The Mathematical Association of Victoria

Trial Exam 2020

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room : any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages,
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the Adjusted 2020 VCE Mathematics Study Design.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (1 mark)

Find the derivative of $h(x) = \dfrac{x^2}{\tan(2x)}$.

Question 2 (2 marks)

If $f(x) = \log_e\left(\sin(2x)\right)$, find $f'\left(\dfrac{\pi}{3}\right)$. Give your answer in the form $\dfrac{a}{\sqrt{b}}$, where $a, b \in Z$.

TURN OVER

Question 3 (3 marks)

Solve $\int_0^a \left(\frac{x}{x^2+4}\right) dx = 3$ for a, where a is a real constant.

Question 4 (4 marks)

Let $f(x) = \frac{1}{9}(3x-1)e^{3x}$.

a. Show that $f'(x) = xe^{3x}$. 1 mark

Another function is defined by $h:[0,\log_e(2)] \rightarrow R,\ h(x) = \frac{3}{7}e^{3x}$.

b. Hence find the area bounded by the curve with equation $y = xh(x)$, the x-axis and the line $x = \log_e(2)$. Express your answer in the form $a\log_e(2)+b$ where a and b are real constants.

3 marks

Question 5 (5 marks)

Consider the functions $f:[0,\infty) \rightarrow R, f(x) = e^{2x}$ and $g(x) = x^2 - 1$ over its maximal domain.

a. Determine, with appropriate mathematical reasoning, whether $g\left(f(x)\right)$ exists. 1 mark

Now consider the function $g_1 : D \rightarrow R, g_1(x) = x^2 - 1$, where D is the maximal domain of g_1 such that $h(x) = f\left(g_1(x)\right)$ exists.

b. Find D. 2 marks

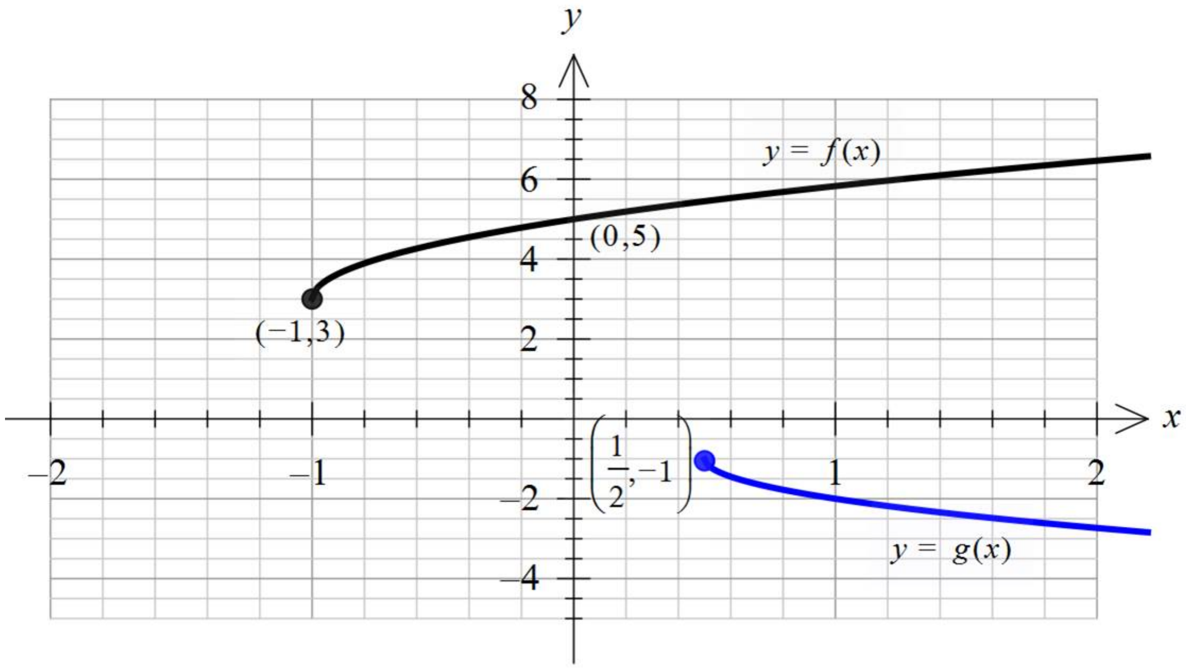
Question 5 - continued

TURN OVER

c. State the rule for h and hence evaluate $h'(2)$. 2 marks

Question 6 (5 marks)

Part of the graphs of $f:[-1,\infty)\to R,\ f(x)=a\sqrt{x+b}+c$, where a, b, and c are real constants, and $g:\left[\frac{1}{2},\infty\right)\to R,\ g(x)=-\sqrt{2x-1}-1$ are shown on the set of axes below.



a. Find a rule for f. 2 marks

A transformation $T : R^2 \to R^2$ that maps the graph of f to g has rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} p \\ q \end{bmatrix}$, where m, n, p and q are non-zero real numbers.

b. Find a set of values for m, n, p and q. 3 marks

TURN OVER

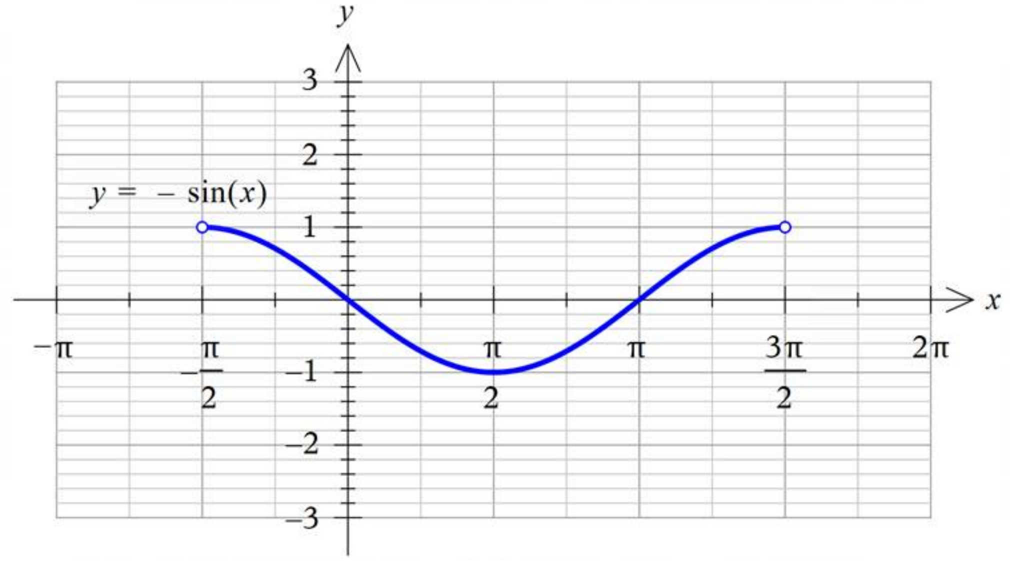
Question 7 (3 marks)
Sue sells a special type of sea shell in a tourist shop. She has 10 sea shells on each display tray.
Some of the sea shells are white and some are yellow. The rare sea shells are the white ones.
Let X be the random variable that represents the number of white sea shells on a display tray in Sue's tourist shop.

The proportion of white sea shells produced by the tourist company is $\frac{1}{10}$. Assume the proportion on display is the same as the proportion produced.

Find the probability that X is at most 2. Give your answer in the form $p\left(q\right)^n$ where p and q are rational numbers and n is a positive even integer.

Question 8 (5 marks)

The graph of $y=-\sin(x)$ for $x\in\left(-\frac{\pi}{2},\frac{3\pi}{2}\right)$ is shown on the set of axes below.



a. Sketch the graph of $y=\tan(x)$ for $x\in\left(-\frac{\pi}{2},\frac{3\pi}{2}\right)\backslash\left\{\frac{\pi}{2}\right\}$ on the set of axes above.

Label any asymptotes with their equations and axial intercepts with their coordinates. 2 marks

b. On the set of axes above, use the process of addition of ordinates to sketch the graph of $y=\tan(x)-\sin(x)$ for $x\in\left(-\frac{\pi}{2},\frac{3\pi}{2}\right)\backslash\left\{\frac{\pi}{2}\right\}$. 2 marks

c. Find the solutions to $\tan(x)=\sin(x)$ for $x\in\left(-\frac{\pi}{2},\frac{3\pi}{2}\right)\backslash\left\{\frac{\pi}{2}\right\}$. 1 mark

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TURN OVER

Question 9 (5 marks)

a. Given $x = -2$ and $x = \frac{1}{2}$ are solutions to the equation $x^2(2x-3) = 11x - 6$, find the third solution for $x \in R$. 2 marks

b. Find the area enclosed by the graphs of $y = 2x^3 - 3x^2$ and $y = 11x - 6$, given that the area of the two bounded regions is the same. 3 marks

Question 10 (7 marks)

Let $f:(-\infty,1)\to R, f(x)=\dfrac{2}{(x-1)^2}-\dfrac{20}{9}$.

a. Find the domain and rule for f^{-1}. 3 marks

b. Find the coordinates of the point(s) of intersection of the graphs of *f* and f^{-1}. 4 marks

END OF QUESTION AND ANSWER BOOK