YEAR 12 *Trial Exam Paper*

2020

MATHEMATICAL METHODS

Written examination 2

Worked solutions

This book presents:

- worked solutions
- mark allocations
- tips.

SECTION A – Multiple-choice questions

Question	Answer
1	*D*
2	*B*
3	*D*
4	*B*
5	*D*
6	*E*
7	*B*
8	*A*
9	*D*
10	*C*
11	*B*
12	*E*
13	*C*
14	*A*
15	*D*
16	*B*
17	*B*
18	*E*
19	*B*
20	*D*

Question 1

Answer: D

Explanatory notes

Period:

$$\frac{2\pi}{4\pi/5} = \frac{5}{2}$$

Range:

$$-1 \le \sin\left(\frac{4\pi}{5}x\right) \le 1$$

$$-2 \le \sin\left(\frac{4\pi}{5}x\right) \le 2$$

$$1 \le 3 - 2\sin\left(\frac{4\pi}{5}x\right) \le 5$$

So the period and range is $\frac{5}{2}$ and $[1, 5]$ respectively.

Tip

- *The period of both sin(nπ) and cos(nπ) is* $\frac{2\pi}{n}$.

Question 2

Answer: B

Explanatory notes

Multiplying the first equation by k and the second by 2 gives

$$2kx + k^2 y = 5k$$
$$2kx + 16y = -20$$

Subtracting the second equation from the first equation gives

$$\left(k^2 - 16\right) y = 5k + 20$$

If we let $k = 4$, then we have $0y = 40$, which is inconsistent. If we let $k = -4$, then we have $0y = 0$, which is consistent.

From this we conclude that no solution exists if $k = 4$.

Alternatively, we can use the determinant of the 2×2 coefficient matrix

$$\begin{vmatrix} 2 & k \\ k & 8 \end{vmatrix} = 16 - k^2 = 0 \Rightarrow k = \pm 4$$

If $k = 4$, then the equations become

$$2x + 4y = 5$$
$$4x + 8y = -10$$

These equations have no solution.

If $k = -4$, then the equations become

$$2x - 4y = 5$$
$$-4x + 8y = -10$$

These equations have an infinite number of solutions.

Tip

- *Using the determinant is an efficient way of solving these types of problem.*

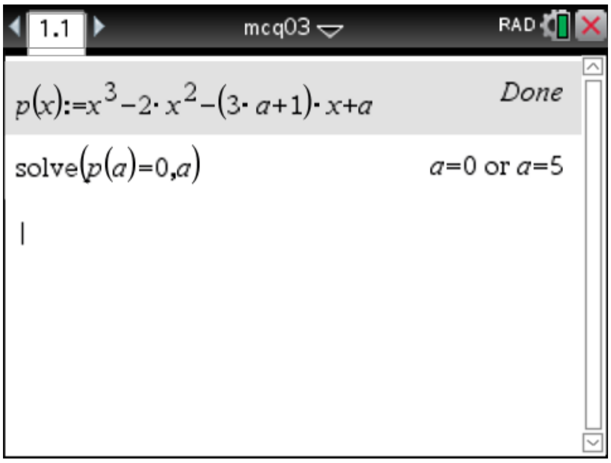
Question 3

Answer: D

Explanatory notes

If $x-a$ is a factor of $p(x)=x^3-2x^2-(3a+1)x+a$, then $p(a)=0$.

Use CAS to solve $p(a)=0$:



Since $a\neq 0$, $a=5$.

Question 4

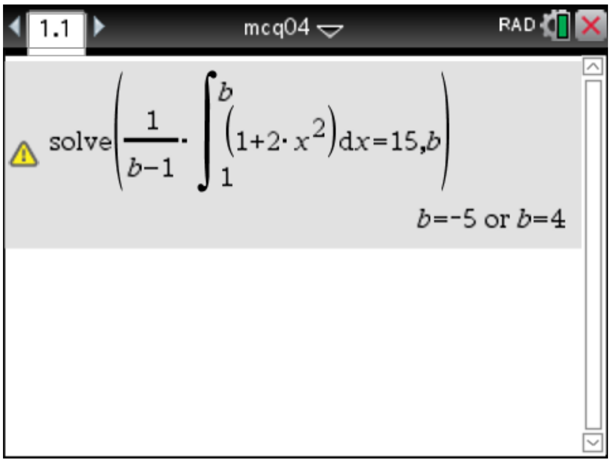
Answer: B

Explanatory notes

The average value of $1+2x^2$ for the interval $[1, b]$, $b>1$ is

$$\frac{1}{b-1}\int_1^b\left(1+2x^2\right)dx$$

Use CAS to determine the value of b:



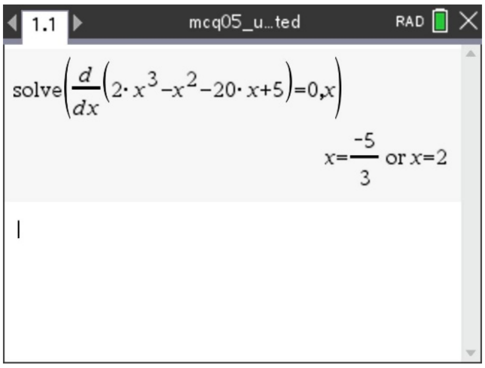
Since $b>1$, $b=4$.

Question 5

Answer: D

Explanatory notes

We require *f* to be 1 – 1 for the inverse to exist. Use CAS to find the turning points:



The turning points of the cubic occur when $x=-\frac{5}{3}$ and when $x=2$.

Therefore, *f* is 1 – 1 on the intervals $\left(-\infty, -\frac{5}{3}\right]$, $\left[-\frac{5}{3}, 2\right)$ and $[2, \infty)$. Only the first interval is available as an option.

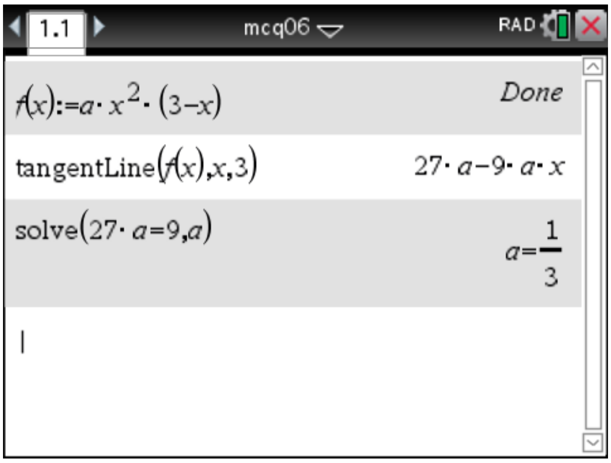
Question 6

Answer: E

Explanatory notes

Define $f(x)=ax^2(3-x)$ in CAS and use the **tangentLine** function to find that the equation of the tangent to *f* at $x=3$ is $y=27a-9ax$. Since this is in gradient–intercept form, the expression for the *y*-intercept can be seen.

If the *y*-intercept is 9, then $27a=9$ and $a=\frac{1}{3}$.



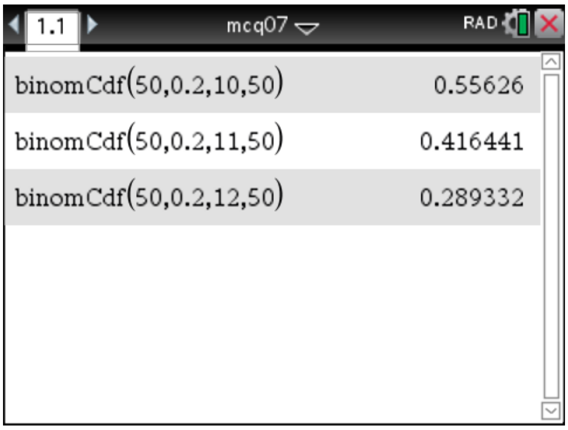
Question 7

Answer: B

Explanatory notes

Let $X \sim \text{Bi}(50, 0.20)$. We want to find the smallest value of n such that $\Pr(X \geq n) = 0.3$.

This can be done using trial and error.

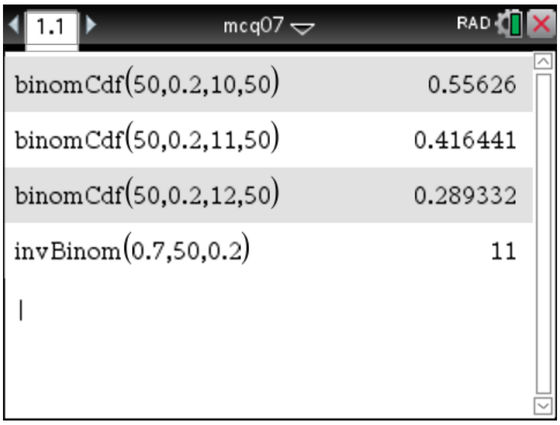


It can be found that $n = 12$.

Alternatively, the inverse binomial function can be used. In order to use this function, the equation must first be rearranged:

$\Pr(X \geq n) = 0.3 \Rightarrow \Pr(X \leq n-1) = 0.7.$

Then $n - 1 = 11 \Rightarrow n = 12.$



Question 8

Answer: A

Explanatory notes

Rearrange $y = \frac{-2}{\sqrt{3x-1}} + 1$ into the form $\frac{y-1}{-2} = \frac{1}{\sqrt{3x-1}}$.

The image is $y' = \frac{1}{\sqrt{x'}}$, so set

$x' = 3x - 1$

$y' = \frac{y-1}{-2} = -\frac{1}{2}y + \frac{1}{2}$

Therefore the transformation has the form

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$$

Question 9

Answer: D

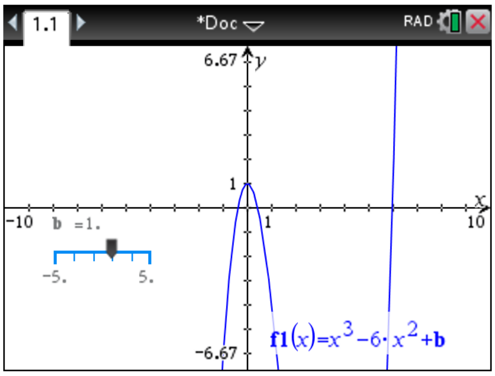
Explanatory notes

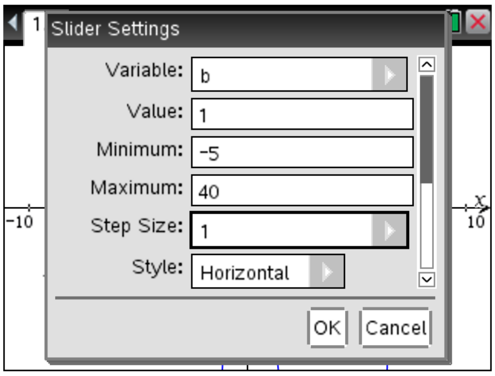
If $f(x) = x^3 - 6x^2 + b$, $f'(x) = 3x^2 - 12x = 3x(x-4)$. Thus, the turning points occur when $x = 0$ and $x = 4$.

Now $f(0) = b$ and $f(4) = b - 32$, from which it follows that $b > 0$ and $b - 32 < 0 \Rightarrow b < 32$.

Therefore there are exactly three x-intercepts if $b \in (0, 32)$.

Alternatively, sketch the function using sliders. Since the possible solutions include the numbers 0 and 32, set up your sliders to cover these values. The solution can then be found visually.





Tip

- *It can be helpful to draw a quick sketch of a typical cubic graph.*

Question 10

Answer: C

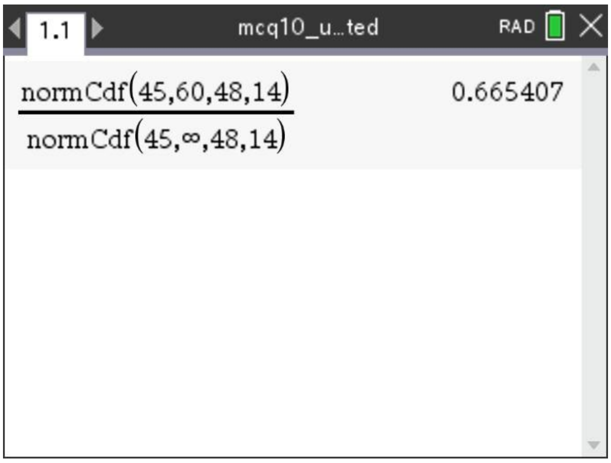
Explanatory notes

This is a conditional probability question.

Let $X \sim N\left(48, 14^2\right)$.

The probability that a trip takes less than 60 minutes (1 hour), given that it takes more than 45 minutes, is

$$\Pr(X < 60 \mid X > 45) = \frac{\Pr(45 < X < 60)}{\Pr(X > 45)} = 0.6654$$



Question 11

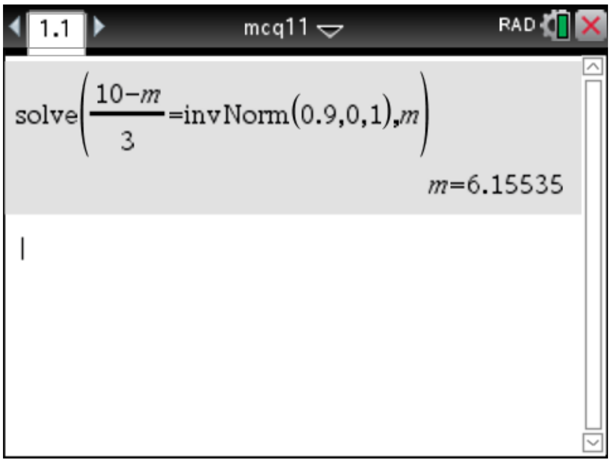
Answer: B

Explanatory notes

Let $X \sim \text{N}\left(\mu, 3^2\right)$ and $Z \sim \text{N}(0,1)$.

$$\Pr(X < 10) = 0.9 \text{ so } \Pr\left(Z < \frac{10-\mu}{3}\right) = 0.9.$$

Use CAS to find that $\mu = 6.2$ correct to one decimal place.



Question 12

Answer: E

Explanatory notes

We have that $g(2) = f(4) + 3 = -4$ and so $f(4) = -7$. Therefore, the graph of f passes through the point $(4, -7)$.

Question 13

Answer: C

Explanatory notes

Since $\int_2^6 f(x)\,dx = 5$, the average value of $f(x)$ on $[2, 6]$ is $\frac{5}{4}$ and the average value of $f(2x)$ on $[1, 3]$ is $\frac{5}{4}$. Therefore $\int_1^3 f(2x)\,dx = 2 \times \frac{5}{4} = \frac{5}{2}$ and

$$\begin{aligned}\int_1^3 \left(f(2x) + x\right) dx &= \frac{5}{2} + \int_1^3 x\,dx \\ &= \frac{5}{2} + \left[\frac{1}{2}x^2\right]_1^3 \\ &= \frac{5}{2} + \frac{9}{2} - \frac{1}{2} \\ &= \frac{13}{2}\end{aligned}$$

Question 14

Answer: A

Explanatory notes

The graph of $f(x)$ has stationary points when $x = -2$, $x = 0$ and $x = 2$. The stationary point at the origin is a stationary point of inflection.

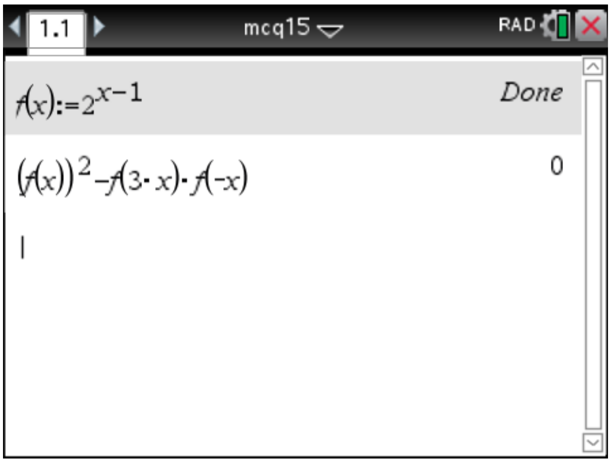
Note that the gradient of $f(x)$ is positive when $x < -2$ and when $x > 2$. Therefore, option A is correct.

Question 15

Answer: D

Explanatory notes

One way to determine which option is correct is to test each one. This can be done quickly in CAS.



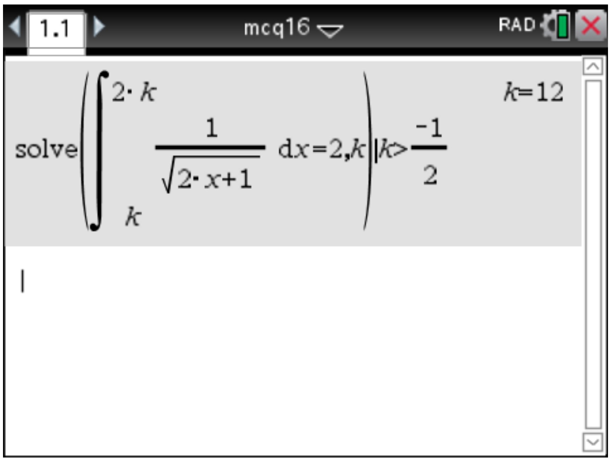
It can be found that $f(x)=2^{x-1}$ satisfies the relation.

Question 16

Answer: B

Explanatory notes

Use CAS to evaluate the integral and solve the equation.



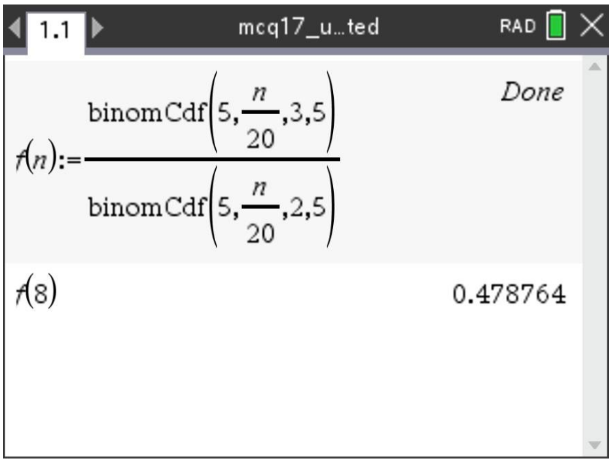
It can be found that $k = 12$.

Question 17

Answer: B

Explanatory notes

The number of blue marbles in the bag is 8.



Question 18

Answer: E

Explanatory notes

The gradient of the graph of the tangent to the function g at $x = 3$ is $g'(3) = \dfrac{1}{f'(5)} = -\dfrac{1}{2}$.

As the tangent passes through the point $(3, 5)$, the equation of the tangent is

$$y = -\frac{1}{2}(x-3) + 5 = -\frac{1}{2}x + \frac{13}{2}$$

Tip

- *Remember that the x values of the inverse function are the y values of the original function, and vice versa.*

Question 19

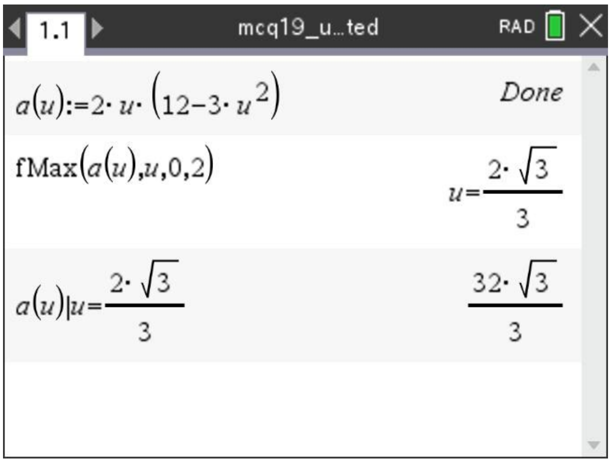
Answer: B

Explanatory notes

Let $a(u) = 2u\left(12 - 3u^2\right)$. Use the fMax function of the calculator to find the value of u for which the area is maximised.

$$u = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

The maximum area is $\frac{32\sqrt{3}}{3} = \frac{32}{\sqrt{3}}$.



Question 20

Answer: D

Explanatory notes

Let $f(x) = y = 2^{-x}$. The area of the bars is

$$0.5 \times \left(f(0.5) + f(1) + f(1.5) + f(2)\right) = 0.90533$$

The area between the x-axis and the graph of $y = 2^{-x}$ over the interval $[0, 2]$ is

$$\int_0^2 2^{-x}\,dx = \frac{3}{4\log_e 2}$$

The error is

$$\frac{3}{4\log_e 2} - 0.90533 = 0.176691.$$

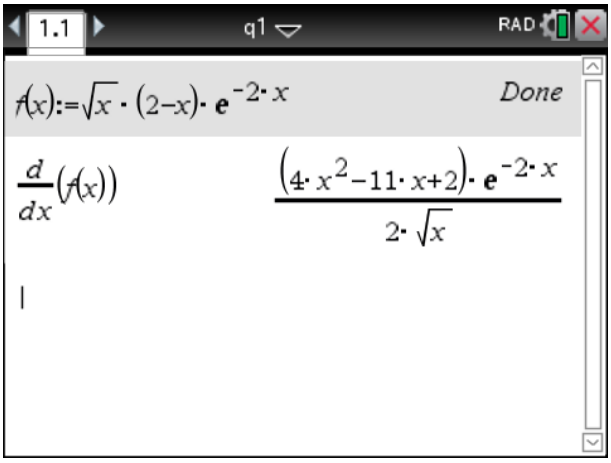
SECTION B

Question 1a.

Worked solution

If a function will be used several times, it is always advisable to define it in your calculator.

Having done this, we can use CAS to differentiate the function.



$$f'(x)=\frac{\left(4x^2-11x+2\right)e^{-2x}}{2\sqrt{x}}$$

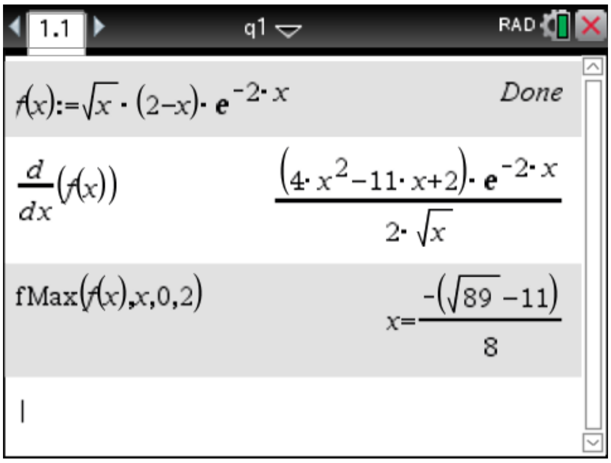
Mark allocation: 1 mark

- 1 mark for the correct answer in the required form

Question 1b.i.

Worked solution

CAS can be used to find the function maximum.



The function is maximum when $x=\frac{11-\sqrt{89}}{8}$.

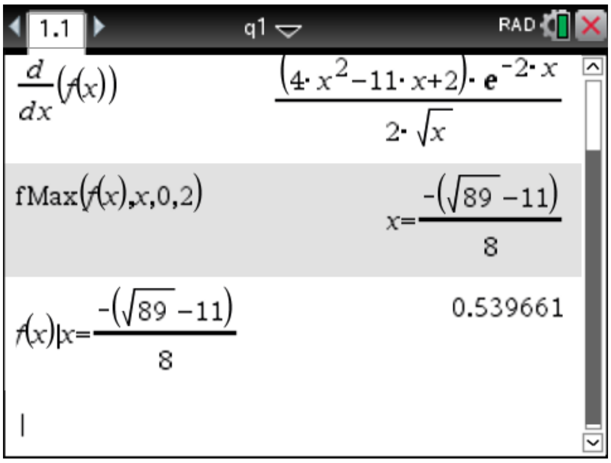
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 1b.ii.

Worked solution

Evaluate the function at the value of x found previously.



The function maximum is 0.5397 correct to four decimal places.

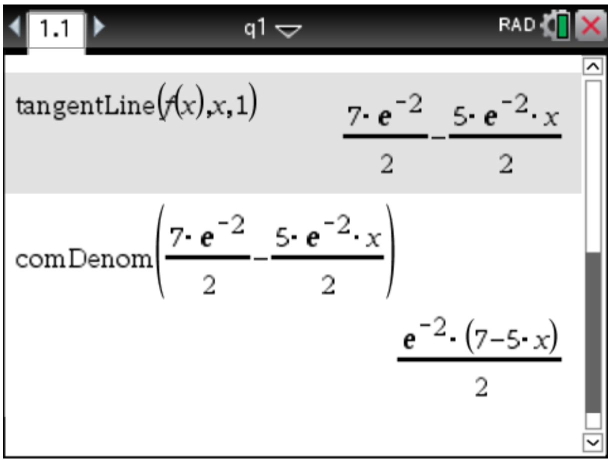
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 1c.i.

Worked solution

The **tangentLine** function of CAS can be used to find the tangent.



As the answer was required to be in a particular form, the common denominator command (**comDenom**) has been used.

The equation of the tangent is $y=\dfrac{7-5x}{2e^2}$.

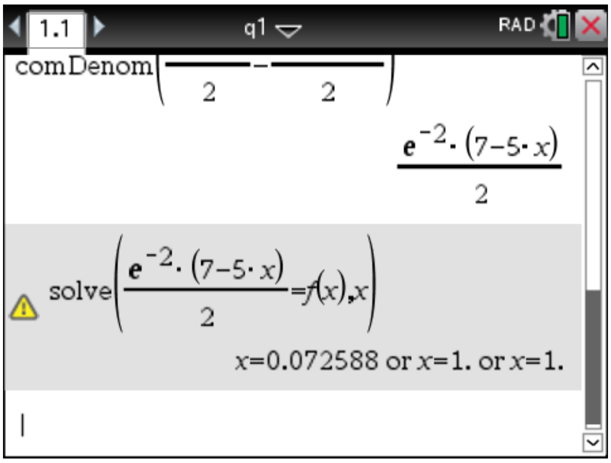
Mark allocation: 1 mark

- 1 mark for the correct answer in the required form

Question 1c.ii.

Worked solution

Use CAS to find the value of the x-coordinate of P.



The x-coordinate of P is 0.0726, correct to four decimal places.

Mark allocation: 1 mark

- 1 mark for the correct answer

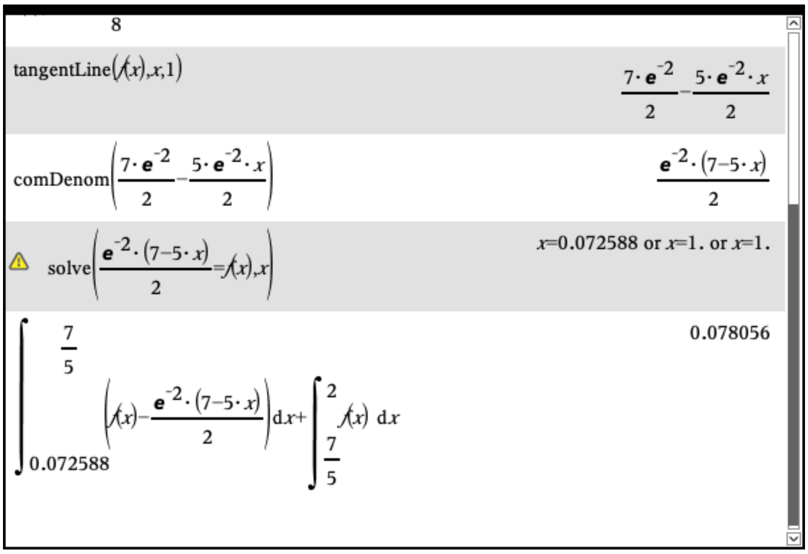
Question 1c.iii.

Worked solution

Note that the tangent passes through the x-axis (point Q) when $x = \dfrac{7}{5}$.

The area of the shaded region is therefore

$$\int_{0.0726}^{\frac{7}{5}} \left(f(x) - \frac{7-5x}{2e^2} \right) dx + \int_{\frac{7}{5}}^{2} f(x)\,dx = 0.0781$$



Mark allocation: 2 marks

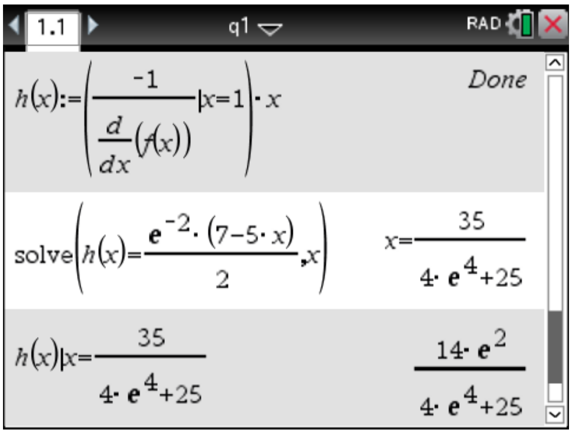
- 1 mark for setting up the two integrals
- 1 mark for the correct answer

Question 1d.i.

Worked solution

The shortest distance must be along a line perpendicular to the tangent passing through the origin. Since we know the gradient of the tangent, we can calculate the slope of a perpendicular line, and since it passes through the origin, its y-intercept must be 0. Therefore the equation of the line perpendicular to the tangent and that passes through the origin is $y = \frac{2e^2 x}{5}$. This can be found using CAS or from inspecting the tangent.

Use CAS to find the point of intersection of the two lines.



Alternatively, you can use the distance formula to calculate the distance from the origin to a point on the tangent line, then use **fMin** in the CAS to find the point which minimises this distance.

The coordinates of R are $\left(\frac{35}{4e^4+25}, \frac{14e^2}{4e^4+25}\right)$.

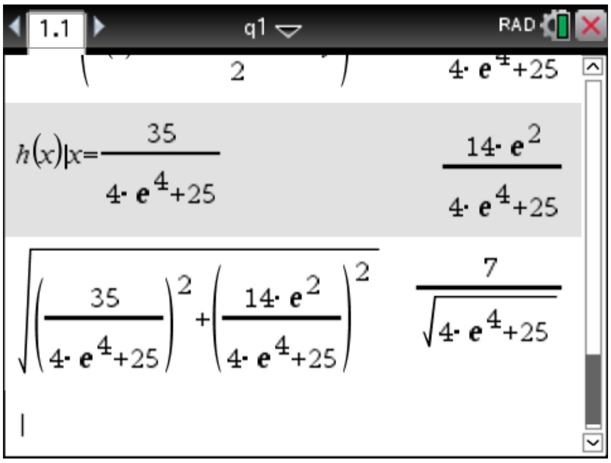
Mark allocation: 2 marks

- 0 marks for incorrect coordinates or no relevant working
- 1 mark for relevant working towards the solution with 0 or 1 correct coordinate given
- 2 marks for relevant working and correct coordinates given

Question 1d.ii.

Worked solution

Use the distance formula to find the required distance.



The distance from R to the origin is $\frac{7}{\sqrt{4e^4+25}}$.

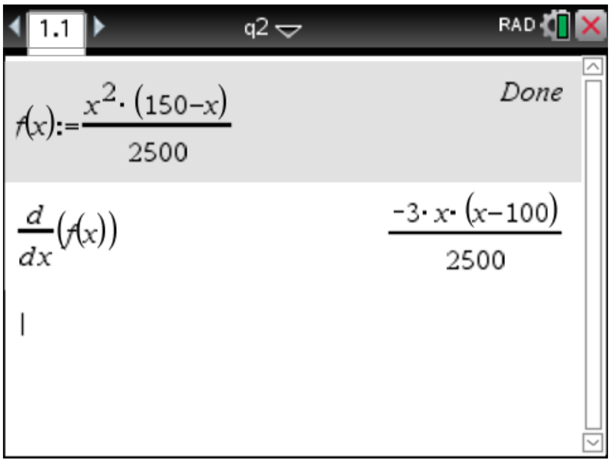
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2a.

Worked solution

$$\frac{dy}{dx} = -\frac{3x(x-100)}{2500}$$



Mark allocation: 1 mark

- 1 mark for the correct answer

Tip

- *Although this is easy to find by hand, it is worth using CAS to find your answer so that you have the function defined in your calculator. In this case we have let y = f(x).*

Question 2b.

Worked solution

Since $\frac{dy}{dx} = 0$ when $x = 100$, the function is strictly decreasing for $x \in [100, 150]$.

Mark allocation: 1 mark

- 1 mark for the correct answer

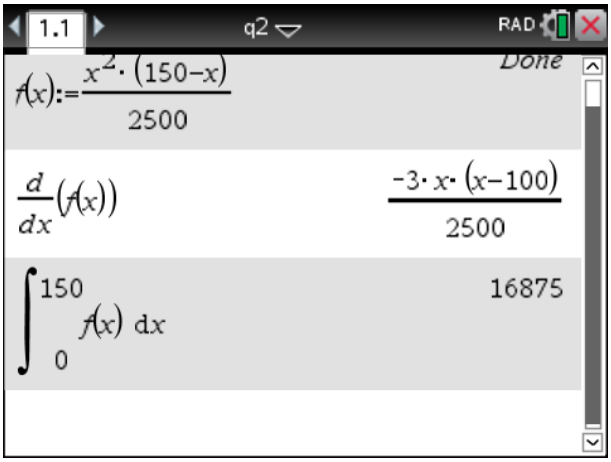
Tip

- *The set of values for which this function is strictly decreasing includes the end points and the turning point.*

Question 2c.

Worked solution

Integrate using CAS to find that the area of the vertical stabiliser is $16\,875\,\text{cm}^2$ (units are not required).



Mark allocation: 1 mark

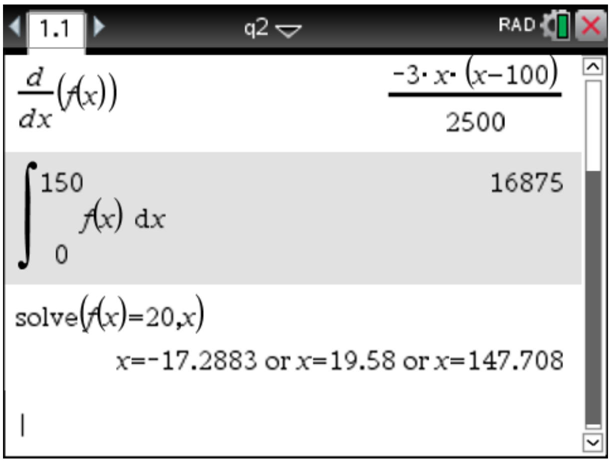
- 1 mark for the correct answer

Question 2d.i.

Worked solution

The value of c is found by solving $f(x) = 20$ for x using CAS.

The value of c is 147.708.



Mark allocation: 1 mark

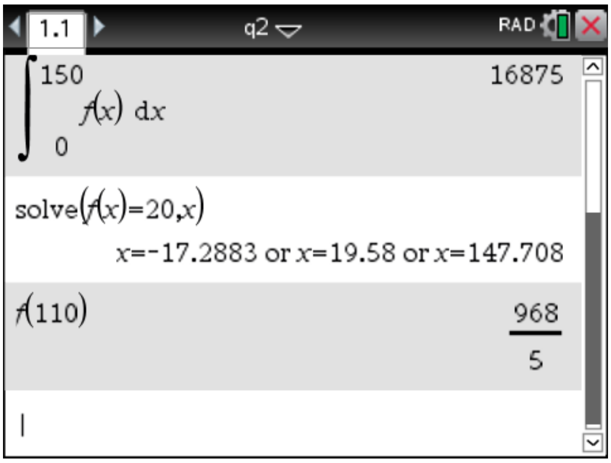
- 1 mark for the correct answer

Question 2d.ii.

Worked solution

The value of b is found by evaluating $f(110)$.

$$b = \frac{968}{5} = 193.6$$



Mark allocation: 1 mark

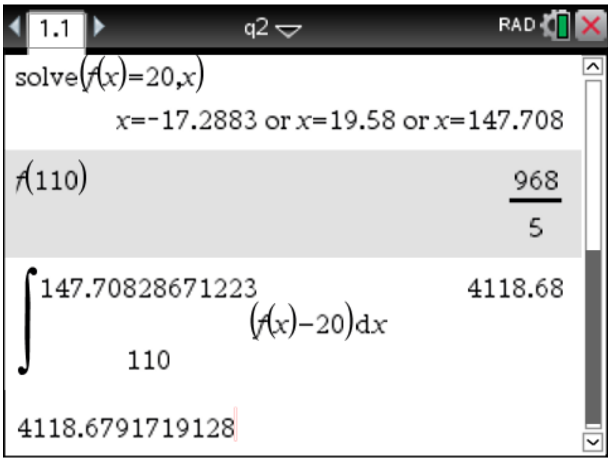
- 1 mark for the correct answer

Question 2e.

Worked solution

The area of the rudder is found by evaluating the integral

$$\int_{110}^{147.708} \left(f(x) - 20 \right) dx = 4118.679$$



Mark allocation: 2 marks

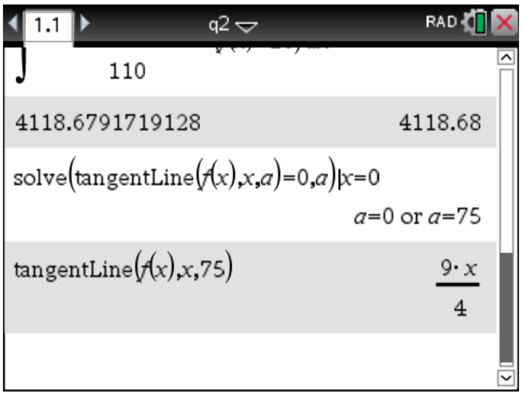
- 1 mark for formulating the integral
- 1 mark for the correct answer, to three decimal places

Question 2f.i.

Worked solution

The line segment *OP* lies on the tangent to the curve that passes through the origin. To find this tangent, first find the general equation for the tangent to the curve (at a point $x = a$, for example) and then determine the value of a when the tangent passes through the origin (that is, when $x = 0$).

This can be done quickly using CAS. We find that we require the tangent to the curve when $x = 75$.



The equation of the tangent is $y = \dfrac{9x}{4}$.

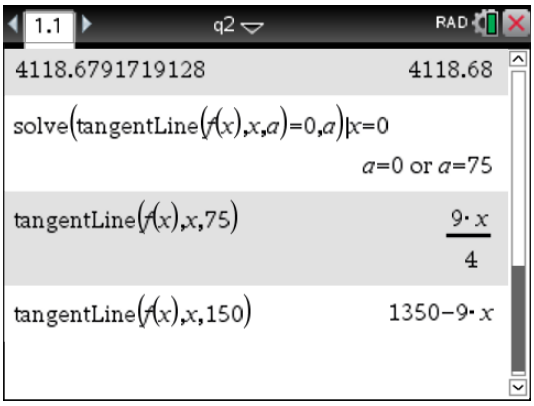
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2f.ii.

Worked solution

The tangent to the curve when $x = 150$ is $y = 1350 - 9x$.



Mark allocation: 1 mark

- 1 mark for the correct answer

Tip

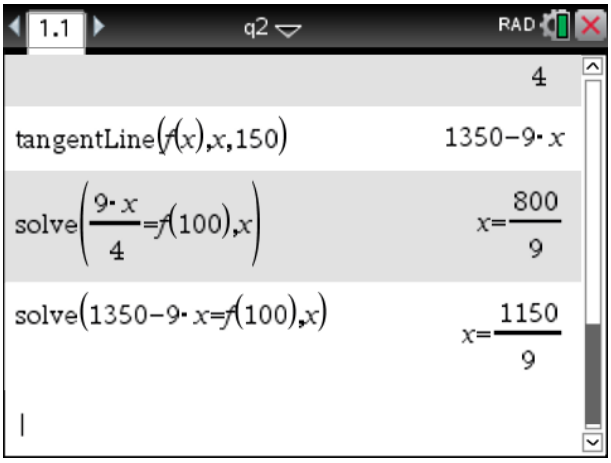
- *If the function is defined on a restricted domain, then the tangent line does not exist at the endpoints. To avoid this problem, do not define the function on a restricted domain.*

Question 2f.iii.

Worked solution

Note that $\frac{9x}{4} = f(100) = 200 \Rightarrow x = \frac{800}{9}$

$1350 - 9x = f(100) = 200 \Rightarrow x = \frac{1150}{9}$



Mark allocation: 2 marks

- 1 mark for each equation leading to the required solution

Tip

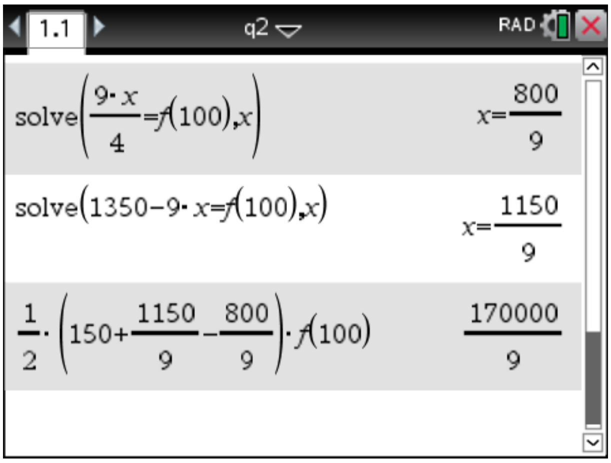
- *This is a 'show that' question, evidence needs to be given to justify each mark.*

Question 2f.iv.

Worked solution

The area of the trapezium *OPQR* is

$$\frac{1}{2}\left(150+\left(\frac{1150}{9}-\frac{800}{9}\right)\right)\times f(100)=\frac{170000}{9}$$



Mark allocation: 2 marks

- 1 mark for using the formula for the area of a trapezium
- 1 mark for the correct answer

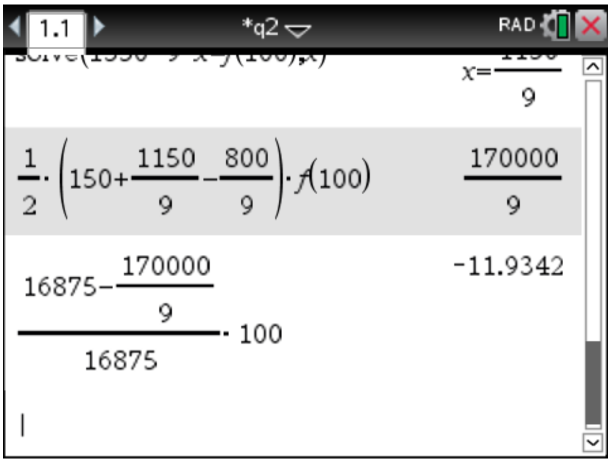
Question 2f.v.

Worked solution

The error is the true value minus the approximate value. To find this as a percentage of the actual area, we divide the error by the actual area and multiply by 100.

$$\frac{16875-\frac{170000}{9}}{16875}\times 100=-11.93$$

Therefore the error in the approximation is 11.93%.



Mark allocation: 2 marks

- 1 mark for calculating the error as the actual area minus the approximate area
- 1 mark for the correct answer (expressed as a positive value)

Question 3a.

Worked solution

The amplitude is 15 and the period is 1. Therefore $a = 15$.

$$\frac{2\pi}{n} = 1 \Rightarrow n = 2\pi$$

Mark allocation: 1 mark

- 1 mark for both values (providing that justifications are given)

Question 3b.

Worked solution

The centre of wheel B is 50 cm horizontally from the centre of wheel A.

The amplitude is 30 and the period is $\frac{2}{3}$.

Therefore,

$b = 50$

$c = 30$

$$\frac{2\pi}{m} = \frac{2}{3} \Rightarrow m = 3\pi$$

Mark allocation: 1 mark

- 1 mark for all values (providing that justifications are given)

Question 3c.

Worked solution

The period is 2.

Note: Wheel A makes 2 complete revolutions in 2 seconds and wheel B makes 3 complete revolutions in 2 seconds.

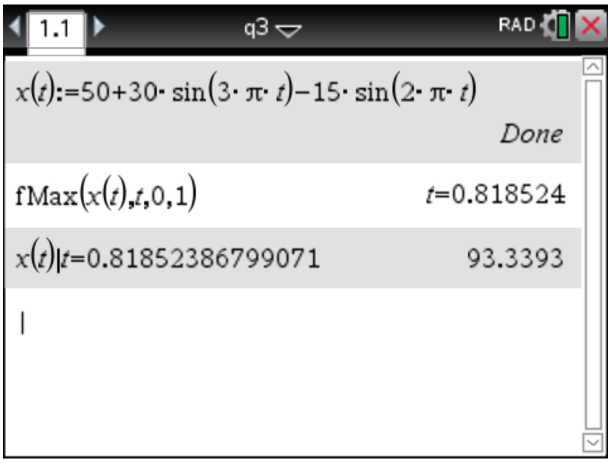
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3d.

Worked solution

Use the function maximum function (**fMax**) in CAS.



The maximum value of $x(t)$ is 93.34 cm and this first occurs when $t = 0.82$ seconds.

Mark allocation: 2 marks

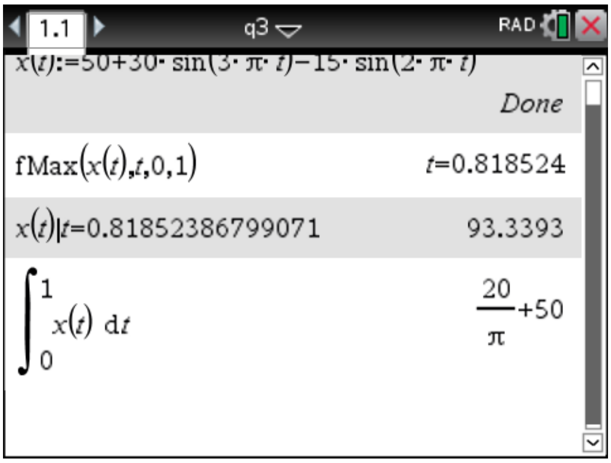
- 1 mark for correctly calculating the maximum value
- 1 mark for correctly calculating the time

Question 3e.

Worked solution

The area is found by evaluating the integral

$$\int_0^1 x(t)\,dt = \frac{20}{\pi} + 50$$



Mark allocation: 2 marks

- 1 mark for formulating the correct integral
- 1 mark for the correct answer

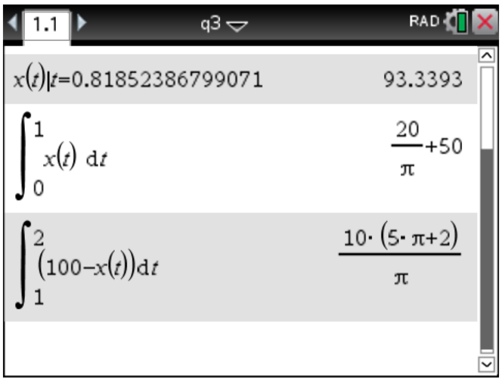
Question 3f.

Worked solution

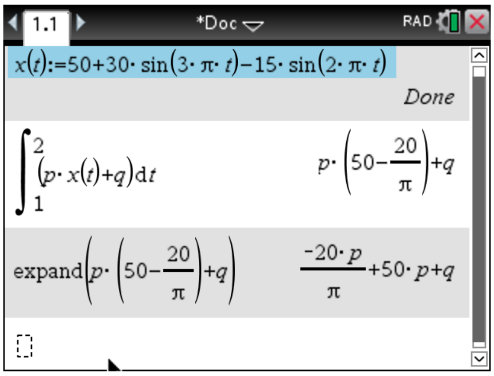
By inspecting the graph, it can be seen that a reflection of the function $x(t)$ in the t axis followed by a shift upwards of 100 units produces a function with the required property.

$$\int_1^2 (100 - x(t))\,dt = \frac{20}{\pi} + 50.$$

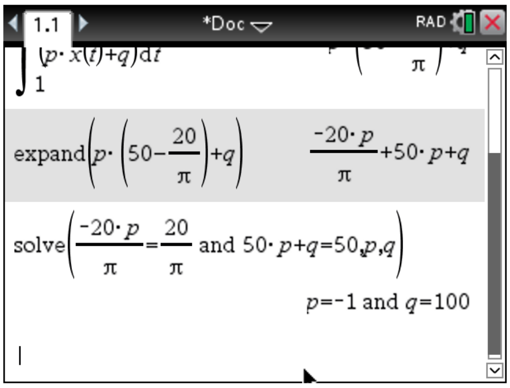
Therefore $p = -1$ and $q = 100$.



Alternatively, calculate the integral using CAS.



Then equate the terms containing π and separately equate the remaining terms (since the pronumerals cannot be irrational).



Mark allocation: 2 marks

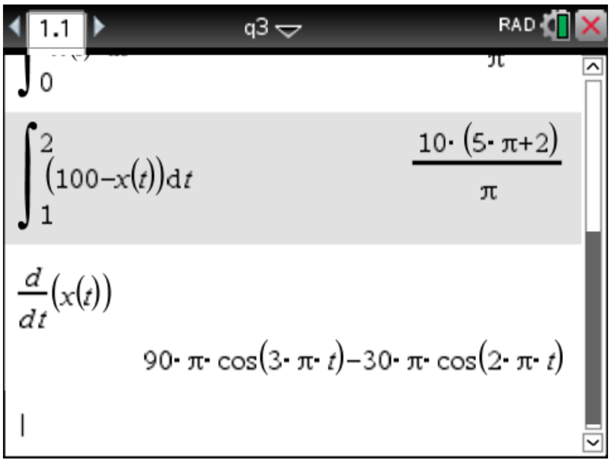
- 1 mark for correctly calculating p
- 1 mark for correctly calculating q

Question 3g.i.

Worked solution

$$\frac{dx}{dt} = 90\pi\cos(3\pi t) - 30\pi\cos(2\pi t)$$

CAS may be used to check that the differentiation has been performed correctly.



Mark allocation: 1 mark

- 1 mark for the correct derivative

Tip

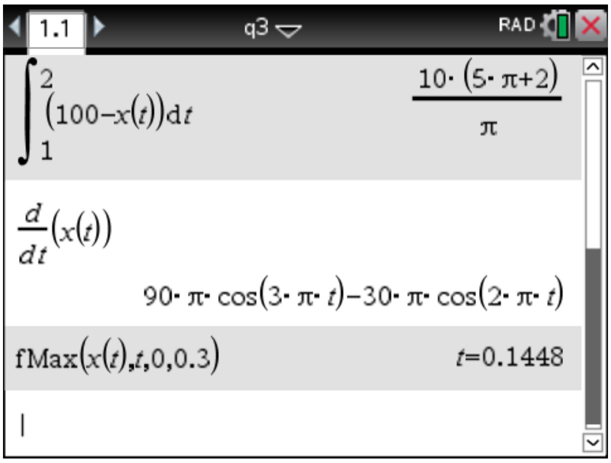
- *It is not necessary to factorise the result.*

Question 3g.ii.

Worked solution

The least value of t such that $t > 0$ and $\frac{dx}{dt} = 0$ is $t = 0.14$.

This may be found by inspecting the graph of the function and finding the first local maximum.



Mark allocation: 1 mark

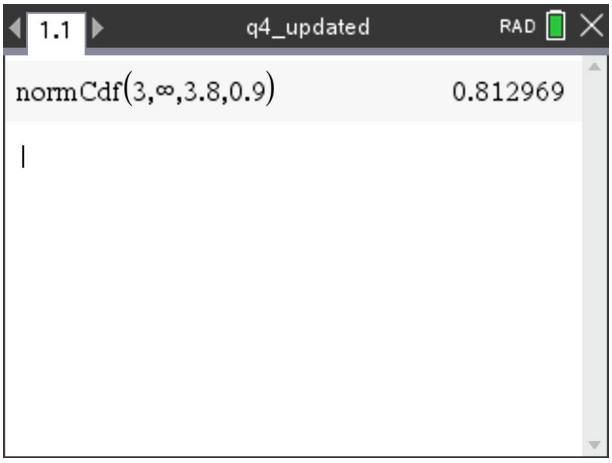
- 1 mark for the correct answer

Question 4a.

Worked solution

$X \sim (3.8,\ 0.9^2)$

Then $\Pr(X > 3) = 0.813$.



Mark allocation: 1 mark

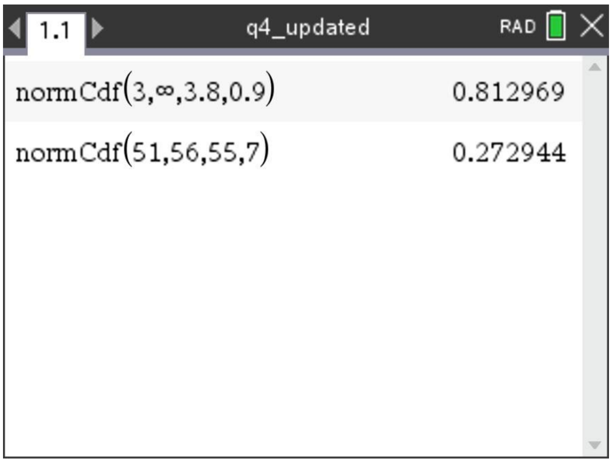
- 1 mark for the correct answer

Question 4b.

Worked solution

Let $A \sim N(55,\ 7^2)$.

Then $\Pr(51 < A < 56) = 0.273$.



Mark allocation: 2 marks

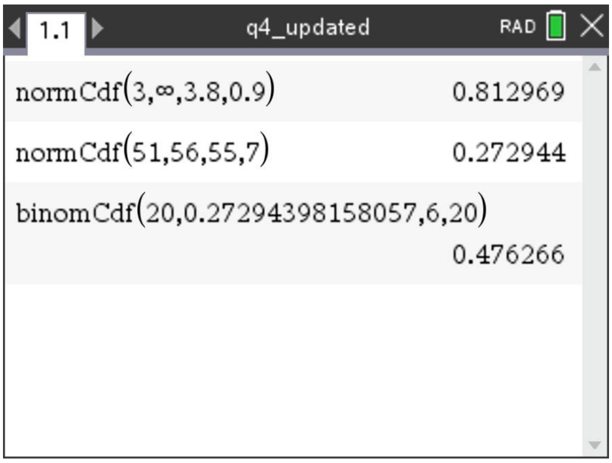
- 1 mark for two probability statements
- 1 mark for the correct answer

Question 4c.

Worked solution

Let $W \sim \text{Bi}(20,\ 0.273)$.

Then $\Pr(W > 5) = 0.476$.



Mark allocation: 2 marks

- 1 mark for binomial distribution, 20 trials, using the answer from **part b.**
- 1 mark for the correct answer

Question 4d.

Worked solution

$\Pr(B < 59) = 0.15$ and $\Pr(B > 68) = 0.2 \Rightarrow \Pr(B < 68) = 0.8$

Let $Z \sim N(0, 1)$ be the standard normal distribution. Then

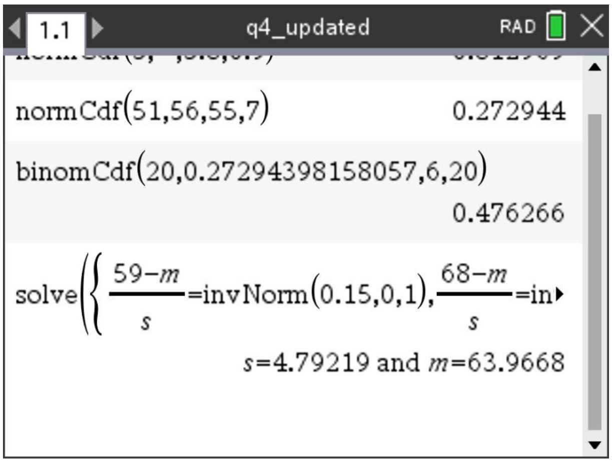
$$\Pr\left(Z < \frac{59-\mu}{\sigma}\right) = 0.15$$

$$\Pr\left(Z < \frac{68-\mu}{\sigma}\right) = 0.8$$

This gives two equations that can be solved to find μ and σ.

$\mu = 63.967$

$\sigma = 4.792$



Mark allocation: 4 marks

- 1 mark for writing probability statements
- 1 mark for converting to the standard normal
- 1 mark for solving to find μ and σ
- 1 mark for the correct answer

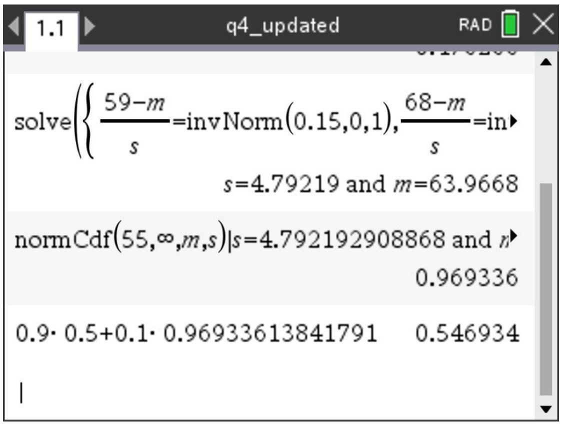
Question 4e.

Worked solution

We have $\Pr(A > 55) = 0.5$ and $\Pr(B > 55) = 0.969$.

Since 90% of the trees are type A trees and 10% of the trees are type B trees, the probability that a randomly selected tree has a diameter that is greater than 55 cm is

$0.9 \times 0.5 + 0.1 \times 0.969 = 0.547$



Mark allocation: 1 mark

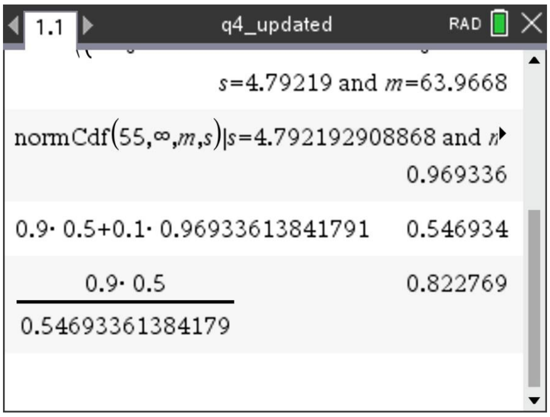
- 1 mark for finding $\Pr(B > 55) = 0.969$ and using this in the formula to find the given answer

Question 4f.

Worked solution

This is a conditional probability question:

$$\Pr(\text{type A tree} \mid \text{diameter} > 55)$$
$$= \frac{\Pr(\text{type A tree} \cap \text{diameter} > 55)}{\Pr(\text{diameter} > 55)}$$
$$= \frac{0.9 \times 0.5}{0.547}$$
$$= 0.823$$



Mark allocation: 2 marks

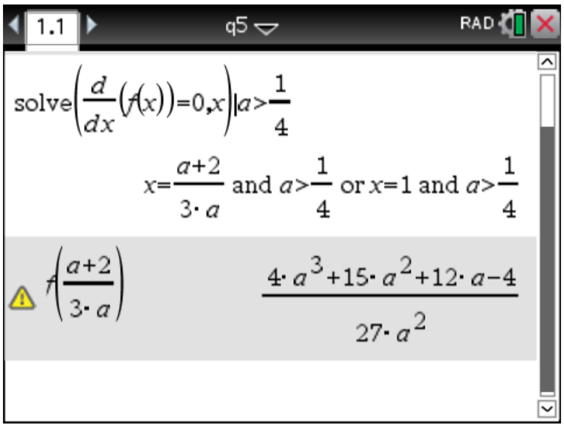
- 1 mark for correct conditional probability
- 1 mark for the correct answer

Question 5a.

Worked solution

The turning point at P occurs when $x = \dfrac{a+2}{3a}$. Therefore the coordinates of the point P are

$$\left(\frac{a+2}{3a}, \frac{4a^3+15a^2+12a-4}{27a^2}\right)$$



Mark allocation: 1 mark

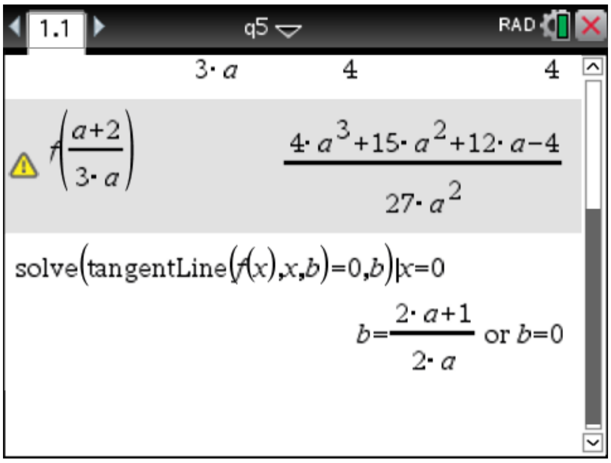
- 1 mark for the correct answer

Question 5b.

Worked solution

Find the tangent line to f at any point on the curve, say $x = b$. The tangent line passes through the origin for certain values of b.

This can be obtained quickly using CAS.



The x-coordinate of Q is $\dfrac{2a+1}{2a}$.

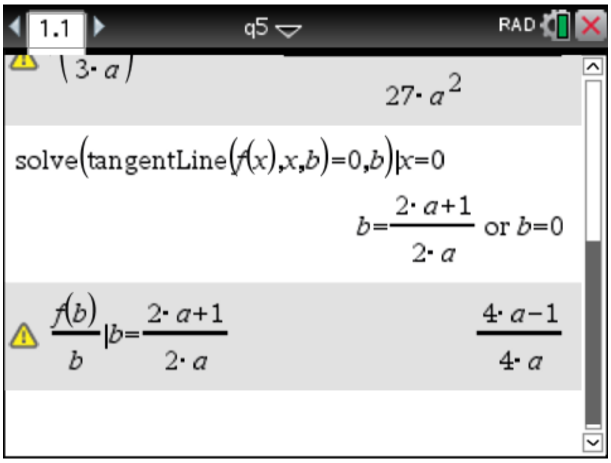
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 5c.

Worked solution

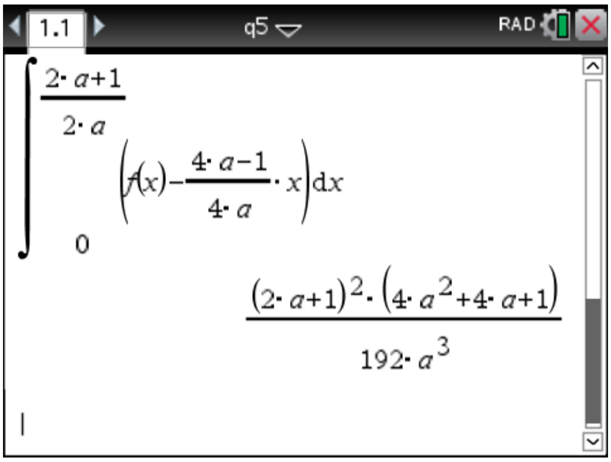
The gradient of the tangent is $\frac{f(b)}{b}$ where $b = \frac{2a+1}{2a}$.



Therefore the equation of the tangent to *f* that passes through the origin is $y = \left(\frac{4a-1}{4a}\right)x$.

The area of the shaded region is

$$\int_0^{\frac{2a+1}{2a}} \left(f(x) - \left(\frac{4a-1}{4a}\right)x \right) dx = \frac{(2a+1)^2\left(4a^2+4a+1\right)}{192a^3} = \frac{(1+2a)^4}{192a^3}$$



Mark allocation: 3 marks

- 1 mark for the equation of the tangent
- 1 mark for the integral with correct terminals
- 1 mark for the correct answer

Tip

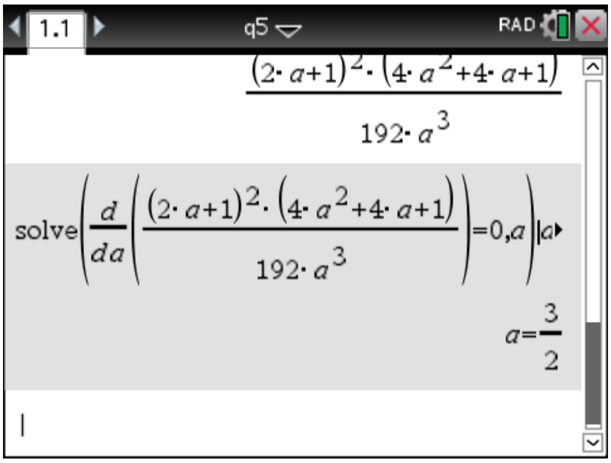
- *Note that CAS does not automatically simplify the result.*

Question 5d.

Worked solution

Solve $\frac{d}{da}\left(\frac{(1+2a)^4}{192a^3}\right)=0$.

If $a>\frac{1}{4}$, then $a=\frac{3}{2}$.



Mark allocation: 2 marks

- 1 mark for differentiating the result from **part c.** and setting it to zero
- 1 mark for the correct value of a

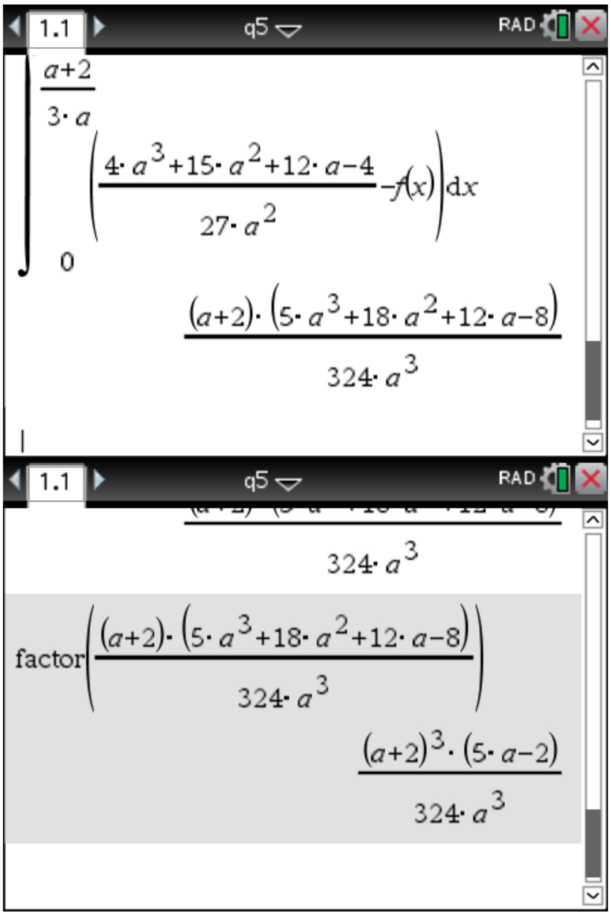
Question 5e.

Worked solution

The area of B is equal to

$$\int_0^{\frac{a+2}{3a}}\left(\frac{4a^3+15a^2+12a-4}{27a^2}-g(x)\right)dx=\frac{(a+2)\left(5a^3+18a^2+12a-8\right)}{324a^3}$$

$$=\frac{(a+2)^3(5a-2)}{324a^3}$$



Mark allocation: 3 marks

- 2 marks for the integral with the difference between the functions, and for the correct terminals
- 1 mark for the correct answer

Question 5f.

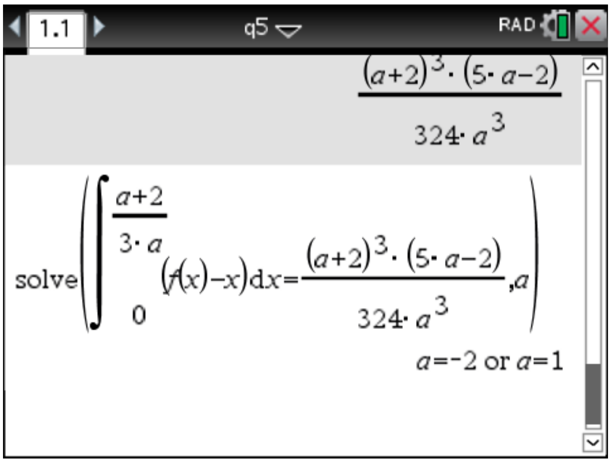
Worked solution

The area of A is

$$\int_0^{\frac{a+2}{3a}} \left(g(x)-x\right)dx$$

Equating this area to the area found in **part e.** gives

$$\int_0^{\frac{a+2}{3a}} \left(g(x)-x\right)dx = \frac{(a+2)^3(5a-2)}{324a^3} \Rightarrow a=1$$



Mark allocation: 2 marks

- 1 mark for equating the integral for the area A with the area found in **part e.**
- 1 mark for the correct answer

END OF WORKED SOLUTIONS

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