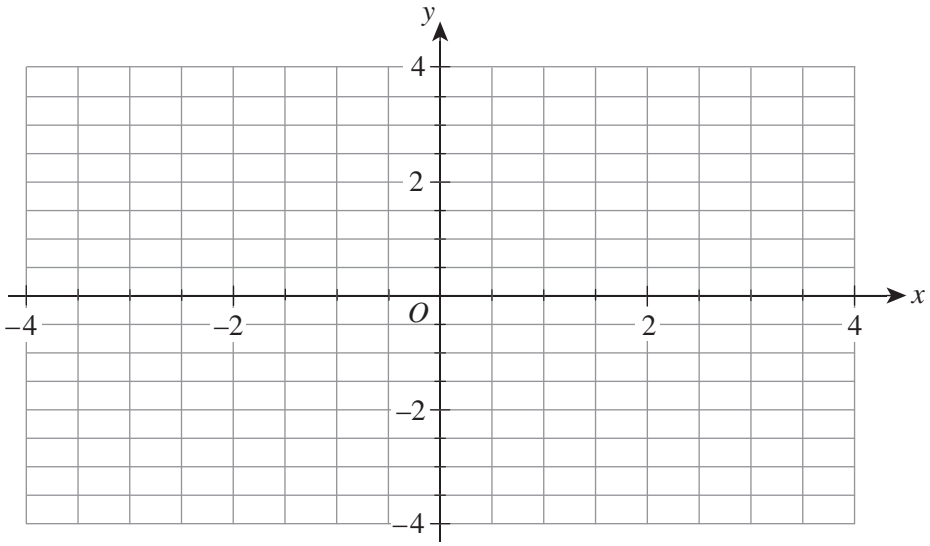
Question 7 (5 marks)

Let $f : R \to R, f(x) = e^{3x} - 2$.

a. Find the rule and domain of the inverse function f^{-1}. 2 marks

b. On the axes below, sketch the graph of $y = f(f^{-1}(x))$ for its maximal domain. 1 mark

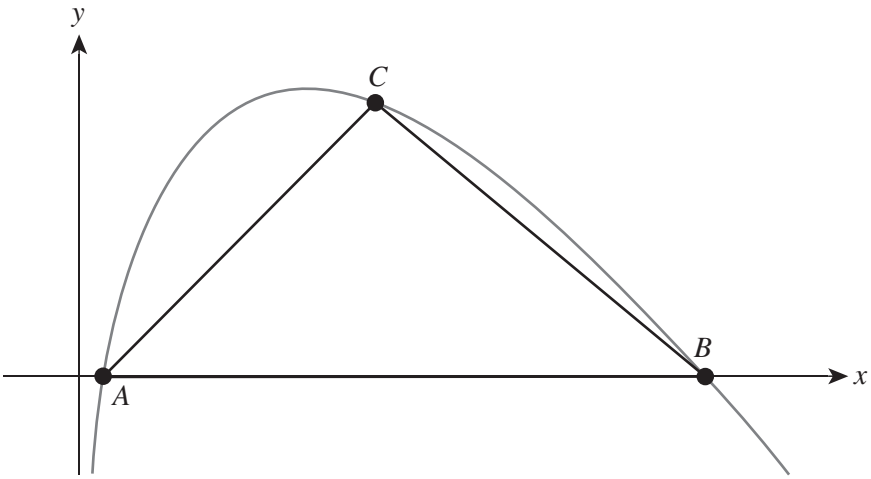


c. Find $f(-f^{-1}(3x))$ in the form $\frac{ax+b}{cx+d}$, where $a, b, c, d \in Z$. 2 marks

Question 8 (4 marks)

Let $f : R^+ \cup \{0\} \to R, f(x) = 6\sqrt{x} - x - 5$.

Part of the graph of $y = f(x)$ is shown below, where points A and B are the x-intercepts of the graph and the point C lies on the curve between A and B.



a. Find the interval for which the graph of f is strictly decreasing. 2 marks

b. Find the maximum area of the triangle ABC. 2 marks

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2020

VCE Mathematical Methods Units 3&4

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.

A question and answer booklet is provided with this formula sheet.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$	

Probability distribution		Mean	Variance
Bernoulli	$P(X = x) = \begin{cases} p & x = 1 \\ 1 - p & x = 0 \end{cases}$	$\mu = p$	$\sigma^2 = p(1 - p)$
binomial	$P(X = x) = \binom{n}{x} p^x (1 - p)^{n - x}$	$\mu = np$	$\sigma^2 = np(1 - p)$
normal	$\Pr(X \le a) = \int_{-\infty}^{a} \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2} dx$	μ	σ^2

END OF FORMULA SHEET