Neap

Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 16 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

If $Q(x) = 3x^3 - ax^2 + bx + c$, $Q(-1) = -6$, $Q(3) = 66$ and $Q(-3) = -126$, the values of a, b and c are, respectively

A. 4, 5, 6

B. –4, 5, 6

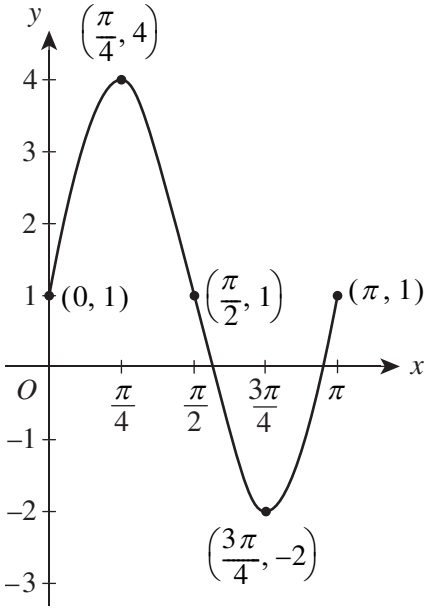
C. 5, –4, 6

D. –5, 4, 6

E. –1, 3, –3

Question 2

Consider the graph shown below.



A possible equation for the graph is

A. $y = -3\sin(2x) + 1$

B. $y = 3\sin(x) + 1$

C. $y = -3\cos(2x) + 1$

D. $y = 3\sin(2x) + 1$

E. $y = -3\sin(x) + 1$

Question 3

For the function $f: (-\infty, q] \to R,\ f(x) = x^4 - 2x^3 - 4x^2 + 2x + 6$ to have an inverse, the largest possible value of q is

A. -2

B. -1

C. 1

D. 2

E. 6

Question 4

If $f(x) = 3x^2 - 6x + 4,$ then $f(x) \geq x$ when

A. $x = 1$ and $\frac{4}{3}$

B. $x \leq 1 \cup x \leq \frac{4}{3}$

C. $x \leq 1 \cup x \geq \frac{4}{3}$

D. $1 \leq x \leq \frac{4}{3}$

E. $x = \frac{4}{3}$

Question 5

The implied domain of $y = -\sqrt{36 - x^2}$ is

A. $-6 \leq x \leq 6$

B. $-36 \leq x \leq 36$

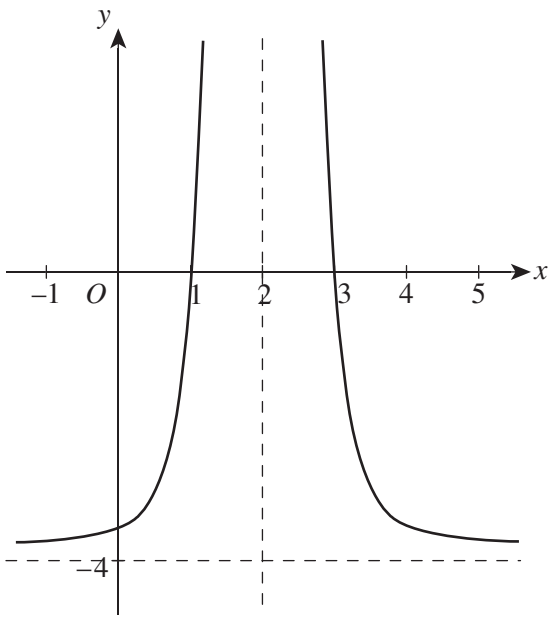
C. $-6 \leq x \leq 0$

D. $0 \leq x \leq 6$

E. $x \leq 6$

Question 6

Consider the function shown below.



The equation of the function is

A. $y = \dfrac{3}{(x-2)} - 4$

B. $y = \dfrac{3}{(x-4)} - 2$

C. $y = \dfrac{4}{(x-2)^2} - 2$

D. $y = \dfrac{3}{(x-2)^2} - 4$

E. $y = -\dfrac{3}{(x-2)^2} - 4$

Question 7

The centre and radius of the circle $x^2 + y^2 - 10x + 4y = -25$ are, respectively

A. $(5, 2)$ and 5.

B. $(-5, 2)$ and 5.

C. $(5, -2)$ and 5.

D. $(5, -2)$ and 4.

E. $(5, -2)$ and 2.

Question 8

The value(s) of m for which the equations $y = 3x^2 + mx - 2$ and $y = x - 5$ have no solution, a unique solution, or two solutions are, respectively

A. $-5 \leq m \leq 7$, $m = -5$ or 7, and $m \leq -5 \cup m \geq 7$

B. $m \leq -5 \cup m \geq 7$, $m = -5$ or 7, and $-5 \leq m \leq 7$

C. $-5 \leq m \leq 7$, $m = -5$ or 7, and $m \leq -5 \cap m \geq 7$

D. $m < -5 \cup m > 7$, $m = -5$ or 7, and $-5 < m < 7$

E. $-5 < m < 7$, $m = -5$ or 7, and $m < -5 \cup m > 7$

Question 9

A set of data is shown in the table below.

x	2	3	4	5
y	6.75	10.125	15.1875	22.78125

The equation for the data in the form of $y = a \times b^x$ is

A. $y = 3 \times 1.5^x$

B. $y = 1.5 \times 3^x$

C. $y = 3 \times 0.5^x$

D. $y = 0.5 \times 1.5^x$

E. $y = 1.5 \times 0.5^x$

Question 10

The number of solutions to the equation $\sin(3x) = -\frac{1}{2}$, where $(-\pi \leq x \leq \pi)$ is

A. 2

B. 3

C. 4

D. 5

E. 6

Question 11

If $f(x) = -2x^3 + \frac{2}{x^2} + 3x + 4$, then $f'(x)$ is equal to

A. $-6x^2 - \frac{4}{x^{-3}} + 3$

B. $-6x^2 - \frac{4}{x^3} + 3$

C. $-6x^2 - \frac{2}{x^3} + 3$

D. $-6x^2 - \frac{2}{x^{-3}} + 3$

E. $-\frac{x^4}{2} - \frac{2}{x} + \frac{3}{2}x^2 + 4x + c$

Question 12

If $\frac{dy}{dx} = 4x^3 - 2x(x-3)$, then a possible antiderivative is

A. $12x^2 - 4x + 6$

B. $x^4 - \frac{1}{3}x^3 + 3x^2 - 3$

C. $x^4 - \frac{2}{3}x^3 - 3x^2 - 3x$

D. $x^4 - \frac{2}{3}x^3 + 3x^2 - 3$

E. $x^4 - \frac{2}{3}x^3 + 3x^2 - 3x - 3$

Question 13

If $f(-1) = -3$ and $f'(-1) = 0$, $f'(x) < 0$ for $x < -1$, and $f'(x) > 0$ for $x > -1$, then the graph of $f(x)$ has

A. a point of inflection at $(-1, -3)$.

B. a local maximum at $(-1, -3)$.

C. a local minimum at $(-1, -3)$.

D. a local maximum at $(-3, -1)$.

E. a local minimum at $(-3, -1)$.

Question 14

If $y = 9$ when $x = 2$ and $\frac{dy}{dx} = 4x + 3$, then the equation of y is

A. $y = 4x^2 + 3x$

B. $y = 2x^2 + 3x$

C. $y = 2x^2 + 3x - 5$

D. $y = 2x^2 + 3x + 5$

E. $y = 2x^2 + 3x - 187$

Question 15

A line joins the points $(2, -5)$ and $(1, -2)$.

The angle that the line makes with the positive direction of the x-axis, correct to three decimal places, is

A. $-71.565°$

B. $71.565°$

C. $108.434°$

D. $108.435°$

E. $178.751°$

Use the following information to answer Questions 16 and 17.

Events A and B are independent. $\Pr(A) = 0.55$ and $\Pr(A \cap B) = 0.363$.

Question 16

$\Pr(B)$ is equal to

A. 0.19

B. 0.20

C. 0.45

D. 0.55

E. 0.66

Question 17

$\Pr(A|B)$ is equal to

A. 0.09

B. 0.45

C. 0.55

D. 0.66

E. 1.52

Use the following information to answer Questions 18 and 19.

Jane has cereal for breakfast 65% of the time and toast on all other days. Ken has toast for breakfast 45% of the time and cereal on all other days.

Question 18

What is the probability that both Jane and Ken have cereal for breakfast on any given day?

A. 0.1575
B. 0.1925
C. 0.2925
D. 0.2975
E. 0.3575

Question 19

What is the probability that only one of them has cereal for breakfast on any given day?

A. 0.4850
B. 0.5500
C. 0.8075
D. 0.8425
E. 0.8752

Question 20

A partial Karnaugh map is shown below.

	N	N'	
F	0.45		0.63
F'			
	0.81		

$\Pr(F' \cap N')$ is

A. 0.01
B. 0.18
C. 0.19
D. 0.36
E. 0.55

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided. Write in blue or black pen.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

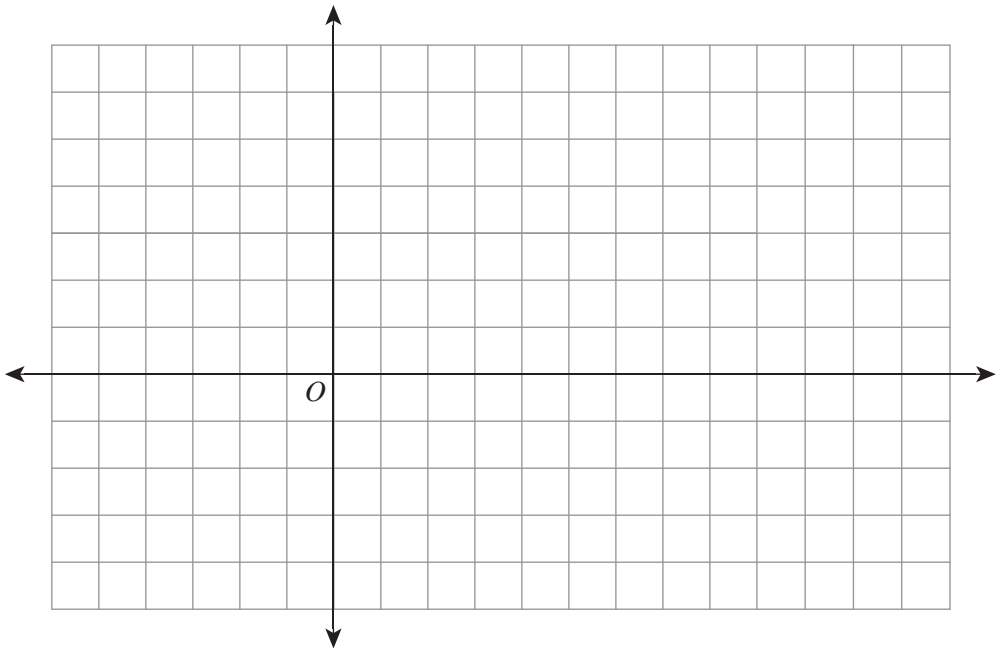
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (15 marks)

Let $f : R\backslash\{3\} \to R, f(x) = -\dfrac{2}{x-3} + 1$.

a. Sketch the graph of $f(x)$ on the axes below, labelling all asymptotes and axes intercepts with their coordinates. 4 marks



b. What type of relation is this? 1 mark

_____

c. Show that the rule of the inverse function is $f^{-1}(x) = \frac{2}{1-x} + 3$. 3 marks

d. State the domain and range of the inverse function. 2 marks

e. Describe the transformations that map the graph of $f(x)$ onto $g(x) = \frac{1}{x}$, where $x \neq 0$. 3 marks

f. Using an algebraic method, show that the point $(4, -1)$ lies on the graph of $f(x)$. 2 marks

Question 2 (15 marks)

A pier extends from the shore out over a lake. The depth (D) of water, in metres, at time t hours after 6.00 am on a particular day is given by the equation: $D(t) = 20 + 3\cos\left(\frac{\pi t}{4}\right)$, $0 \le t \le 16$.

a. State the amplitude of $D(t)$. 1 mark

_____

b. State the period of $D(t)$. 1 mark

_____

_____

c. Find the depth of the water at 6.00 am. 1 mark

_____

_____

_____

_____

_____

d. What are the maximum and minimum depths of water during the 16 hours? 2 marks

_____

_____

_____

e. **i.** How many high tides are there during the 16-hour period? 1 mark

_____

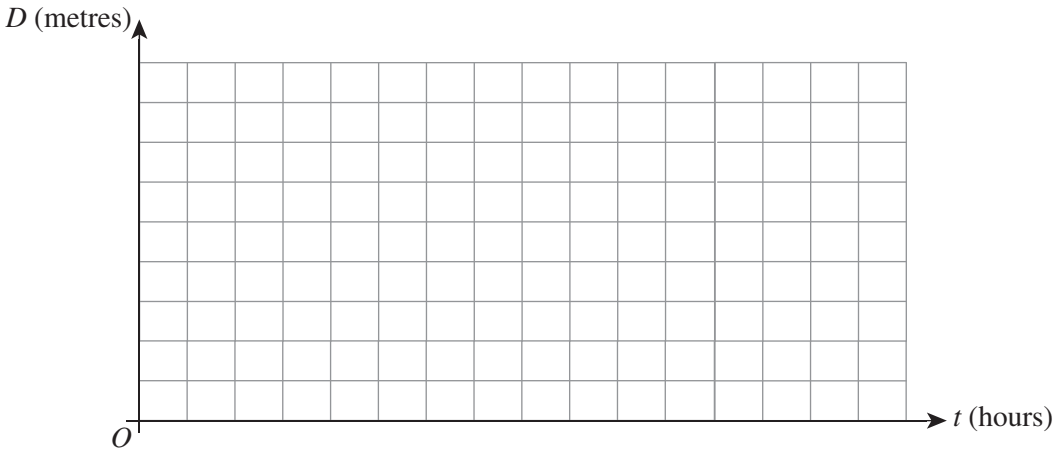
_____

ii. When do these high tides occur? 1 mark

_____

_____

f. Sketch the graph of $D(t)$ on the axes below, labelling the start and end points and maximum and minimum points with their coordinates. 3 marks



g. **i.** Solve $D(t) = 19$, correct to two decimal places. 2 marks

ii. A ship can only approach the pier when the minimum depth of water is 19 metres.
During what times can the ship sail to and from the pier? Give your answer in 12-hour system form (for example, 6.15 am). 3 marks

Question 3 (14 marks)

Let $f(x) = -x^4 - 4x^3 - x^2 + 8x + 4$.

a. Find the x-intercepts of $f(x)$, correct to two decimal places. 2 marks

_____

_____

_____

_____

b. Find the coordinates of the stationary points of $f(x)$, correct to two decimal places, and state the nature of each stationary point. 3 marks

_____

_____

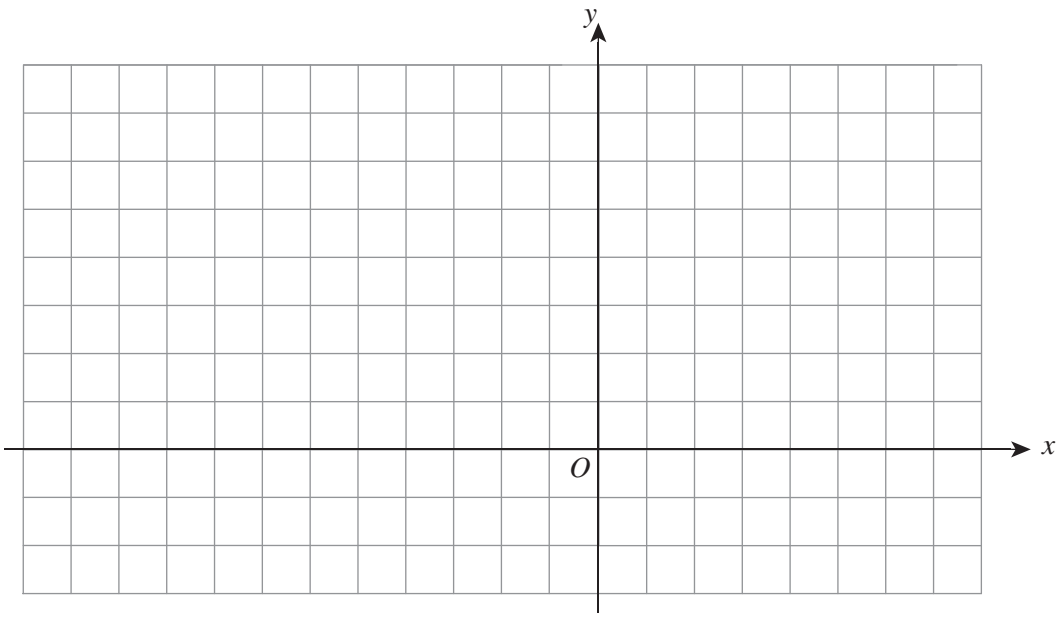
_____

_____

_____

_____

c. Sketch the graph of $f(x)$ on the axes below, labelling axes intercepts and maximum and minimum points with their coordinates. 4 marks



d. Find the average rate of change between $x = 0$ and $x = 0.5$, correct to three decimal places. 2 marks

e. Find the instantaneous rate of change at $x = \frac{1}{2}$. 2 marks

f. Find $\int_{-1.2}^{1.2} f(x)dx$, correct to three decimal places. 1 mark

Question 4 (16 marks)

A box contains four red marbles, five blue marbles and three green marbles.

a. Three marbles are randomly selected **with** replacement.

i. What is the probability of selecting three marbles of different colours? 1 mark

ii. What is the probability of selecting three green marbles? 1 mark

iii. What is the probability of selecting three marbles of the same colour? 2 marks

b. Two marbles are now selected at random **without** replacement.

i. How many different possible combinations of colours could be selected? 1 mark

ii. What is the probability of selecting two green marbles? 1 mark

iii. What is the probability of selecting two marbles of the same colour? 1 mark

iv. What is the probability of selecting two marbles of different colours? 2 marks

v. If the first marble selected is green, what is the probability that the second marble selected is green? 1 mark

_____

_____

_____

c. All the marbles are now removed from the box and lined up in a row.

i. How many different possible combinations could be made? 1 mark

_____

_____

ii. If the red marbles must be grouped together, how many different possible combinations could be made? 2 marks

_____

_____

iii. If the marbles must be grouped together based on colour, how many different possible combinations can be made? 1 mark

_____

_____

d. A new set of twelve marbles is placed in the box. The marbles are all different colours. Five marbles are randomly selected.

i. How many combinations could be made? 1 mark

_____

_____

ii. If one marble must be red and one marble must be green, how many combinations can be made? 1 mark

_____

_____

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: _____

Teacher's Name: _____

Instructions

Use a pencil for all entries. If you make a mistake, erase the incorrect answer – do not cross it out. Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like **this** example: A **B** C D E

Use pencil only

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

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Trial Examination 2020

VCE Mathematical Methods Units 1&2

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	

END OF FORMULA SHEET