The Mathematical Association of Victoria
Trial Examination 2020

MATHEMATICAL METHODS

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 22 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
  Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the Adjusted 2020 VCE Mathematics Study Design.

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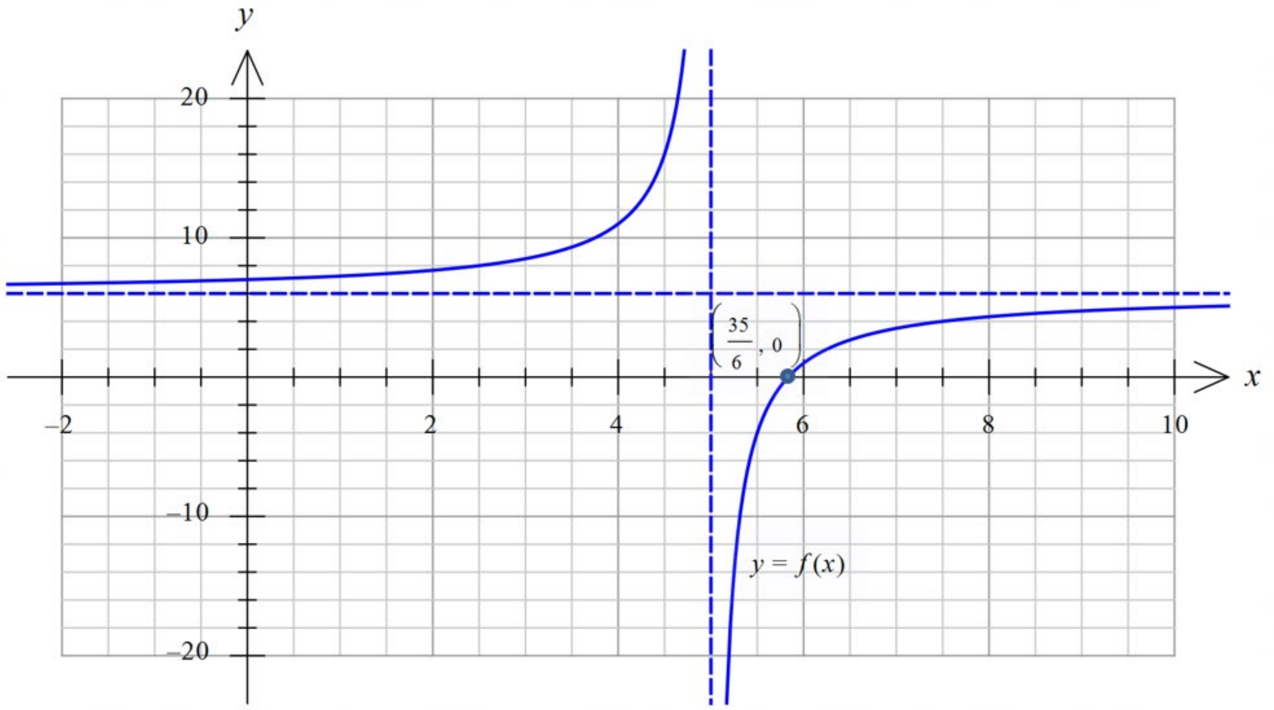
SECTION A- Muliple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1



An equation for the graph above could be

A. $f(x) = -\frac{1}{x-5} + 6$

B. $f(x) = \frac{5}{5-x} + 6$

C. $f(x) = -\frac{1}{x+5} - 6$

D. $f(x) = -\frac{5}{6(x-5)} + 6$

E. $f(x) = \frac{5}{x-5} + 6$

Question 2

The point $\left(\frac{1}{2},-3\right)$ lies on the graph of a function with the rule $y=-4(2x-1)^4-3$.

At the point $\left(\frac{1}{2},-3\right)$

A. the gradient is undefined.
B. the gradient is zero.
C. the curve is discontinuous.
D. there is a stationary point of inflection.
E. there is a minimum turning point.

Question 3

The solution(s) to the equation $\cos(2x)=\cos(x)$ can be expressed as

A. $x=\frac{2\pi k}{3}$ $\quad k\in Z$
B. $x=\frac{2\pi k}{3}+\pi k,\ x=2\pi k$ $\quad k\in Z$
C. $x=\pi k$ $\quad k\in Z$
D. $x=0,\ x=\frac{2\pi}{3},\ x=\frac{4\pi}{3},\ x=2\pi$
E. $x=\frac{2\pi k}{3}$ $\quad k\in R$

Question 4

Given $f:[-3,4)\to R, f(x)=2x+1$ and $g:[-4,2]\to R, g(x)=x^2+2x$.

The range of $h(x)=f(x)+g(x)$ is

A. $[-2,13]$
B. $[-3,2]$
C. $[-4,4)$
D. $[-3,13]$
E. $[-2,4]$

Question 5

Under a sequence of transformations the image of $y=\frac{1}{2x}$ is $y_T=-\frac{3}{x-1}+6$. The transformations applied, in order, to the graph of $y=\frac{1}{2x}$ could be

A. a reflection in the x-axis and then translations in the positive direction of the x-axis by 1 unit and the y-axis by 6 units, a dilation from the y-axis by a factor of 2, followed by a dilation from the x-axis by a factor of 3.

B. a dilation from the x-axis by a factor of $\frac{1}{2}$, followed by a dilation from the y-axis by a factor of 3, then a reflection in the x-axis and translations in the positive direction of the x-axis by 1 unit and the y-axis by 6 units.

C. a dilation from the y-axis by a factor of 2, followed by a dilation from the x-axis by a factor of 3, then a reflection in the x-axis and translations in the negative direction of the x-axis by 1 unit and the positive direction of the y-axis by 6 units.

D. a dilation from the y-axis by a factor of $\frac{1}{2}$, followed by a dilation from the x-axis by a factor of 3, then a reflection in the x-axis and translations in the positive direction of the x-axis by 1 unit and the y-axis by 6 units.

E. a dilation from the x-axis by a factor of 2, followed by a dilation from the y-axis by a factor of 3, then a reflection in the x-axis and translations in the positive direction of the x-axis by 1 unit and the y-axis by 6 units.

Question 6

Consider the following simultaneous equations

$$2x+ky=a$$
$$kx+3y=7$$

where k and a are real constants.

The above simultaneous equations will have no solutions when

A. $k=\sqrt{6}$ and $a=\frac{14}{\sqrt{6}}$ only

B. $k=\sqrt{6}$ and $a=\frac{14}{\sqrt{6}}$ or $k=-\sqrt{6}$ and $a=-\frac{14}{\sqrt{6}}$

C. $k=\sqrt{6}$ and $a\in R\backslash\left\{\frac{14}{\sqrt{6}}\right\}$ or $k=-\sqrt{6}$ and $a=R\backslash\left\{-\frac{14}{\sqrt{6}}\right\}$

D. $k\in R\backslash\left\{-\sqrt{6},\sqrt{6}\right\}$ and $a\in R$

E. $k=\sqrt{6}$ and $a\in R\backslash\left\{\frac{14}{\sqrt{6}}\right\}$ only

SECTION A - continued
TURN OVER

Question 7

The graph of the derivative of the inverse function of *f* where $f(x)=2\log_e\left(1-4x\right)+1$ has

A. an asymptote with equation $x=\frac{1}{4}$ and an *x*-axis intercept at $\frac{1}{4}\left(1-e^{-\frac{1}{2}}\right)$.

B. an asymptote with equation $y=\frac{1}{4}$ and an *x*-axis intercept at 1.

C. an asymptote with equation $y=0$ and a *y*-axis intercept at $-\frac{1}{8}e^{-\frac{1}{2}}$.

D. an asymptote with equation $x=0$ and an *x*-axis intercept at $-\frac{1}{8}e^{-\frac{1}{2}}$.

E. asymptotes with equations $x=\frac{1}{4}$ and $y=0$.

Question 8

If the transformation $T: R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

maps the curve with equation $f(x)=a(x-b)^3+c$, where *a*, *b* and *c* are real constants, to the curve with equation $g(x)=x^{\frac{1}{3}}+1$, then the values of *a*, *b* and *c* are respectively

A. 1, 3 and -3
B. 1, 2 and -3
C. 1, 3 and -2
D. 1, 4 and 2
E. 1, 4 and -2

Question 9

Consider $f(x)=e^{x^2+2x+1}$ and $g(x)=\log_2(x)$ over their maximum domains.

The rule and range for $g\left(f(x)\right)$ are respectively

A. $g\left(f(x)\right)=\log_2(e)(x+1)^2$ and $\left[1,\infty\right)$

B. $g\left(f(x)\right)=(x+1)^2$ and $\left[0,\infty\right)$

C. $g\left(f(x)\right)=\frac{1}{\log_e(2)}(x+1)^2$ and $\left[0,\infty\right)$

D. $g\left(f(x)\right)=e^{(\log_2(x))^2+2\log_2(x)+1}$ and $\left[1,\infty\right)$

E. $g\left(f(x)\right)=\log_2(e)(x+1)^2$ and R

Question 10

Which one of the following statements is true for the polynomial $Q(x) = -2x^3 + 3x^2 - x - 6$?

A. The graph of $Q(x)$ has a stationary point at $x = \frac{3-\sqrt{3}}{6}$.

B. $x-1$ is a factor of $Q(x)$.

C. The graph of $Q(x)$ has a stationary point of inflection.

D. The derivative of $y = Q(x)$ is $\frac{dy}{dx} = -6x^2 + 6x - 6$.

E. The graph of $y = Q(x)$ has three x-intercepts.

Question 11

Let $f : R^+ \to R$ be a differentiable function. For all real values of x, the derivative of $f\left(\log_e(2x)\right)$ with respect to x equals

A. $\frac{1}{2x} f'(x)$

B. $\frac{1}{x} f'(x)$

C. $\frac{1}{x} f'\left(\log_e(2x)\right)$

D. $2f'\left(\log_e(2x)\right)$

E. $f'\left(\log_e(2x)\right)$

Question 12

The graph of $f(x) = ax^6 + bx^5 + x^4 - 3$, where a and b are real constants will have three stationary points when

A. $a > -\frac{25b^2}{96}$

B. $a \le \frac{25b^2}{96}$

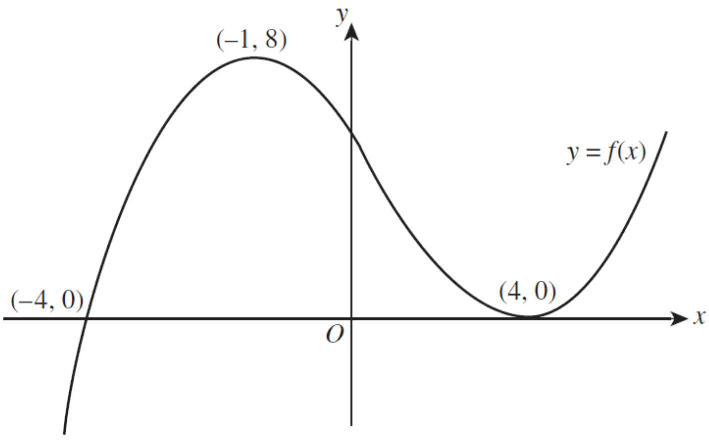
C. $a = \frac{25b^2}{96}$

D. $a < \frac{25b^2}{96}$

E. $a > \frac{25b^2}{96}$

Question 13

Part of the graph of polynomial *f* is shown below.



It is **incorrect** to state that

A. $f'(x)=0$ at $x=-1$ and $x=4$.

B. the graph of the derivative of *f* is strictly increasing for $x\in(-\infty,-1]\cup[4,\infty)$.

C. the graph of *f* is strictly decreasing over the interval $[-1,4]$.

D. $f'(x)>0$ for $x\in(-\infty,-1)\cup(4,\infty)$.

E. $f(x)=0$ at $x=-4$ and $x=4$.

Question 14

The average value of the function $y=\cos^2(x)$ over the interval $\left[0,\frac{\pi}{2}\right]$ is

A. $\frac{\pi^2}{8}$

B. $\frac{\pi}{4}$

C. $\frac{2}{\pi}$

D. $-\frac{2}{\pi}$

E. $\frac{1}{2}$

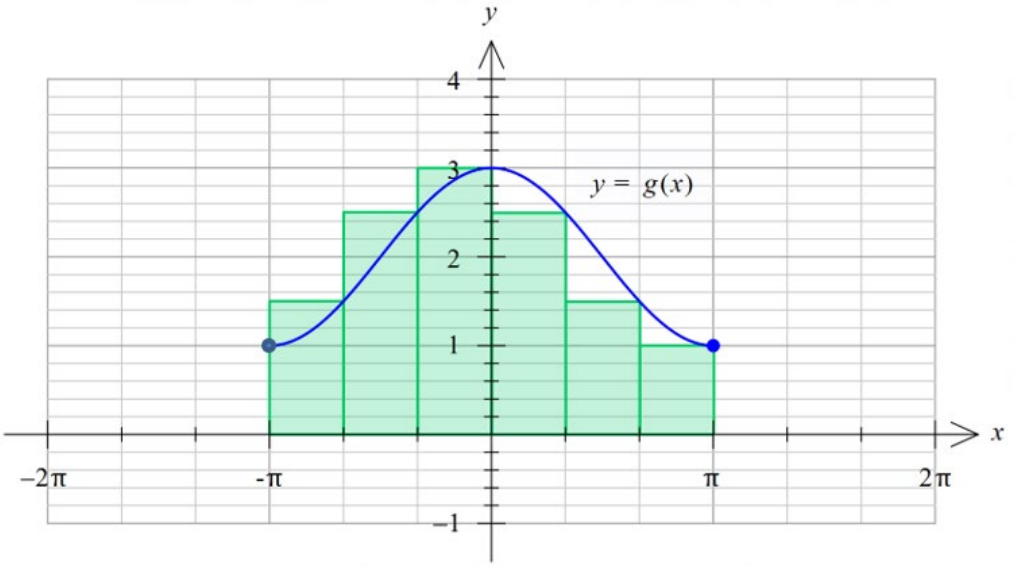
Question 15

Given $\int_{-1}^{6} f(x)=3$ then $\int_{6}^{-1} 1-2f(x)dx$ equals

A. -1
B. -13
C. 13
D. -8
E. 8

Question 16

The graph of $g:[-\pi,\pi]\to R, g(x)=\cos(x)+2$ is shown below.



The definite integral $\int_{-\pi}^{\pi} g(x)dx$ is approximated using the rectangles shown above. The approximation can be found by evaluating

A. $\frac{2\pi}{3}\left(g\left(-\frac{2\pi}{3}\right)+g\left(-\frac{\pi}{3}\right)+g(0)+g(\pi)\right)$

B. $2g\left(-\frac{2\pi}{3}\right)+2g\left(-\frac{\pi}{3}\right)+g(0)+g(\pi)$

C. $\frac{\pi}{3}\left(2g\left(\frac{\pi}{3}\right)+2g\left(\frac{2\pi}{3}\right)+g(\pi)\right)$

D. $\frac{\pi}{3}\left(2g\left(-\frac{2\pi}{3}\right)+2g\left(-\frac{\pi}{3}\right)+g(0)+g(\pi)\right)$

E. $\frac{\pi}{3}\left(2g(-\pi)+2g\left(-\frac{2\pi}{3}\right)+g\left(-\frac{\pi}{3}\right)+g(\pi)\right)$

Question 17

Let $f(x)=a(x-b)^2(x+c)$ where a, b and c are positive real constants.

The area bounded by the graph of f and the x-axis can be expressed as

A. $\int_a^b f(x)dx$

B. $\int_b^c f(x)dx$

C. $\int_b^{-c} f(x)dx$

D. $\int_a^b f(x)dx+\int_b^c f(x)dx$

E. $\int_{-c}^b f(x)dx$

Question 18

Vanessa is a netball player. The probability that she shoots a goal in a match is 0.7. The number of shots she needs to make to ensure the probability that she shoots more than 20 goals in a match is greater than 0.95 is

A. at most 35.
B. more than 35.
C. at least 34.
D. at least 35.
E. less than 36.

Question 19

The continuous random variable X has a normal distribution with mean 40 and variance 9. The continuous random variable Z has the standard normal distribution. The probability that Z lies between -3 and 1 is equal to

A. $\Pr(1<Z<3)$

B. $\Pr(37<X<49)$

C. $\Pr(13<X<49)$

D. $\Pr(X<37)+\Pr(X>49)$

E. $\Pr(31<X<49)$

Question 20

At a certain cafe 55% of customers buy flat whites, 35% buy cappuccinos and 10% buy short blacks. If they buy a flat white there is a 30% chance they will also buy a muffin. If they buy a cappuccino there is a 40% chance they will buy a muffin and if they buy a short black there is a 60% chance they will buy a muffin too. Given that a customer buys a muffin, what is the probability they also bought a short black ?

A. $\frac{12}{73}$

B. $\frac{1}{10}$

C. $\frac{1}{60}$

D. $\frac{1}{13}$

E. $\frac{73}{200}$

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (16 marks)

Consider the function $f(x) = a(x-b)^3(x-c)$ where a, b, c are real constants. The graph of *f* passes through the point $(-1,\ 8)$.

a. Write down the derivative of the function *f* in terms of a, b and c. Express your answer in fully factorised form. 1 mark

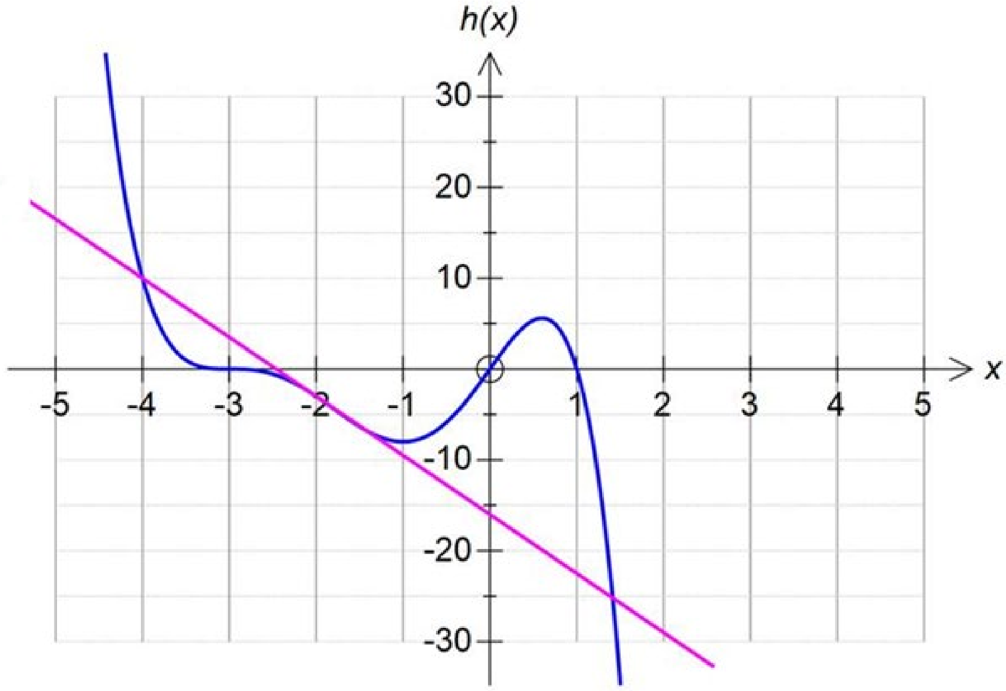
Let $a = -\frac{1}{2},\ b = -3,\ c = 1$.

b. Verify, by substituting the set of values $a = -\frac{1}{2},\ b = -3,\ c = 1$ into *f*, that the *x*-intercepts are at $x = -3$ and $x = 1$, and that $f'(0) = 0$. 1 mark

c. State the coordinates, and nature, of the stationary point(s) for the graph of $y = f(x)$. 2 marks

SECTION B - Question 1 - continued

The diagram below shows the graph of $h(x) = xf\left(x\right)$ and the tangent to the graph at $x = -2$.



The tangent to the graph at $x = -2$ also intersects the graph of *h* at another three points.

d. Label these three points on the graph with their coordinates. 2 marks

The equation of perpendicular line at $x = -2$ is $y = \frac{2}{13}x - \frac{35}{13}$.

e. Sketch this perpendicular line on the graph above. 1 mark

f. Find the coordinates of the points where the perpendicular line intersects the graph of the polynomial of degree 5, $y = h(x)$. Give any non-integer values correct to two decimal places. 2 marks

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g. **i.** Write down the sum of the definite integrals that equals the area enclosed by the perpendicular line to the curve at $x = -2$ and the graph of $y = h(x)$. Give any non-integer values for the terminals correct to two decimal places. 2 marks

ii. Find the area described in **part g.i.** Give your answer correct to one decimal place. 1 mark

The tangent to the graph of $y = h(x)$ at $x = -2$ runs parallel to another line which passes through three points on the graph of $y = h(x)$. One of the points is at $x = -\frac{1}{2}$.

h. **i.** Find the equation of this line. 2 marks

ii. Find the shortest distance between the lines. Give your answer correct to two decimal places. 2 marks

SECTION B - continued

Question 2 (12 marks)
A virus, called the C-virus, can be be modelled by a range of mathematical functions.
An appropriate function displays the fast increase of confirmed cases of the C-virus at the beginning of a certain period of time and then a slowing down of confirmed cases.

One suggested model for a certain town, MathsTown, is $C(t)=\dfrac{1000}{1+e^{-\frac{1}{5}(t-10)}}$ where $C(t)$ is the number of C-Virus confirmed cases after *t* days, where $t \geq 0$.

a. **Show** that $C(t)=\dfrac{1000}{1+e^{-\frac{1}{5}(t-10)}}$ can be expressed as $C(t)=\dfrac{1000e^{\frac{1}{5}(t-10)}}{1+e^{\frac{1}{5}(t-10)}}$. 1 mark

b. Using the quotient rule, show that $C'(t)=\dfrac{200e^{\frac{1}{5}(t-10)}}{\left(1+e^{\frac{1}{5}(t-10)}\right)^2}$. 1 mark

c. Hence show that the graph of $y=C(t)$ has no stationary points. 1 mark

SECTION B -- Question 2 - continued
TURN OVER

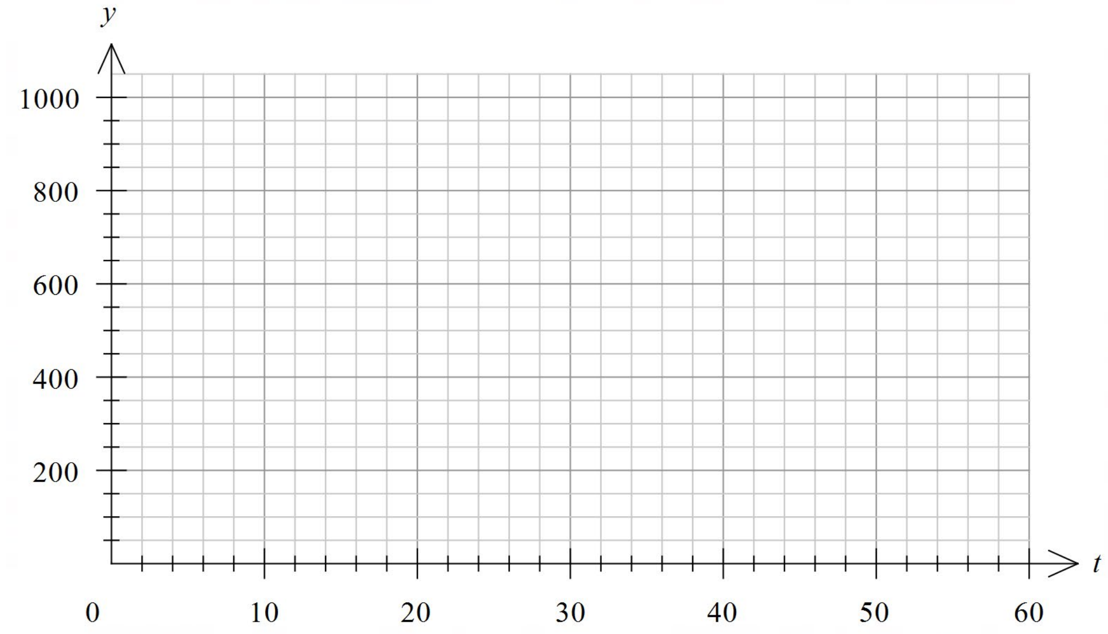
d. Find the value of *t* where the graph of $y = C(t)$ has its maximum gradient. 1 mark

e. What is this maximum gradient for the graph of $y = C(t)$? 1 mark

f. State the transformations, in an appropriate order, that will transform the graph of $y = \frac{e^t}{1+e^t}$

to its image $C(t) = \frac{1000e^{\frac{1}{5}(t-10)}}{1+e^{\frac{1}{5}(t-10)}}$. 2 marks

g. Sketch, on the grid below, the graph of $y = C(t) = \dfrac{1000e^{\frac{1}{5}(t-10)}}{1+e^{\frac{1}{5}(t-10)}}$. Label the point where the gradient is a maximum with its coordinates, and any asymptotes with their equations. 2 marks



h. Will the number of confirmed cases ever reach 1000 in Maths Town according to the equation that models the number of confirmed cases of C-Virus after *t* days, $C(t) = \dfrac{1000}{1+e^{-\frac{1}{5}(t-10)}}$ where $t \geq 0$? Give a mathematical reason for your answer. 1 mark

i. Social distancing restrictions will be lifted in Maths Town the day after the average rate of change per day of confirmed cases falls below 1 case per day, for 10 consecutive days. Use this model to determine the day restrictions will be lifted. 2 marks

SECTION B - continued
TURN OVER

Question 3 (11 marks)

The gestation period of a certain species of female dog is normally distributed with a mean of 62.9 days and a standard deviation of 1.6 days.

a. Find the probability that a randomly chosen female dog of this species will have a gestation period of less than 61 days. Give your answer correct to four decimal places. 1 mark

b. Given that the gestation period of a randomly chosen female dog of this species is less than 61 days, what is the probability that it is more than 58 days? Give your answer correct to three decimal places. 2 marks

c. If 30 of these female dogs are randomly selected, what is the probability that more than five of them will have a gestation period less than 61 days? Give your answer correct to four decimal places. 2 marks

SECTION B - Question 3 - continued

d. Write down the mean and standard deviation for the distribution in **part c.** Give your answers correct to three decimal places. 2 marks

Puppies love to play with toilet paper.

A dog owner, Nadia, shops at the supermarket every day during March. She only shops once a day. The probability she buys no toilet paper is 0.7. The probability she buys one packet is 0.2. The probability she buys two packets is 0.05 and the probability she buys more than two packets is 0.05.

e. What is the probability Nadia will only purchase two packets of toilet paper in the first week (7 days) of March? Give your answer correct to four decimal places. 2 marks

On a particular day, the probability that Nadia's puppy, Milo, will play with a roll of toilet paper and eat his dinner is p. The probability he will not eat his dinner but play with the toilet paper is $\frac{3p-2}{3}$. The probability he will eat his dinner but not play with the toilet paper is $\frac{7-8p}{8}$.

f. If the events of playing with a roll of toilet paper and eating the dinner are independent of each other, find the probability he will do neither. 2 marks

Question 4 (14 marks)

Two water waves meet in the ocean. One of the waves is modelled by the function $w_1 = 6\cos(t) + 8$ and the other $w_2 = 5\sin\left(2t + \frac{\pi}{2}\right) + 6$, where w is the height of the wave in metres from the bottom of the ocean and t the time in seconds. When the two waves meet, they create a resulting wave, w_r, where $w_r = w_1 + w_2$.

a. What is the period and amplitude of w_2? 1 mark

b. Write w_2 as a cosine function. 1 mark

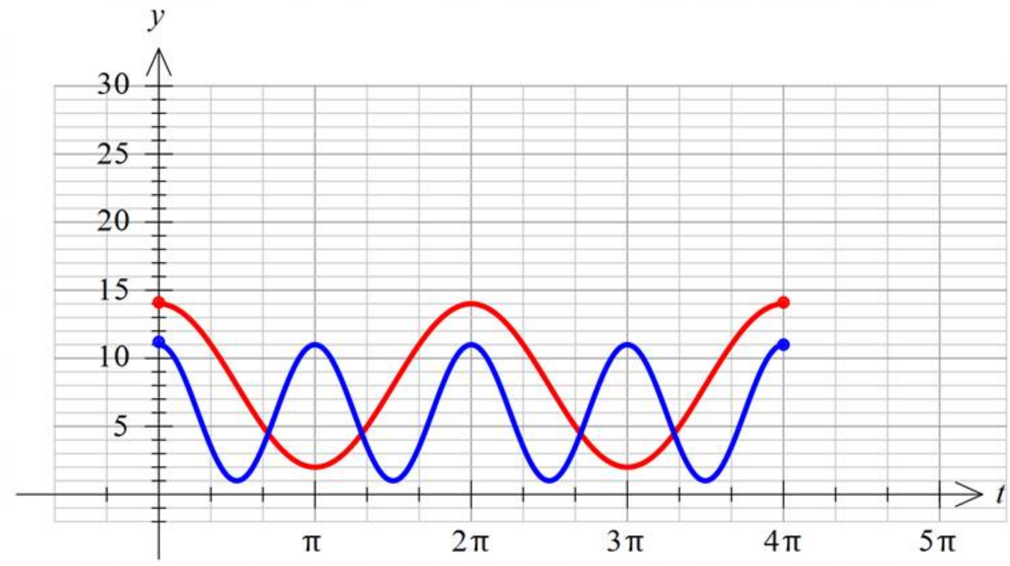
c. Write down the rule for the resultant wave w_r as a cosine function. 1 mark

d. What is the period and range of w_r? 2 marks

e. The graph of w_r has local maxima at $(a, 13)$. Write down the two smallest values of a. 1 mark

SECTION B - Question 4 - continued

The graphs of w_1 and w_2 are shown on the set of axes below for $0 \le t \le 4\pi$.



f. Label both graphs and sketch the graph of w_r on the same set of axes. Label the endpoints and local maxima of w_r with their coordinates. 3 marks

Answer the following questions for w_r for the time period $0 \le t \le 4\pi$.

g. Write down the coordinates of the minimum turning points. 1 mark

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h. When is the height of the wave changing the fastest? Give your answers in seconds correct to one decimal place. 2 marks

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i. For what percentage of the time is the height of the wave greater than 20 metres? Give your answer as a percentage correct to two decimal places. 2 marks

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SECTION B - continued
TURN OVER

Question 5 (7 marks)

Consider the functions *f* and *g* with rules $f(x)=a\sqrt{3-x}$ and $g(x)=-(x-2)^3+3$, where $a\in R\setminus\{0\}$ over their maximal domains.

a. For what value of *a* will there be one solution to $f(x)=g(x)$? Give your answer correct to three decimal places. 2 marks

b. For $a<0$, find the rule, in terms of *a*, and domain of the inverse of *f*. 1 mark

c. Find the coordinates of the point of intersection of *f* and f^{-1} in terms of *a*, for $a<0$. 1 mark

d. For $a>0$, find the smallest value of *a* for which there are three points of intersection for *f* and the inverse function. 1 mark

e. For $a>0$, find the values of *a* for which there are three points of intersection for *f* and the inverse function. 2 marks

END OF QUESTION AND ANSWER BOOK