EXTRA
EXPERIENCED TEACHER RESOURCES & ASSESSMENTS

Student Name: ______________________________________

MATHEMATICAL METHODS
UNITS 3&4 Exam 1
2020 Written Trial Examination

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = \sqrt{3e^{2x} - e^{4x}}$, find $\frac{dy}{dx}$ 2 marks

b. Let $f(x) = x^2 \tan(3x)$
Evaluate $f'(\pi)$ 2 marks

Question 2 (4 marks)

Consider the functions $f: R \to R, f(x) = 3x^2 + x$ and $g: R \to R, g(x) = 2x - 1$.

a. State the rule of $f\left(g(x)\right)$ in the form $ax^2 + bx + c$. 1 mark

b. Let $h: R \to R, h(x) = f\left(g(x)\right)$.

Find the equation of the line perpendicular to the graph, $h(x)$, when $x = \frac{1}{2}$.

Give your answer in the form $ax + by = c$. 3 marks

Question 3 (4 marks)

Let $y = 3x\sin(2x)$.

a. Find $\frac{dy}{dx}$ 1 mark

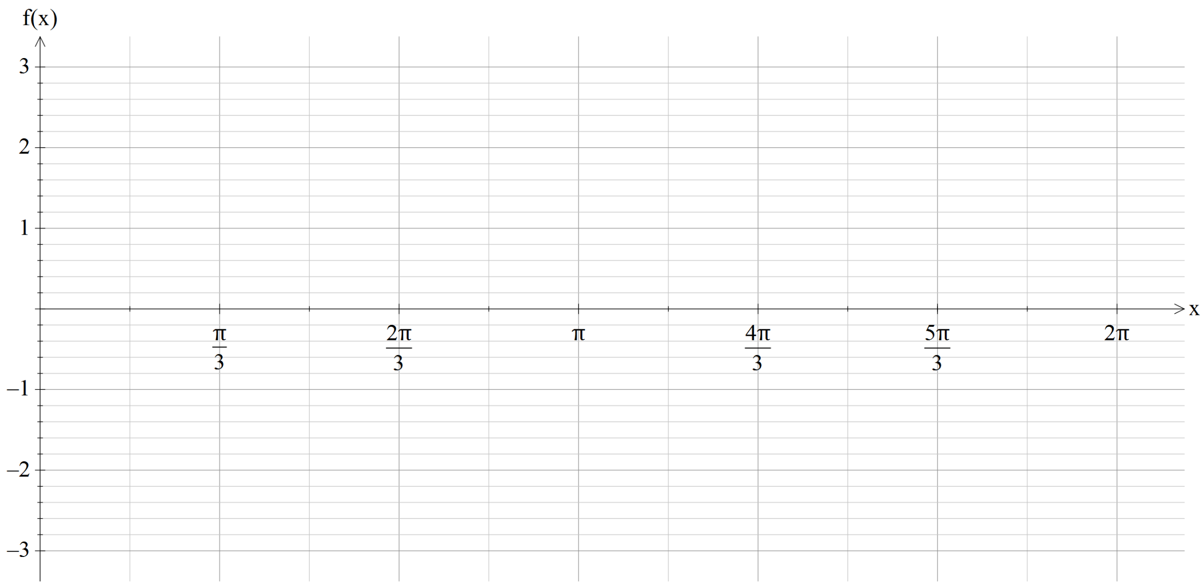
b. Hence, calculate $\int_0^{\pi/4} 3x\cos(2x)\ dx$. 3 marks

Question 4 (5 marks)

Let $f:[0,2\pi]\rightarrow R, f(x)=2\sin\left(2\left(x-\frac{\pi}{12}\right)\right)-1.$

a. Find the x – intercepts for $f(x)$ 2 marks

b. Sketch the graph of $f(x)$, labelling any endpoints, axial intercepts and turning points with their coordinates. 3 marks

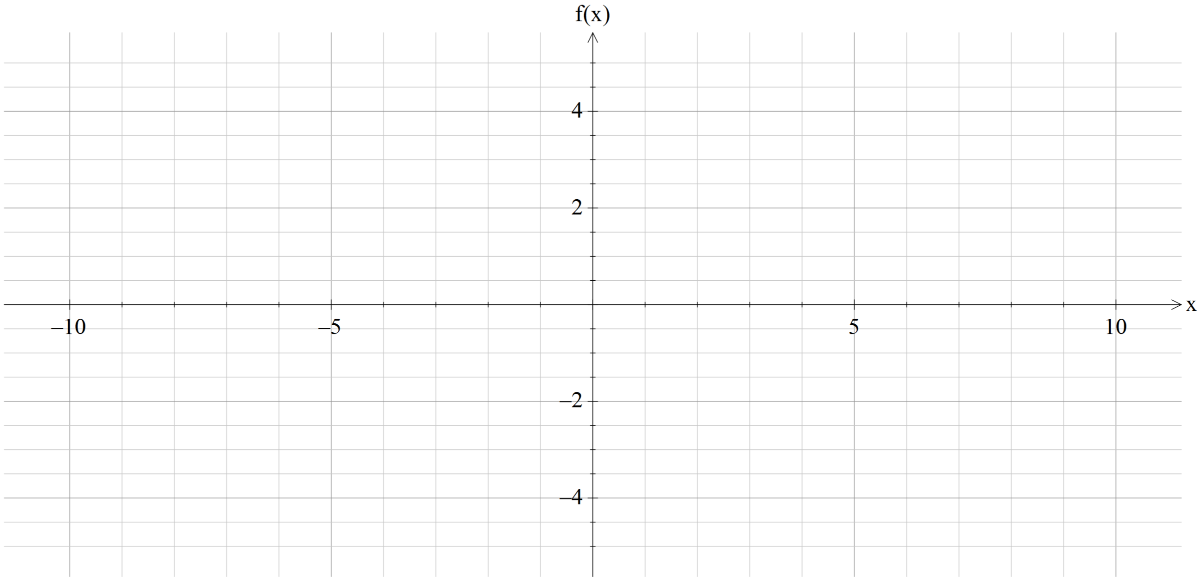


Question 5 (7 marks)

Let $f: R\setminus\{1\} \to R, f(x) = \dfrac{5-3x}{1-x}$.

a. Find the rule of $f^{-1}(x)$, stating the domain. 2 marks

b. Sketch the graph of $f^{-1}(x)$, labelling axial intercepts with their coordinates and asymptotes with their equations. 3 marks



c. Find the area bounded by the graph of $f^{-1}(x)$, the x–axis, the line $x = 5$ and $x = 7$.
Express your answer in exact form or in the form $a + \log_e(b)$. 2 marks

Question 6 (3 marks)

A fashion store employs seven people consisting of four females and three males. Only three staff can be rostered on at any time.

a. Find the probability of the staff on duty all being the same sex. 2 marks

b. During the day, hundreds of people come through the store. Three in five customers purchase something during the day. On a particular day, 400 customers enter the shop. 1 mark

What is the standard deviation of the Binomial Distribution of customers who buy goods at the store?

Question 7 (3 marks)

A set of linear equations are given as

$$(m+1)x - 2y = k + 1$$
$$-mx + 4y = 2 - k$$

Find the values of *m* and *k* which would give infinite solutions. 3 marks

Question 8 (5 marks)

For events A and B from a sample space, $\Pr(A \mid B) = \frac{2}{q}$ and $\Pr(B \mid A) = \frac{1}{q+1}$.

Let $\Pr(A \cap B) = \frac{1}{3}$.

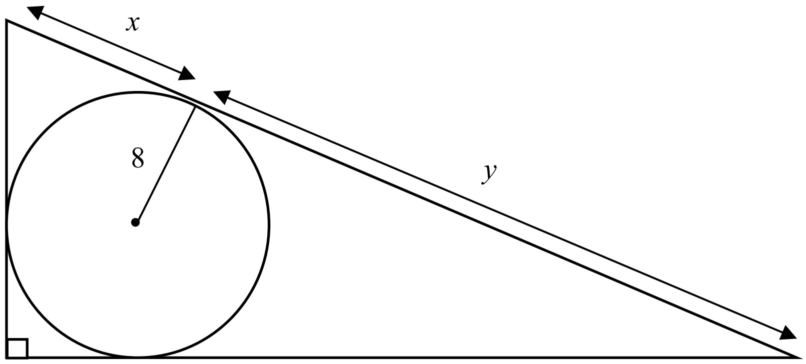
a. Find $\Pr(A)$ in terms of q. 2 marks

b. Find $\Pr(B)$ in terms of q. 1 mark

c. Given that A and B are independent events, solve for q. 2 marks

Question 9 (5 marks)

A right-angled triangle fits a circle of radius of 8 units, shown in the figure below.



a. Show that the y dimension of the right-angles triangle can be expressed as 2 marks

$$y = \frac{128}{x-8} + 8$$

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b. Find the value of x that gives the maximum area of this triangle. 3 marks

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END OF QUESTION AND ANSWER BOOK

Mathematical Methods formulas

Measurement

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}(\sin(ax))=a\ \cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\ \sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \frac{\Pr(A\cap B)}{\Pr(B)}$	mean	$\mu = \mathrm{E}(X)$

END OF FORMULA SHEET