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YEAR 12 *Trial Exam Paper*

2020

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

Let $f:\left(-\infty,\frac{1}{3}\right)\rightarrow R,\ f(x)=\sqrt{1-3x}$.

a. **i.** Find $f'(x)$.

1 mark

ii. Find an antiderivative of $f(x)$.

1 mark

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b. Let $g:(-\pi,\pi) \to R,\ g(x) = \dfrac{\sin(x)}{\cos(x)+1}$.

Evaluate $g'\left(\dfrac{\pi}{2}\right)$.

2 marks

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Question 2 (3 marks)

Let $f:\left(-\infty,\frac{1}{3}\right)\to R,\ f(x)=\sqrt{1-3x}$.

a. Find the rule of f^{-1}.

2 marks

b. State the domain of f^{-1}.

1 mark

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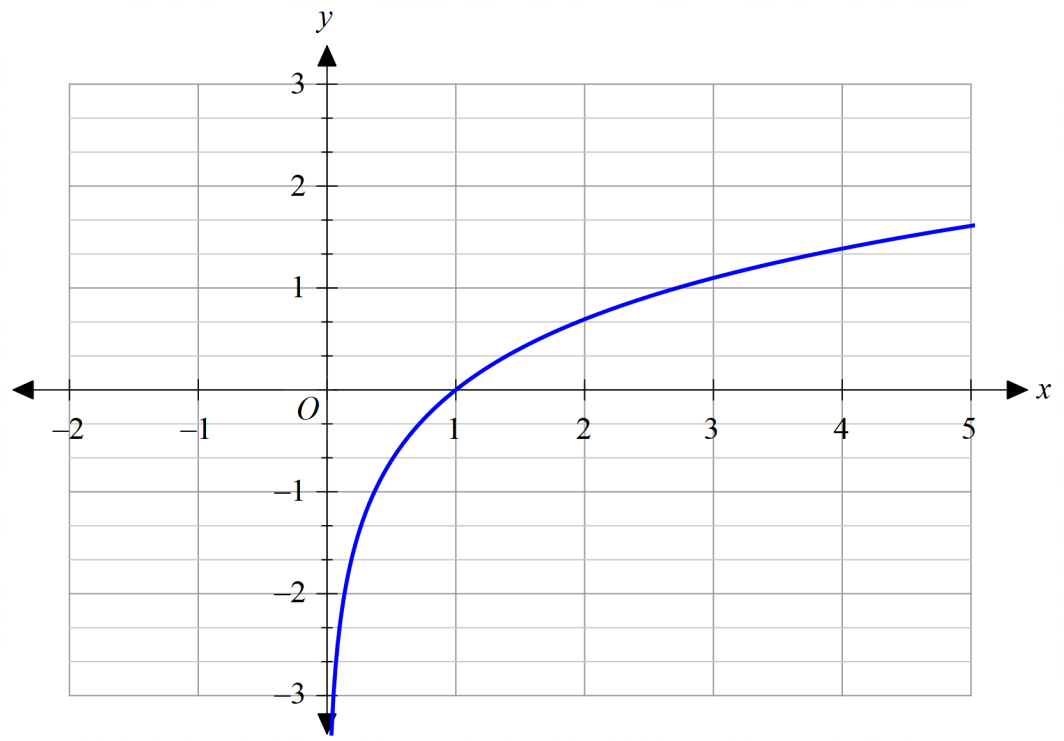
Question 3 (4 marks)

a. Solve $\log_e(x) = 1 - 2\log_e(x)$.

1 mark

b. The function $f : (0, \infty) \to R,\ f(x) = \log_e(x)$ is shown on the axes below.



Let $g : (0, \infty) \to R,\ g(x) = 1 - 2f(x)$.

Sketch the graph of g on the axes above. Label all points of intersection of the graphs of f and g. Label any asymptotes with the appropriate equation and label each of the axis intercepts, with their coordinates.

3 marks

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Question 4 (3 marks)

Susan models the time it takes her to get to school each day with the random variable T, normally distributed with a mean of 12 minutes. She estimates that $\Pr(T > 15) = 0.05$.

a. Find the probability that it takes between 9 and 15 minutes for Susan to get to school.

1 mark

b. Find the probability that it takes Susan more than 15 minutes to get to school, given that it has taken her at least 12 minutes.

2 marks

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Question 5 (6 marks)

Let $f:[0,\infty) \to R,\ f(x) = \dfrac{4}{2x-1} - 1$.

a. Evaluate $f\left(\dfrac{3}{2}\right)$.

1 mark

b. Find the equation of the tangent to the graph of f at $x = \dfrac{3}{2}$.

2 marks

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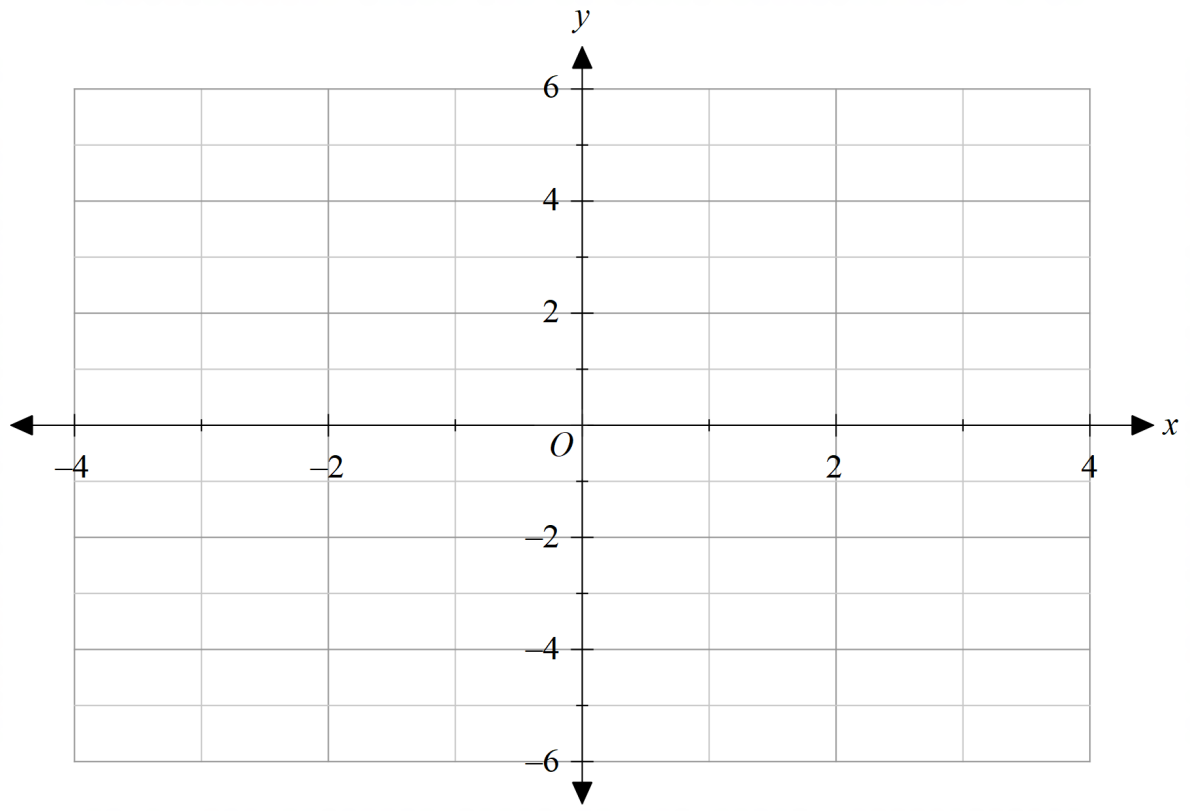
c. Sketch the graph of $y = f(x)$ on the axes below. Label each of the axis intercepts with their coordinates and label all asymptotes with their equations. Sketch the tangent to the graph of f at $x = \frac{3}{2}$, and label its point of intersection with $f(x)$.

3 marks



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Question 6 (2 marks)

A board game uses a customised eight-sided die to resolve conflict within the game. Three sides of the die have a victory symbol. Ten of these dice are rolled. Let N be the number of victory symbols rolled.

a. Find $\mathrm{E}(N)$.

1 mark

b. What is the variance of N?

1 mark

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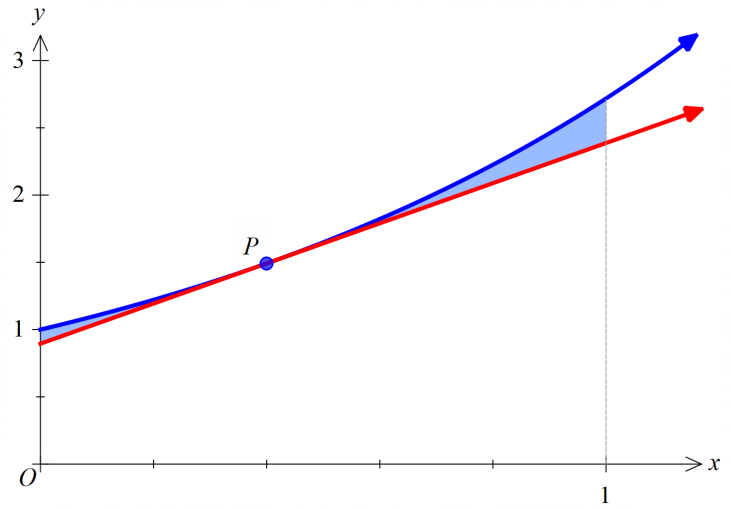
Question 7 (4 marks)

The graph of the relation $y = e^x$ is shown on the axes below. P is a point on the graph of this relation with coordinates $\left(p, e^p\right)$, and the tangent at P is shown.



a. Show that the equation of the tangent at P is $y = e^p x + e^p(1 - p)$.

1 mark

b. Find the value of p for which the area bounded by $y = e^x$, its tangent at $x = p$ and the lines $x = 0$ and $x = 1$ are minimised.

3 marks

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Question 8 (6 marks)

Consider the functions $f : R \to R,\ f(x) = \sin\left(\frac{x}{2}\right)$ and $g : R \to R,\ g(x) = a\sin(\pi x) + a$, where a is a real number.

a. **i.** For which values of a is $g\left(f(x)\right) \geq 0$ for all values of x?

1 mark

ii. For which value of x is $g\left(f(x)\right)$ at its maximum on the interval $[0, \pi]$, when $g\left(f(x)\right) \geq 0$?

3 marks

b. For which values of a is $f\left(g(x)\right) \geq 0$ for all values of x?

2 marks

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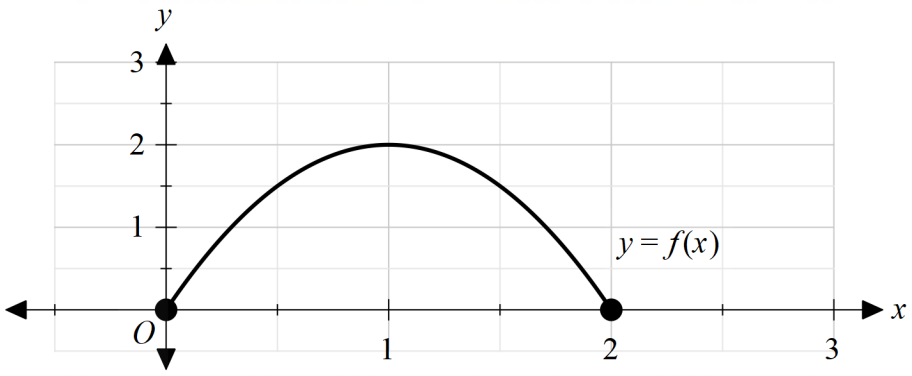
Question 9 (8 marks)

The graph of $f:[0, 2] \to R, f(x) = -2x^2 + 4x$ is shown below.



a. Show that the area enclosed by f and the x-axis is equal to $\frac{8}{3}$ square units.

1 mark

Let g be the function obtained by applying the transformation T to the function f, where

$$T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } a > 0 \text{ is a real constant.}$$

b. State the domain of g in terms of a.

1 mark

c. For what value of a is the area enclosed by the graph of $y = g(x)$ and the x-axis, over the domain of g, equal to the area enclosed by f and the x-axis?

2 marks

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d. **i.** Solve $f(x) = 1$.

1 mark

ii. Show that when $a = 3$, then $f(x) = g(x)$ has a unique solution.

1 mark

iii. Let D be the intersection of the domains of f and g.
For what values of a is $g(x) > f(x)$ for $x \in D$?

2 marks

END OF QUESTION AND ANSWER BOOK

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