EXPERIENCED TEACHER RESOURCES & ASSESSMENTS

Student Name: ___________________________________

MATHEMATICAL METHODS
UNITS 3&4 Exam 2
2020 Written Trial Examination

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator
  or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 19 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Multiple Choice Questions

Instructions for Section A
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions (last page).
Choose the response that is **correct** for the questions.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The set values of k for which $2x^2+(k+1)x+k=0$ has two real solutions is:

A. $k\in\left(-\infty,3+2\sqrt{2}\right)$

B. $k\in R$

C. $k\in\left(3-2\sqrt{2},\infty\right)$

D. $k\in\left(-\infty,3-2\sqrt{2}\right)\cup\left(3+2\sqrt{2},\infty\right)$

E. $k\in\left\{3-2\sqrt{2},3+2\sqrt{2}\right\}$

Question 2

Let $g(x)=\dfrac{\log_e(ax)}{x^2}$, what a is non-zero real constant.

The derivative, $g'(x)$, is given by:

A. $\dfrac{1}{2x^2}$

B. $\dfrac{2x\log_e(ax)-x}{x^4}$

C. $\dfrac{1-2\log_e(ax)}{x^3}$

D. $2x^2$

E. $x+2x\log_e(ax)$

Question 3

A box contains five white tiles and six black tiles. Two tiles are drawn at random from the box without replacement. The probability that the tiles are the **same** colour is:

A. $\dfrac{2}{11}$

B. $\dfrac{5}{22}$

C. $\dfrac{3}{11}$

D. $\dfrac{5}{11}$

E. $\dfrac{6}{13}$

Question 4

If $x-b$ is a factor of $3x^4-2x^3-x^2$, then the value of b could be:

A. $\frac{-1}{3},0,1$

B. $0,1$

C. $\frac{1}{3},0$

D. $0,\frac{1}{3},1$

E. $0,1,3$

Question 5

The simultaneous linear equations $y-(m+2)x=4$ and $my-3x=k+1$ have no solutions when:

A. $m=3$ and $k=-13$ or $m=1$ and $k\neq 3$

B. $m=3$ and $k\neq -13$ or $m\neq 1$ and $k\neq 3$

C. $m=1$ and $k=3$ or $m=1$ and $k=3$

D. $m=1$ and $k=-13$ or $m\neq 1$ and $k\neq 3$

E. $m=-3$ and $k\neq -13$ or $m=1$ and $k\neq 3$

Question 6

Let $f(x)=\sqrt{4-x}$ and $g(x)=x^2$.

For $f(g(x))$ to exist, the domain and range of $g(x)$ respectively, must be changed to:

A. $[-\infty,0]$ and $[4,\infty]$

B. R and $[0,\infty]$

C. $[0,4]$ and $[-2,2]$

D. $[0,2]$ and $[-4,0]$

E. $[-2,2]$ and $[0,4]$

Question 7

The point $A(4,-1)$ lies on the graph of the functions $f(x)$. A transformation maps the graph of $f(x)$ to the graph of $g(x)$, where $g(x)=-3f(x+2)+1$.

If that same transformation maps the point A to the point P, the coordinates of the point P are:

A. $(4,2)$

B. $(2,4)$

C. $(-2,4)$

D. $(-4,2)$

E. $(-1,4)$

Question 8

A discrete random variable has a binomial distribution with a mean of 3.15 and a variance of 1.7325.
The values of n (the number of independent trials) and p (the probability of success in each trial) are:

A. $n = 9$ and $p = 0.45$

B. $n = 7$ and $p = 0.55$

C. $n = 7$ and $p = 0.45$

D. $n = 5$ and $p = 0.5$

E. $n = 7$ and $p = 0.65$

Question 9

A six-sided die is loaded such that the chance of throwing a one is $\frac{x}{5}$, the chance of a two is $\frac{1}{5}$, and the chance of a three is $\frac{1}{5}(1+x)$. The chance of a four, five or six is $\frac{1}{6}$, and the die is thrown twice. The probability of getting a sum of six on the dice is:

A. $\frac{1}{75}\left(3x^2+11x+8\right)$

B. $\frac{1}{30}\left(3x^2+11x+8\right)$

C. $3x^2+11x+8$

D. $\frac{1}{75}\left(6x^2+17x+11\right)$

E. $\frac{1}{30}\left(6x^2+17x+11\right)$

Question 10

The average value of the function with the rule $f(x) = 2x^3 + 3x$ over the interval $[0, m]$, where $m > 0$ is:

A. $2m^2+3$

B. $\frac{2m^3+3}{m}$

C. $\frac{2}{m}(m+3)$

D. $\frac{m}{2}\left(m^2+3\right)$

E. $6m^2+3$

Question 11

The tangent line to the graph of $y = xe^{2x}$ can be drawn at $x = 1$.
This tangent will cross the y-axis at:

A. $-2e^2$

B. e^2

C. $\frac{-2}{e^2}$

D. $3e^2$

E. $\frac{e^2}{2}$

Question 12

The domain and range of $f(x)=3\log_e(x+1)-x$, respectively, is given by:

A. $(-1,\infty)$ *and* $\left(\log_e\left(\frac{27}{e^2}\right),\infty\right)$

B. $(-1,\infty)$ *and* $\left(-\infty,\log_e\left(\frac{27}{e^2}\right)\right)$

C. $(-1,\infty)$ *and* $\left(-\infty,\log_e\left(27e^2\right)\right)$

D. $(-1,\infty)$ *and* $\left(\log_e\left(27e^2\right),\infty\right)$

E. $\left(-\infty,\log_e\left(\frac{27}{e^2}\right)\right)$ *and* $(-1,\infty)$

Question 13

If the point $(3a,-b)$ is transformed by:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Then the image coordinate is:

A. $(4+9a,2b-1)$

B. $(9a-4,2b+1)$

C. $(4-9a,-2b-1)$

D. $(3a,-b)$

E. $(-9a,-2b)$

Question 14

Which one of the following is the inverse function of $f:(-\infty,-4]\to R, f(x)=x^2+8x-1$?

A. $f^{-1}:[-17,\infty)\to R, f^{-1}(x)=-4+\sqrt{x+17}$

B. $f^{-1}:[17,\infty)\to R, f^{-1}(x)=-4-\sqrt{x-17}$

C. $f^{-1}:[-17,17)\to R, f^{-1}(x)=4-\sqrt{x+17}$

D. $f^{-1}:[-17,\infty)\to R, f^{-1}(x)=-4-\sqrt{x+17}$

E. $f^{-1}:[0,\infty)\to R, f^{-1}(x)=-4-\sqrt{x+17}$

Question 15

Given that $\frac{d(2x\cos(3x))}{dx} = 2\cos(3x) - 6x\sin(3x)$ then $\int x\sin(3x)\,dx$ is equal to:

A. $\frac{1}{9}\left(\int 2\cos(3x)\ dx - 2x\cos(3x)\right)$

B. $\frac{1}{3}\sin(3x) - \frac{1}{9}\cos(3x) + c$

C. $\frac{1}{3}\left(\int 2\cos(3x)\ dx - 2x\cos(3x)\right)$

D. $\frac{1}{3}\cos(3x) - \frac{1}{9}\sin(3x) + c$

E. $\frac{1}{6}\left(\int 2\cos(3x)\ dx - 2x\cos(3x)\right)$

Question 16

The waiting time at a kiosk, in minutes, is normally distributed with a mean of 8 and a standard deviation of 1.2.

When a customer arrives at the kiosk, the probability that they wait longer than 10 minutes is closest to:

A. 0.7408

B. 0.4780

C. 0.8748

D. 0.0478

E. 0.7804

Question 17

Let $h:[0,4] \rightarrow R, h(x) = \frac{-2}{1-x} + 3$

Which one of the following statements about $h(x)$ is true?

A. An endpoint is $\left(4, \frac{5}{3}\right)$

B. The y-intercept is $(0,1)$

C. The asymptotes are $x = -1$ and $y = 3$

D. The range of $h(x)$ is R^+

E. $h(x)$ has a stationary point at $x = 2$

Question 18

The average rate change of $g(x)=x^2\sin(2x)$ over the interval $\left[\frac{-\pi}{2},\frac{\pi}{4}\right]$ is:

A. π

B. $\frac{\pi}{12}$

C. $\frac{\pi}{2}$

D. $\frac{12}{\pi}$

E. $\frac{3\pi}{4}$

Question 19

The area bounded by the graph of $f(x)$, the line x = 0, the line x = 1 and the x-axis is $\frac{2}{e}+1$.

A possible equation for $f(x)$ is:

A. $f(x)=-2e^{x-1}+3$

B. $f(x)=2e^{x+1}+3$

C. $f(x)=-2e^{x-1}-3$

D. $f(x)=-e^{x-1}+3$

E. $f(x)=-2e^{x}+3$

Question 20

Which one of the following statements is false for $f:(0,10]\rightarrow R, f(x)=\log_e(x)-\cos(x)$?

A. It has no y-intercept

B. It has an endpoint at $\frac{1}{\log_{10}(e)}-\cos(10)$

C. It has 3 turning points

D. $f'(x)=\frac{1+x\sin(x)}{x}$

E. It has a stationary point of inflection.

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided. Write using blue or black pen.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

Question 1 (15 marks)

Let $f: R^+ \to R, f(x) = x^2 \log_e(x)$.

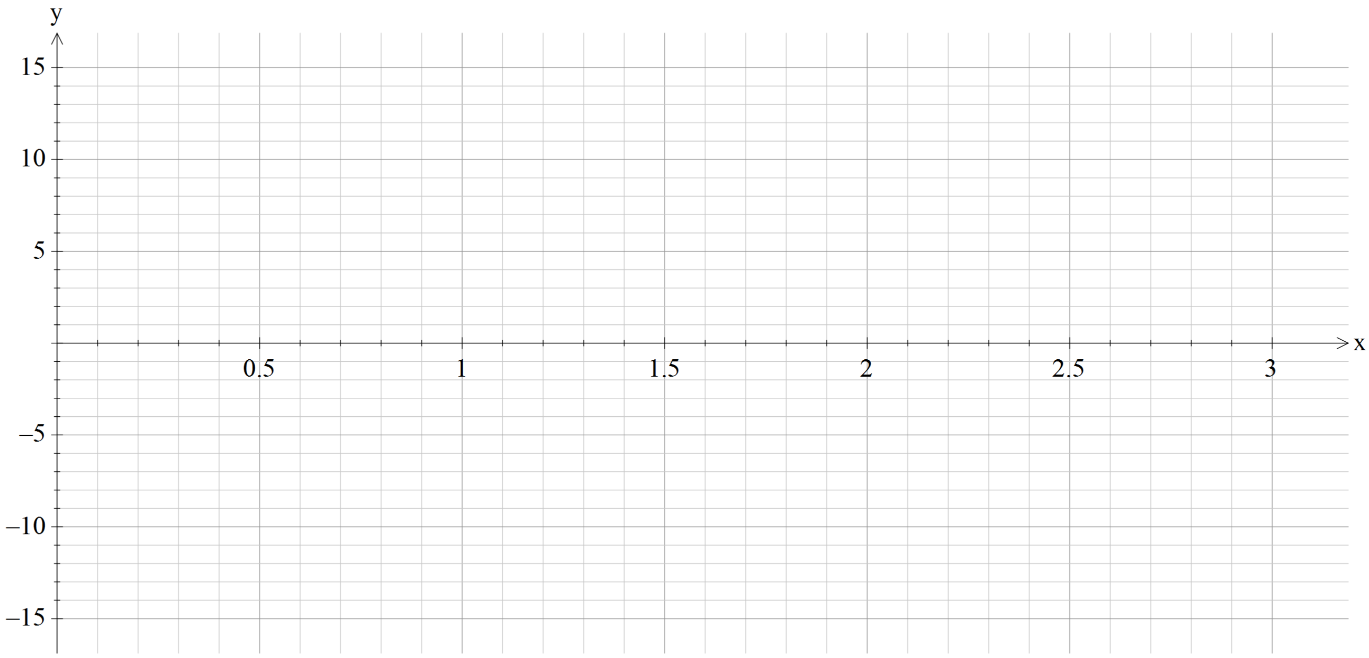
a. Find $\{x : f(x) = 0\}$ 2 marks

b. **i.** Find the stationary point of $f(x)$. 3 marks

ii. Show that the nature of the turning point is a minimum. 2 marks

c. Find the equation of tangent line at $x = e$. 2 marks

d. Sketch $f(x)$ and the tangent line at $x = e$ for $x \in (0,3]$ 3 marks



e. **i.** If $y = x^3 \log_e(x)$ find $\dfrac{dy}{dx}$ 1 mark

ii. Hence, algebraically find $\int_{1}^{2} x^2 \log_e(x)\, dx$. 3 marks

Question 2 (16 marks)

An airport has been analysing their departure processes and has some information on three main stages: taking carry-on luggage, selection for random swabbing and proceeding through customs.

Carry-on luggage statistics has shown to follow a normal distribution with a mean of 7.2kg and a variance of 4.70.

a. If the top 10% of carry-on luggage is rejected as too heavy and needs to be placed in the cargo hold, what is the maximum acceptable weight for carry-on luggage, correct to two decimal places? 2 marks

b. Find the value of c if $\Pr(-c < X < c) = 0.95$ for carry-on luggage weights, correct to three decimal places? 1 mark

c. A low-cost carrier has a different policy on carry-on luggage. Their limits of acceptability are imposed on the lowest 5% and highest 10%, being 3.1327 kg and 7.5223 kg respectively.

Find the mean and standard deviation of this normally distributed policy, correct to two decimal places. 3 marks

Random swabbing is a way of checking for traces of explosives and other potential threats. The airport has a set target of checking 22% of the passengers as they pass through security.

d. If a group of 10 travelers pass through security, find the probability that less than half are swabbed. 2 marks

e. If the airport claims that there is more than a 98% chance of at least two passengers being swabbed when n travelers pass through security, find the smallest possible value of n. 2 marks

From the group of 10 travelers previously mentioned, six of them are locals and the rest are internationals. As they pass through the next stage and approach customs, they are asked to approach the desk four at a time.

f. Find the probability that the four chosen are all internationals. 1 mark

g. Find the probability that more locals are chose than internationals in the selected four travelers to approach the desk. 2 marks

h. Given that at least three locals were chosen to approach the desk, find the probability that no internationals were included in the selected four travelers. 3 marks

Question 3 (15 marks)

Temperature variation for a ski resort can be given by:

$$T(t) = -3\cos\left(\frac{\pi}{12}(t-b)\right)+1, \quad t \in [0,24]$$

where t is given in hours, starting at 9pm on a Sunday, and T is measured in $°C$.

a. Find the amplitude and periods of *T(t)*. 1 mark

A scientist takes a temperature reading at 10pm on the same day, and records it as $\frac{-3\sqrt{2}}{2}+1$

b. Show that the smallest positive value of $b = 4$, if $b > 0$ 2 marks

c. State the initial temperature for the town. 1 mark

d. If the temperature is taken from the initial time for one period, find the t intercepts, correct to three decimal places, hence state the time to the nearest minute. 3 marks

e. Find the average value of the temperature over the first 12 hours. 3 marks

f. Find the stationary points of $T(t)$ over the whole day. 2 marks

g. Find the equation of the normal at $t = 6$. 3 marks

Question 4 (14 marks)

Let $f:(-\infty,a)\to R, f(x)=\dfrac{3}{(x-5)^2}-1$

a. Find the largest possible value of a for $f^{-1}(x)$ to exist 1 mark

b. Find the inverse function, $f^{-1}(x)$ using full functional notation. 2 marks

c. Find the coordinates of the point(s) of intersection between $f(x)=f^{-1}(x)$ correct to two decimal places. 2 marks

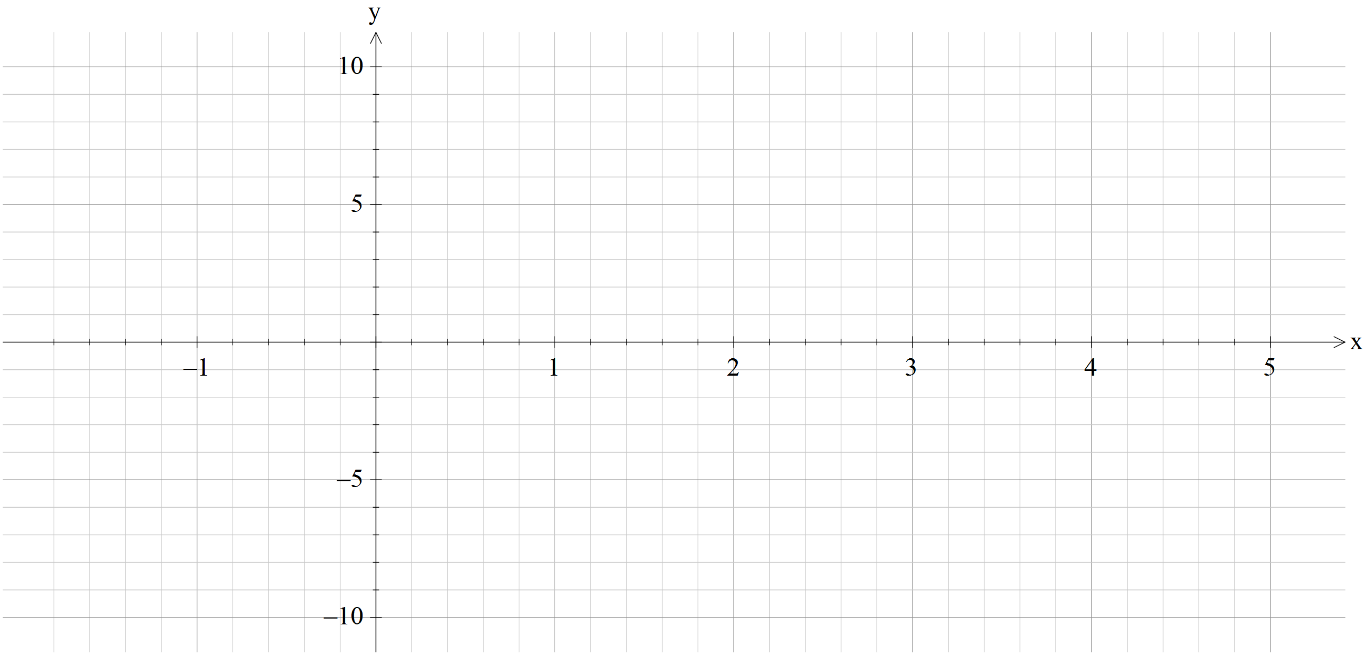
d. The function $h(x)$ is obtained by applying the transformation T to the function $f(x)$, where:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

i. Find the equation $h(x)$ 3 marks

ii. State the equations of the asymptotes for $h(x)$ 1 mark

e. Sketch the graph of $h(x)$ within the relevant domain. 3 marks



f. Find the area for $h(x)$ enclosed between $x = 2.5, 5$ and the x-axis. 2 marks

END OF QUESTION AND ANSWER BOOK

Mathematical Methods formulas

Measurement

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \frac{\Pr(A\cap B)}{\Pr(B)}$	mean	$\mu = \mathrm{E}(X)$

END OF FORMULA SHEET

EXTRA
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Student Name: ________________________________

Multiple Choice Answer Sheet

SECTION A	One answer per line				
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E