Neap

Trial Examination 2020

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

a. $y = \dfrac{1}{(1-2x)^2}$

$= (1-2x)^{-2}$

$\dfrac{dy}{dx} = -2 \times -2 \times (1-2x)^{-3}$

$= \dfrac{4}{(1-2x)^3}$ A1

b. Let $f(x) = x^3\cos(2x)$.

$f'(x) = 3x^2\cos(2x) - 2x^3\sin(2x)$ M1

$f'\left(\dfrac{\pi}{4}\right) = 3\left(\dfrac{\pi}{4}\right)^2\cos\left(\dfrac{\pi}{2}\right) - 2\left(\dfrac{\pi}{4}\right)^3\sin\left(\dfrac{\pi}{2}\right)$

$= 0 - \dfrac{2\pi^3}{64}$

$= -\dfrac{\pi^3}{32}$ A1

Question 2 (2 marks)

$\displaystyle\int_{-1}^{0}\frac{3}{1-3x}\,dx = \left[-\frac{1}{3}\times 3\log_e(1-3x)\right]_{-1}^{0}$ M1

$= -[\log_e(1-3x)]_{-1}^{0}$

$= -(\log_e(1) - \log_e(4))$

$= \log_e(4)$

$\therefore b = 4$ A1

Question 3 (6 marks)

a. $f'(x) = 1 + e^{-\frac{x}{2}}$

$f'(\log_e(9)) = 1 + e^{-\frac{\log_e(9)}{2}}$

$= 1 + e^{\log_e 9^{-0.5}}$ M1

$= 1 + e^{\log_e \frac{1}{3}}$

$= 1 + \dfrac{1}{3}$

$= \dfrac{4}{3}$ A1

b. $f(x) = \int 1 + e^{-\frac{x}{2}} dx$

$= x - 2e^{-\frac{x}{2}} + c$ A1

$f(-2) = -2e \to -2e = -2 - 2e + c$

$c = 2$

$f(x) = x - 2e^{-\frac{x}{2}} + 2$ A1

c. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$x' = 2x + 1$

$\to x = \frac{1}{2}(x' - 1)$

$y' = y + 4$ M1

$\to y = y' - 4$

$y' - 4 = 1 + e^{-\frac{1}{2} \times \frac{1}{2}(x' - 1)}$

$y = e^{-\frac{1}{4}(x - 1)} + 5$ A1

Question 4 (5 marks)

a. $25^m - \frac{1}{5^{1 - 2m}} = 48$

$5^{2m} - \frac{1}{5} \times 5^{2m} = 48$ M1

$\frac{4}{5} \times 5^{2m} = 48$

$5^{2m} = 60$ M1

$\log_e(5^{2m}) = \log_e(60)$

$2m(\log_e(5)) = \log_e(60)$

$m = \frac{\log_e(60)}{2\log_e(5)}$

$= \frac{\log_e(60)}{\log_e(25)}$ A1

b. $$\frac{5}{\log_e(x)+2} = \log_e(x) - 2$$

$$(\log_e(x)+2)(\log_e(x)-2) = 5$$

$$(\log_e(x))^2 - 4 = 5$$ M1

$$(\log_e(x))^2 = 9$$

$$\log_e(x) = \pm 3$$

$x = e^3$ or $x = e^{-3}$ A1

Question 5 (7 marks)

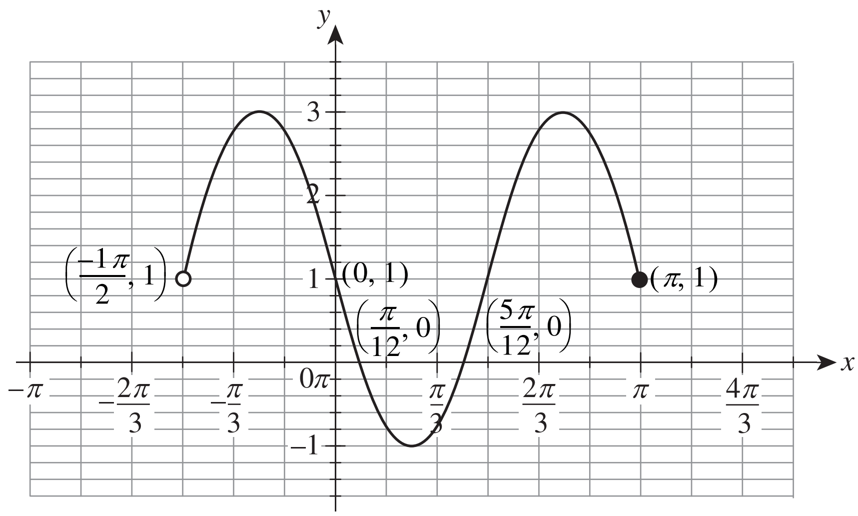
a. $$1 - 2\sin(2x) = 0$$

$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$ M1

$x = \frac{\pi}{12}$ or $x = \frac{5\pi}{12}$ A1

b.



correct intercepts A1
correct endpoints A1
correct shape A1

c. $$f'(x) = -4\cos(2x)$$

$$f'(0) = -4\cos(0)$$ M1

$$= -4$$

$$= 1$$

$\therefore y = -4x + 1$ is the tangent line at the y-intercept A1

Question 6 (8 marks)

a. **i.** $\Pr(\text{at least one faulty}) = 1 - \Pr(\text{none faulty})$

$$= 1 - \frac{5}{8} \times \frac{4}{7}$$ A1

$$= 1 - \frac{5}{14} = \frac{9}{14}$$ M1

ii. Let X be the number of faulty batteries.

$$\Pr(X = 1 \mid X \geq 1) = \frac{\Pr(X = 1 \cap X \geq 1)}{\Pr(X \geq 1)} = \frac{\Pr(X = 1)}{\Pr(X \geq 1)}$$

$$\Pr(X = 1) = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{28}$$ M1

$$\Pr(X \geq 1) = \frac{9}{14}$$

$$\Pr(X = 1 \mid X \geq 1) = \frac{\frac{15}{28}}{\frac{9}{14}}$$

$$= \frac{5}{6}$$ A1

b. Let Y be the random variable for faulty batteries.

$$Y \sim Bi\left(4, \frac{1}{5}\right)$$

$$\Pr(Y \geq 2) = 1 - \Pr(Y = 0) - \Pr(Y = 1)$$

$$= 1 - \left(\frac{4}{5}\right)^4 - {}^4C_1\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)$$ M1

$$= 1 - \frac{256}{625} - \frac{256}{625}$$

$$= \frac{113}{625}$$ A1

c. **i.** $\Pr(Z > -1.5) = \Pr(Z < 1.5)$

$$z = \frac{x - \bar{x}}{\mu}$$

$$1.5 = \frac{b - 540}{70}$$

$$b = 540 + 1.5 \times 70$$

$$= 645$$ A1

ii. $$\Pr(X > 470 \mid X < 540) = \frac{\Pr(470 < X < 540)}{\Pr(X < 540)}$$

$$= \frac{\Pr(-1 < Z < 0)}{\Pr(Z < 0)}$$

$$= \frac{0.5 - 0.16}{0.5}$$

$$= 0.68$$ A1

Question 7 (5 marks)

a. $f: R \to R, f(x) = e^{3x} - 2$

Let $y = e^{3x} - 2$.

For inverse, swap *x* and *y*:

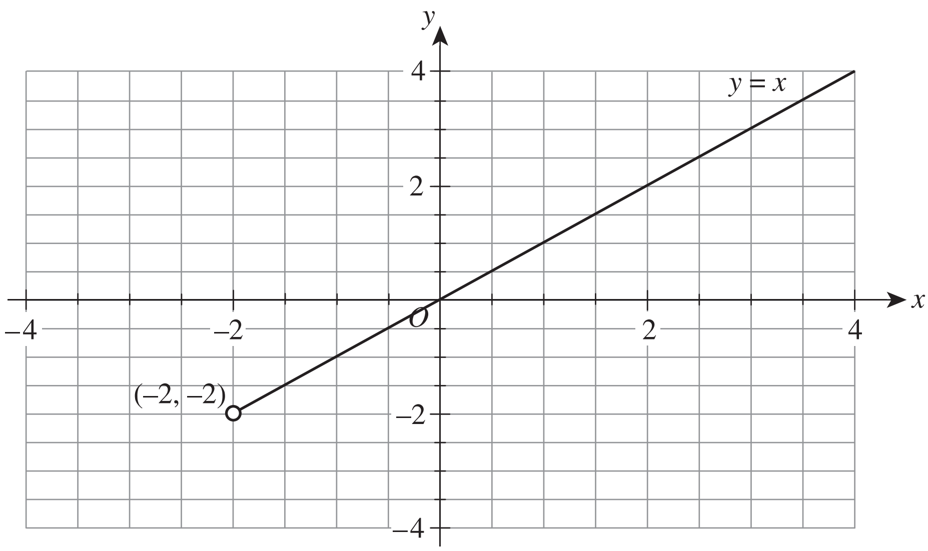
$$x = e^{3y} - 2$$

$$3y = \log_e(x + 2)$$

$$f^{-1}(x) = \frac{1}{3}\log_e(x + 2)$$ A1

The domain is (–2, ∞). A1

b.



correct line, domain and end point (–2, –2) A1

c. $f^{-1}(3x) = \frac{1}{3}\log_e(3x + 2)$

$$f(-f^{-1}(3x)) = e^{\ln\left(\frac{1}{3x+2}\right)} - 2$$ M1

$$= \frac{1}{3x + 2} - 2$$

$$= \frac{1}{3x + 2} - \frac{2(3x + 2)}{3x + 2}$$

$$= \frac{-6x - 3}{3x + 2}$$ A1

Question 8 (4 marks)

a. $f(x) = 6\sqrt{x} - x - 5$

$f'(x) = 3x^{-\frac{1}{2}} - 1$

$= \frac{3}{\sqrt{x}} - 1$

Let $f'(x) = 0$.

$\frac{3}{\sqrt{x}} - 1 = 0$ M1

$x = 9$

The domain is strictly decreasing for $x \in [9, \infty)$. A1

b. The maximum area of the triangle *ABC* occurs when point *C* is the turning point of $f(x)$ at $x = 9 \rightarrow f(9) = 4$.

Point *C* is (9, 4).

Let $f(x) = 0$.

$6\sqrt{x} - x - 5 = 0$

Let $a = \sqrt{x} \rightarrow 6a - a^2 - 5 = 0$.

$a^2 - 6a + 5 = 0$

$(a - 5)(a - 1) = 0$

$(\sqrt{x} - 5)(\sqrt{x} - 1) = 0$ M1

$x = 1$ or $x = 25$

Area of $ABC = \frac{1}{2}bh$

$= \frac{1}{2}(25 - 1) \times 4$

$= 48$ square units A1