EXTRA

EXPERIENCED TEACHER RESOURCES & ASSESSMENTS

Mathematical Methods
UNITS 3&4 Exam 1
2020 Written Trial Examination

Reading time: 15 minutes

Writing time: 1 hour

SOLUTIONS

THIS PAGE IS BLANK

Question 1 (4 marks)

a.

$$y=\sqrt{3e^{2x}-e^{4x}}$$

$$y=\left(3e^{2x}-e^{4x}\right)^{1/2}$$ **M1**

$$\frac{dy}{dx}=\frac{1}{2}\left(3e^{2x}-e^{4x}\right)^{-1/2}\left(6e^{2x}-4e^{4x}\right)$$

$$\frac{dy}{dx}=\frac{6e^{2x}-4e^{4x}}{2\sqrt{3e^{2x}-e^{4x}}}$$ **A1**

$$\frac{dy}{dx}=\frac{\cancel{2}\left(3e^{2x}-2e^{4x}\right)}{\cancel{2}\sqrt{3e^{2x}-e^{4x}}}$$

$$\therefore \frac{dy}{dx}=\frac{e^{2x}\left(3-2e^{2x}\right)}{\sqrt{3e^{2x}-e^{4x}}}$$

b.

$$f(x)=x^2\tan(3x)$$

$$f'(x)=\tan(3x)(2x)+x^2\left(3\sec^2(3x)\right)$$

$$f'(x)=2x\tan(3x)+3x^2\sec^2(3x)$$

$$\therefore f'(x)=2x\tan(3x)+\frac{3x^2}{\cos^2(3x)}$$ **M1**

$$f'(\pi)=2\pi\tan(3\pi)+\frac{3\pi^2}{\left(\cos(3\pi)\right)^2}$$

$$f'(\pi)=0+\frac{3\pi^2}{(-1)^2}$$

$$\therefore f'(\pi)=3\pi^2$$ **A1**

Question 2 (4 marks)

a.

$$f\left(g(x)\right) = 3(2x-1)^2 + (2x-1)$$
$$f\left(g(x)\right) = 3(4x^2 - 4x + 1) + (2x-1)$$
$$f\left(g(x)\right) = 12x^2 - 12x + 3 + 2x - 1$$
$$\therefore f\left(g(x)\right) = 12x^2 - 10x + 2$$

A1

b.

$$h(x) = 12x^2 - 10x + 2$$
$$h'(x) = 24x - 10$$

$$h'\left(\tfrac{1}{2}\right) = 12 - 10 = 2$$
$$\therefore m_{\perp} = -\tfrac{1}{2}$$

M1

$$h\left(\tfrac{1}{2}\right) = 12\left(\tfrac{1}{2}\right)^2 - 10\left(\tfrac{1}{2}\right) + 2$$
$$h\left(\tfrac{1}{2}\right) = 3 - 5 + 2 = 0 \qquad \left(\tfrac{1}{2}, 0\right)$$

M1

$$y - 0 = -\tfrac{1}{2}\left(x - \tfrac{1}{2}\right)$$
$$y = -\tfrac{1}{2}x + \tfrac{1}{4}$$
$$4y = -2x + 1$$
$$\therefore 2x + 4y = 1$$

A1

Question 3 (4 marks)

a.

$$y = 3x\sin(2x)$$

$$\frac{dy}{dx} = \sin(2x)\times 3 + 3x \times 2\cos(2x)$$

$$\therefore \frac{dy}{dx} = 3\sin(2x) + 6x\cos(2x)$$ **A1**

b.

$$\int 3\sin(2x) + 6x\cos(2x)\ dx = 3x\sin(2x)$$

$$\int 3\sin(2x)\ dx + \int 6x\cos(2x)\ dx = 3x\sin(2x)$$

$$\int 6x\cos(2x)\ dx = 3x\sin(2x) - \int 3\sin(2x)\ dx$$

$$\int 6x\cos(2x)\ dx = 3x\sin(2x) + \frac{3}{2}\cos(2x)$$ **M1**

$$\int_0^{\pi/4} 3x\cos(2x)\ dx = \left[\frac{1}{2}\left(3x\sin(2x) + \frac{3}{2}\cos(2x)\right)\right]_0^{\pi/4}$$

$$\int_0^{\pi/4} 3x\cos(2x)\ dx = \left[\frac{3x}{2}\sin(2x) + \frac{3}{4}\cos(2x)\right]_0^{\pi/4}$$

$$\int_0^{\pi/4} 3x\cos(2x)\ dx = \left[\left(\frac{3\pi}{8}\sin\left(\frac{\pi}{2}\right) + \frac{3}{4}\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{3}{4}\cos(0)\right)\right]$$ **M1**

$$\int_0^{\pi/4} 3x\cos(2x)\ dx = \frac{3\pi}{8} + 0 - \frac{3}{4}$$

$$\therefore \int_0^{\pi/4} 3x\cos(2x)\ dx = \frac{3\pi - 6}{8} = \frac{3(\pi - 2)}{8}$$ **A1**

Question 4 (5 marks)

a.

$$2\sin\left(2\left(x-\frac{\pi}{12}\right)\right)-1=0 \qquad 0\le x\le 2\pi$$

$$\sin\left(2\left(x-\frac{\pi}{12}\right)\right)=\frac{1}{2} \qquad \frac{-\pi}{12}\le x-\frac{\pi}{12}\le\frac{23\pi}{12}$$

$$Basic\ Angle=\frac{\pi}{6},1st\ \&\ 2nd\ quadrants \qquad \frac{-\pi}{6}\le 2\left(x-\frac{\pi}{12}\right)\le\frac{23\pi}{6}$$

M1

$$2\left(x-\frac{\pi}{12}\right)=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$$

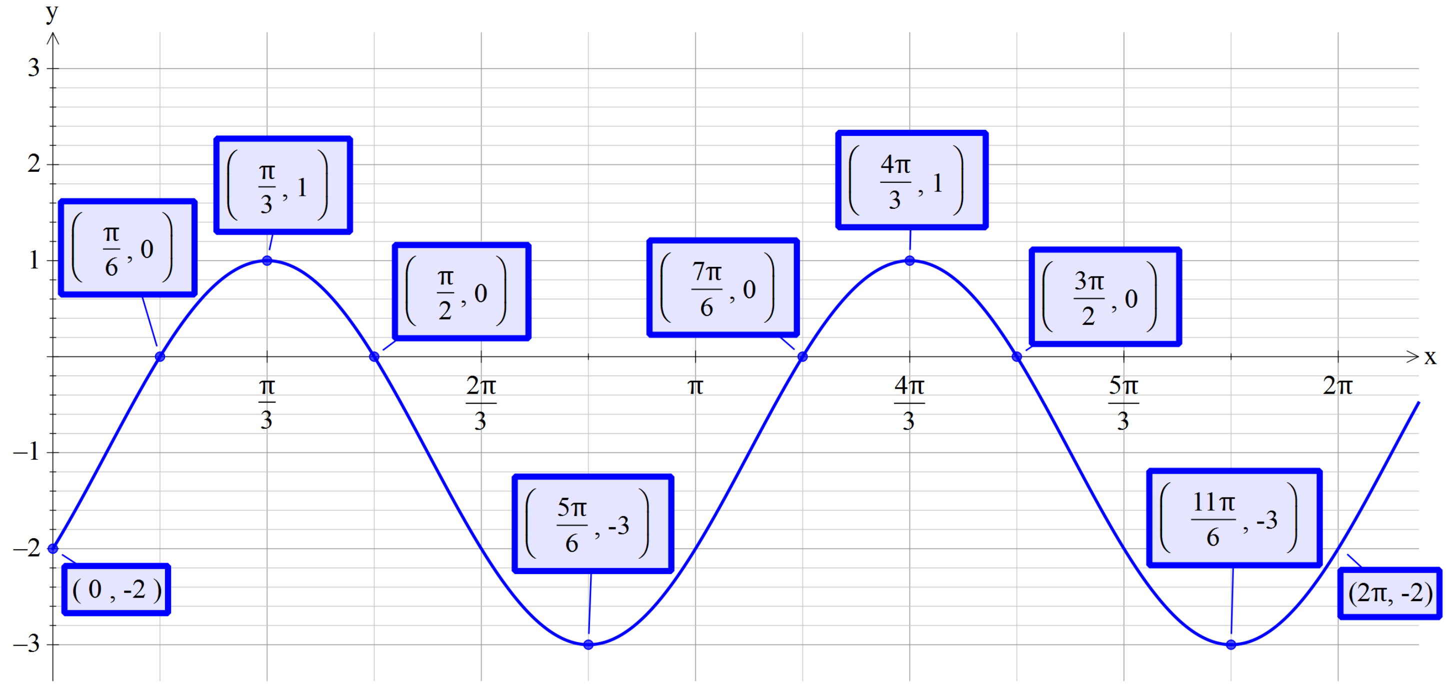
$$x-\frac{\pi}{12}=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$$

$$x=\frac{2\pi}{12},\frac{6\pi}{12},\frac{14\pi}{12},\frac{18\pi}{12}$$

$$\therefore x=\frac{\pi}{6},\frac{\pi}{2},\frac{7\pi}{6},\frac{3\pi}{2}$$

A1

b.



1 mark – Shape coordinates

1 mark – Intercepts

1 mark – Max/min

Question 5 (7 marks)

a.

$$f(x)=\frac{5-3x}{1-x}$$

Let $y=f(x)$, *swap* x & y

$$x=\frac{5-3y}{1-y}$$

$$x-xy=5-3y$$

$$3y-xy=5-x$$

$$y(3-x)=5-x$$ **M1**

$$y=\frac{5-x}{3-x}$$

$$\therefore f^{-1}:R\setminus\{3\}\to R, f^{-1}(x)=\frac{5-x}{3-x}$$ **A1**

OR

$$f(x)=\frac{5-3x}{1-x}=\frac{2}{1-x}+3$$

Let $y=f(x)$, *swap* x & y

$$x=\frac{2}{1-y}+3$$

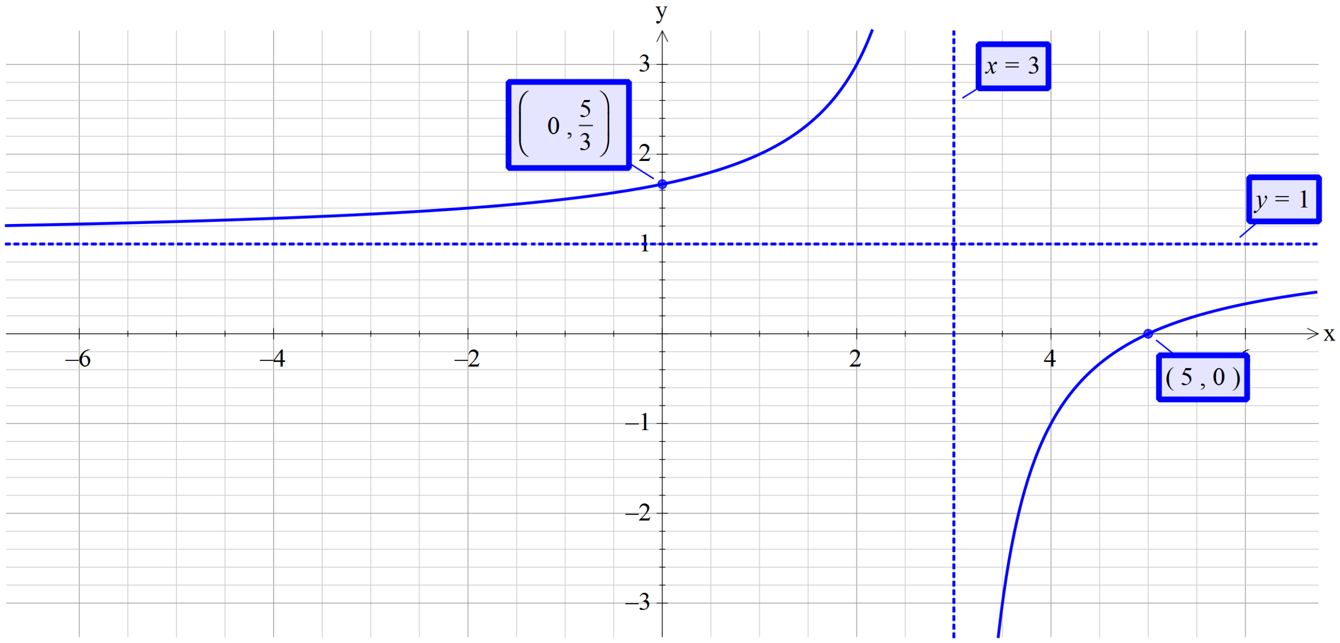
$$x-3=\frac{2}{1-y}$$ **M1**

$$1-y=\frac{2}{x-3}$$

$$y=1-\frac{2}{x-3}$$

$$\therefore f^{-1}:R\setminus\{3\}\to R, f^{-1}(x)=1-\frac{2}{x-3}$$ **A1**

b.



1 mark – Shape

1 mark – Intercepts

1 mark – Asymptotes

c.

$$\int_5^7 1-\frac{2}{x-3}\,dx$$

$$=\left[x-2\log_e(x-3)\right]_5^7 \qquad \textbf{M1}$$

$$=\left[\left(7-2\log_e(4)\right)-\left(5-2\log_e(2)\right)\right]$$

$$=7-2\log_e(4)-5+2\log_e(2)$$

$$=2-\log_e(16)+\log_e(4)$$

$$=2-\log_e(4) \qquad \textbf{A1}$$

Question 6 (3 marks)

a.

$$\Pr(all\ \ males) = \frac{\binom{3}{3}\binom{4}{0}}{\binom{7}{3}} = \frac{1}{35}$$

$$\Pr(all\ \ females) = \frac{\binom{4}{3}\binom{3}{0}}{\binom{7}{3}} = \frac{4}{35}$$ **M1**

$$\Pr(same\ \ sex) = \frac{5}{35} = \frac{1}{7}$$ **A1**

b.

$$SD(X) = \sqrt{np(1-p)}$$

$$\therefore SD(X) = \sqrt{400 \times \frac{3}{5} \times \frac{2}{5}}$$

$$\therefore SD(X) = 4\sqrt{6}$$ **A1**

Question 7 (3 marks)

Equation 1

$2y = (m+1)x - k - 1$

$y = \frac{m+1}{2}x - \frac{k}{2} - \frac{1}{2}$

Equation 2

$4y = 2 - k + mx$

$y = \frac{2-k}{4} + \frac{m}{x}$ **M1**

Equate the gradients of the two straight lines.

$\frac{m+1}{2} = \frac{\cancel{2-k}}{4} + \frac{m}{4}$

$4m + 4 = 2m$

$2m = -4$

$m = -2$ **A1**

Equate the y-intercept of the two straight lines.

$-\frac{k}{2} - \frac{1}{2} = \frac{2-k}{4}$

$4(-k-1) = 2(2-k)$

$-4k - 4 = 4 - 2k$

$-2k = 8$

$k = -4$ **A1**

OR

$\begin{bmatrix} m+1 & -2 \\ -m & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k+1 \\ 2-k \end{bmatrix}$

Let $\det = 0$

$4(m+1) - 2m = 0$ **M1**

$4m + 4 - 2m = 0$

$2m = -4$

$\therefore m = -2$ **A1**

For equation 1,

$y = \frac{(m+1)}{2}x - \frac{k+1}{2}$

For equation 2,

$y = \frac{m}{4}x + \frac{2-k}{4}$

Equate $y-\text{int}$ *for both*

$-\frac{k+1}{2} = \frac{2-k}{4}$

$-4k - 4 = 4 - 2k$

$-8 = 2k$

$\therefore k = -4$ **A1**

Question 8 (5 marks)

a.

$$\Pr\left(B \mid A\right) = \frac{\Pr\left(A \cap B\right)}{\Pr(A)}$$

$$\Pr(A) = \frac{\Pr\left(A \cap B\right)}{\Pr\left(B \mid A\right)}$$ **M1**

$$\Pr(A) = \frac{1}{3} \times \frac{q+1}{1} = \frac{q+1}{3}$$ **A1**

b.

$$\Pr\left(A \mid B\right) = \frac{\Pr\left(A \cap B\right)}{\Pr(B)}$$

$$\Pr(B) = \frac{\Pr\left(A \cap B\right)}{\Pr\left(A \mid B\right)}$$

$$\Pr(B) = \frac{1}{3} \times \frac{q}{2} = \frac{q}{6}$$ **A1**

c.

$$\Pr\left(A \cap B\right) = \Pr(A) \times \Pr(B)$$

$$\frac{1}{3} = \frac{q+1}{3} \times \frac{q}{6}$$ **M1**

$$\frac{1}{3} = \frac{q(q+1)}{18}$$

$$6 = q^2 + q$$

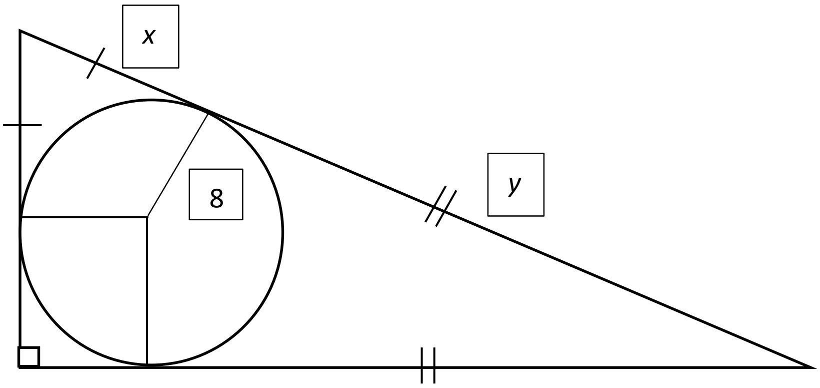
$$q^2 + q - 6 = 0$$

$$(q+3)(q-2) = 0$$

$$q = -3, 2 \quad but \quad q \neq -3$$

$$\therefore q = 2$$ **A1**

Question 9 (5 marks)



a.

$$(x+y)^2=(x+8)^2+(y+8)^2$$

$$\left(\cancel{x^2}+2xy+\cancel{y^2}\right)=\left(\cancel{x^2}+16x+64\right)+\left(\cancel{y^2}+16y+64\right)$$

$$2xy-16y=16x+128$$

$$y(2x-16)=16x+128$$ **M1**

$$y=\frac{16x+128}{2x-16}$$

$$y=\frac{8x+64}{x-8}$$

Using division of polynomials

$$\begin{array}{r} 8 \\ x-8\overline{)8x+64} \\ \underline{8x-64} \\ 128 \end{array}$$ **M1**

$$\therefore y=\frac{128}{x-8}+8$$

b.

$$A=\frac{1}{2}\left(\frac{8x+64}{x-8}+8\right)(x+8)$$

$$A=\frac{1}{2}\left(\frac{8x\cancel{+64}+8x\cancel{-64}}{x-8}\right)(x+8)$$

$$A=\left(\frac{8x}{x-8}\right)(x+8)$$

$$A=\frac{8x^2+64x}{x-8}$$ **M1**

For max *area*

$$\frac{dA}{dx}=\frac{(x-8)(16x+64)-(8x^2+64x)}{(x-8)^2}=0$$

$$16x^2\cancel{+64x}-128x-512-8x^2\cancel{-64x}=0$$

$$8x^2-128x-512=0$$

$$8\left(x^2-16x-64\right)=0$$ **M1**

Solve for x

$$x=\frac{16\pm\sqrt{256-4(1)(64)}}{2(1)}$$

$$x=\frac{16\pm\sqrt{512}}{2}$$

$$x=\frac{16\pm16\sqrt{2}}{2}$$

$$x=8\pm8\sqrt{2} \quad but \;\; x>0$$

$$\therefore x=8+8\sqrt{2}$$ **A1**

END OF SOLUTIONS BOOK