VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Written examination 2

Monday 3 June 2019

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 26 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

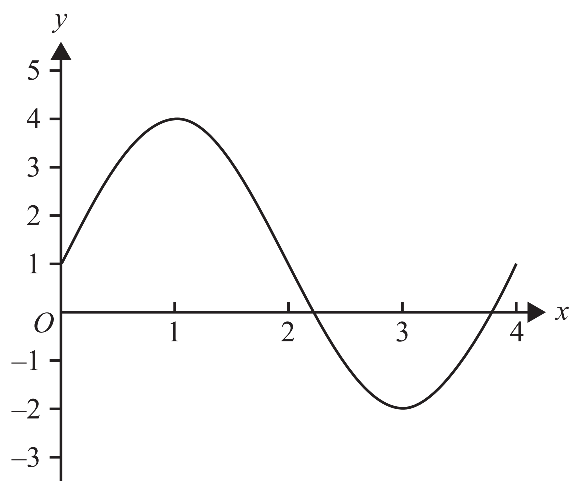
Question 1

The maximal domain of the function with rule $f(x) = x^2 + \log_e(x)$ is

A. R
B. $(0, \infty)$
C. $[0, \infty)$
D. $(-\infty, 0)$
E. $[1, \infty)$

Question 2

The diagram below shows one cycle of a circular function.



The amplitude, period and range of this function are respectively

A. 3, 2 and $[-2, 4]$
B. 3, $\frac{\pi}{2}$ and $[-2, 4]$
C. 4, 4 and $[0, 4]$
D. 4, $\frac{\pi}{4}$ and $[-2, 4]$
E. 3, 4 and $[-2, 4]$

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Question 3

If $x + a$ is a factor of $8x^3 - 14x^2 - a^2x$, where $a \in R\backslash\{0\}$, then the value of a is

A. 7

B. 4

C. 1

D. –2

E. –1

Question 4

The graph of the function $f: D \to R,\ f(x) = \dfrac{2x-3}{4+x}$, where D is the maximal domain, has asymptotes

A. $x = -4,\ y = 2$

B. $x = \dfrac{3}{2},\ y = -4$

C. $x = -4,\ y = \dfrac{3}{2}$

D. $x = \dfrac{3}{2},\ y = 2$

E. $x = 2,\ y = 1$

Question 5

Consider the probability distribution for the discrete random variable X shown in the table below.

x	-1	0	1	2	3
$\Pr(X = x)$	b	b	b	$\dfrac{3}{5} - b$	$\dfrac{3b}{5}$

The value of $\mathrm{E}(X)$ is

A. $\dfrac{76}{65}$

B. 1

C. 0

D. $\dfrac{2}{13}$

E. $\dfrac{86}{65}$

Question 6

Let $f: [0, \infty) \to R,\ f(x) = x^2 + 1$.

The equation $f\left(f(x)\right) = \frac{185}{16}$ has real solution(s)

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

D. $x = \frac{3}{2}$

E. $x = \pm\frac{3}{2}$

Question 7

If $m = \int_1^3 \frac{2}{x}\,dx$, then the value of e^m is

A. $\log_e(9)$

B. -9

C. $\frac{1}{9}$

D. 9

E. $-\frac{1}{9}$

Question 8

The simultaneous linear equations $2y + (m - 1)\,x = 2$ and $my + 3x = k$ have infinitely many solutions for

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

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Question 9

At the start of a particular week, Kim has three red apples and two green apples. She eats one apple every day. On Monday, Tuesday and Wednesday of that week, she randomly selects an apple to eat.

In this three-day period, the probability that Kim does not eat an apple of the same colour on any two consecutive days is

A. $\frac{1}{10}$

B. $\frac{1}{5}$

C. $\frac{3}{10}$

D. $\frac{2}{5}$

E. $\frac{6}{25}$

Question 10

Let f be the probability density function $f:\left[0, \frac{2}{3}\right] \to R,\ f(x) = kx(2x+1)(3x-2)(3x+2)$.
The value of k is

A. $\frac{308}{405}$

B. $-\frac{308}{405}$

C. $-\frac{405}{308}$

D. $\frac{405}{308}$

E. $\frac{960}{133}$

Question 11

The function $f: D \to R,\ f(x) = 5x^3 + 10x^2 + 1$ will have an inverse function for

A. $D = R$

B. $D = (-2, \infty)$

C. $D = \left(-\infty, \frac{1}{2}\right]$

D. $D = (-\infty, -1]$

E. $D = [0, \infty)$

Question 12

The transformation $T: R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x+1} - 3$ onto the graph of $y = \sqrt{x}$, has rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

Question 13

The graph of $f(x) = x^3 - 6(b-2)x^2 + 18x + 6$ has exactly two stationary points for

A. $1 < b < 2$

B. $b = 1$

C. $b = \frac{4 \pm \sqrt{6}}{2}$

D. $\frac{4-\sqrt{6}}{2} \le b \le \frac{4+\sqrt{6}}{2}$

E. $b < \frac{4-\sqrt{6}}{2}$ or $b > \frac{4+\sqrt{6}}{2}$

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Question 14

If $2\log_e(x) - \log_e(x+2) = \log_e(y)$, then x is equal to

A. $\dfrac{y+\sqrt{y^2+8y}}{2}$

B. $\dfrac{y\pm\sqrt{y^2+8y}}{2}$

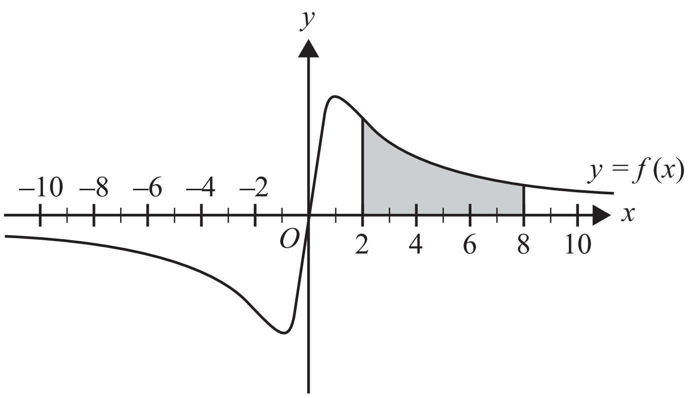
C. $\dfrac{y\pm\sqrt{y^2-8y}}{2}$

D. $\dfrac{-1\pm\sqrt{4y-7}}{2}$

E. $\dfrac{-1+\sqrt{4y-7}}{2}$

Question 15

The area bounded by the graph of $y = f(x)$, the line $x = 2$, the line $x = 8$ and the x-axis, as shaded in the diagram below, is $3\log_e(13)$.



The value of $\int_4^{10} 3f(x-2)\,dx$ is

A. $3\log_e(13)$

B. $9\log_e(13)$

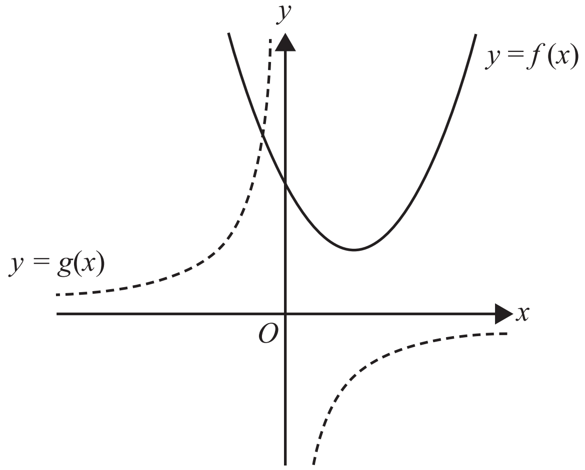
C. $6\log_e(39)$

D. $\log_e(13)$

E. $9\log_e(11)$

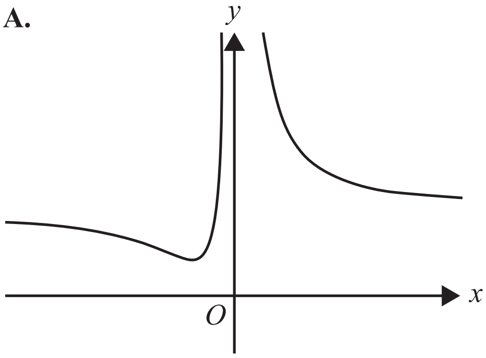
Question 16

Parts of the graphs of $y = f(x)$ and $y = g(x)$ are shown below.

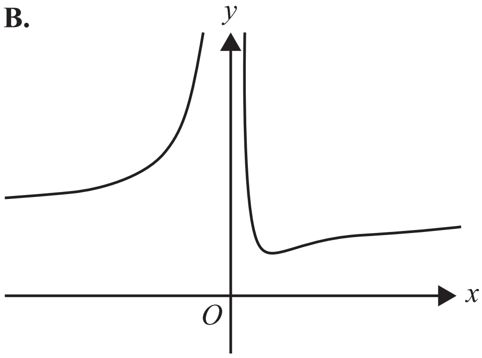


The corresponding part of the graph of $y = g(f(x))$ is best represented by

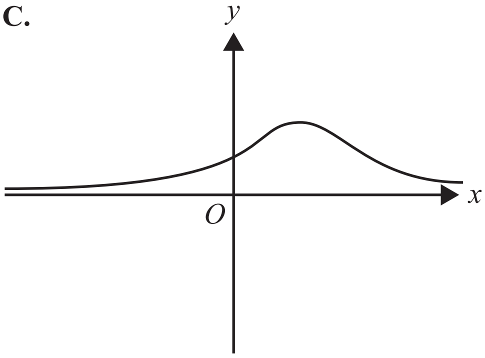
A.



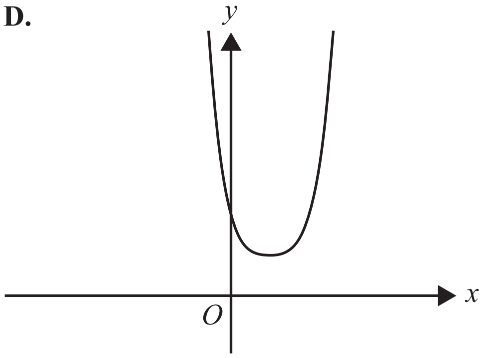
B.



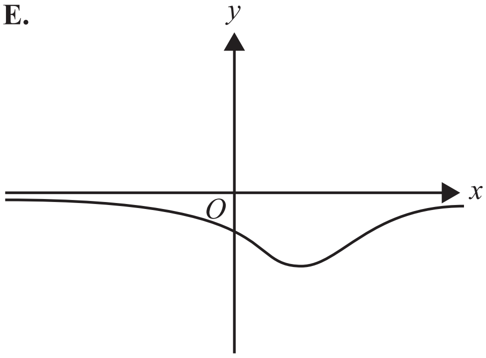
C.



D.



E.



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Question 17

The graph of the function g is obtained from the graph of the function f with rule $f(x)=\cos(x)-\frac{3}{8}$ by a dilation of factor $\frac{4}{\pi}$ from the y-axis, a dilation of factor $\frac{4}{3}$ from the x-axis, a reflection in the y-axis and a translation of $\frac{3}{2}$ units in the positive y direction, in that order.

The range and the period of g are respectively

A. $\left[-\frac{1}{3}, \frac{7}{3}\right]$ and 2

B. $\left[-\frac{1}{3}, \frac{7}{3}\right]$ and 8

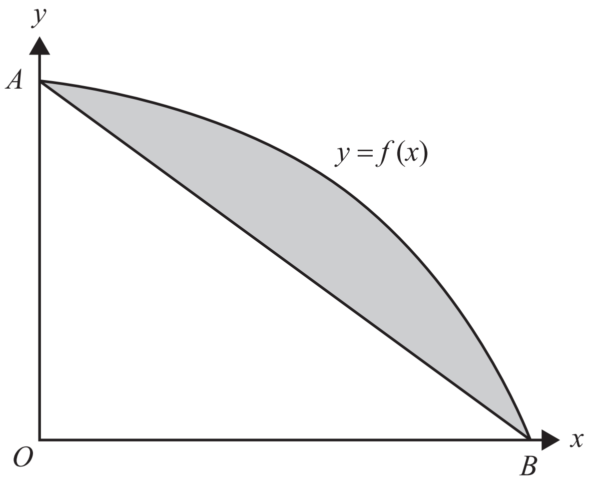
C. $\left[-\frac{7}{3}, \frac{1}{3}\right]$ and 2

D. $\left[-\frac{7}{3}, \frac{1}{3}\right]$ and 8

E. $\left[-\frac{4}{3}, \frac{4}{3}\right]$ and $\frac{\pi^2}{2}$

Question 18

Part of the graph of the function f, where $f(x) = 8 - 2^{x-1}$, is shown below. It intersects the axes at the points A and B. The line segment joining A and B is also shown on the graph.



The area of the shaded region is

A. $16-\frac{15}{\log_e(2)}$

B. $17-\frac{15}{2\log_e(2)}$

C. $\frac{7}{\log_e(2)}-\frac{159}{16}$

D. $16-\frac{15}{2\log_e(2)}$

E. $\frac{17}{2\log_e(2)}-15$

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Question 19

A random sample of computer users was surveyed about whether the users had played a particular computer game. An approximate 95% confidence interval for the proportion of computer users who had played this game was calculated from this random sample to be (0.6668, 0.8147).

The number of computer users in the sample is closest to

A. 5

B. 33

C. 135

D. 150

E. 180

Question 20

Let $f(x) = (ax + b)^5$ and let g be the inverse function of f.

Given that $f(0) = 1$, what is the value of $g'(1)$?

A. $\dfrac{5}{a}$

B. 1

C. $\dfrac{1}{5a}$

D. $5a(a + 1)^4$

E. 0

END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

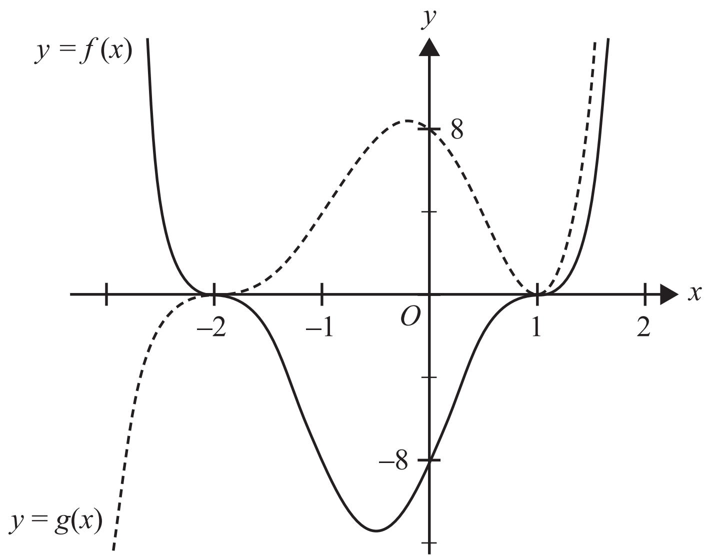
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (9 marks)

Parts of the graphs of $f(x) = (x - 1)^3(x + 2)^3$ and $g(x) = (x - 1)^2(x + 2)^3$ are shown on the axes below.



The two graphs intersect at three points, (–2, 0), (1, 0) and (c, d). The point (c, d) is not shown in the diagram above.

a. Find the values of c and d. 2 marks

b. Find the values of x such that $f(x) > g(x)$. 1 mark

c. State the values of x for which

i. $f'(x) > 0$ 1 mark

ii. $g'(x) > 0$ 1 mark

d. Show that $f(1 + m) = f(-2 - m)$ for all m. 1 mark

e. Find the values of h such that $g(x + h) = 0$ has exactly one negative solution. 2 marks

f. Find the values of k such that $f(x) + k = 0$ has no solutions. 1 mark

Question 2 (14 marks)

The wind speed at a weather monitoring station varies according to the function

$$v(t) = 20 + 16\sin\left(\frac{\pi t}{14}\right)$$

where v is the speed of the wind, in kilometres per hour (km/h), and t is the time, in minutes, after 9 am.

a. What is the amplitude and the period of $v(t)$? 2 marks

b. What are the maximum and minimum wind speeds at the weather monitoring station? 1 mark

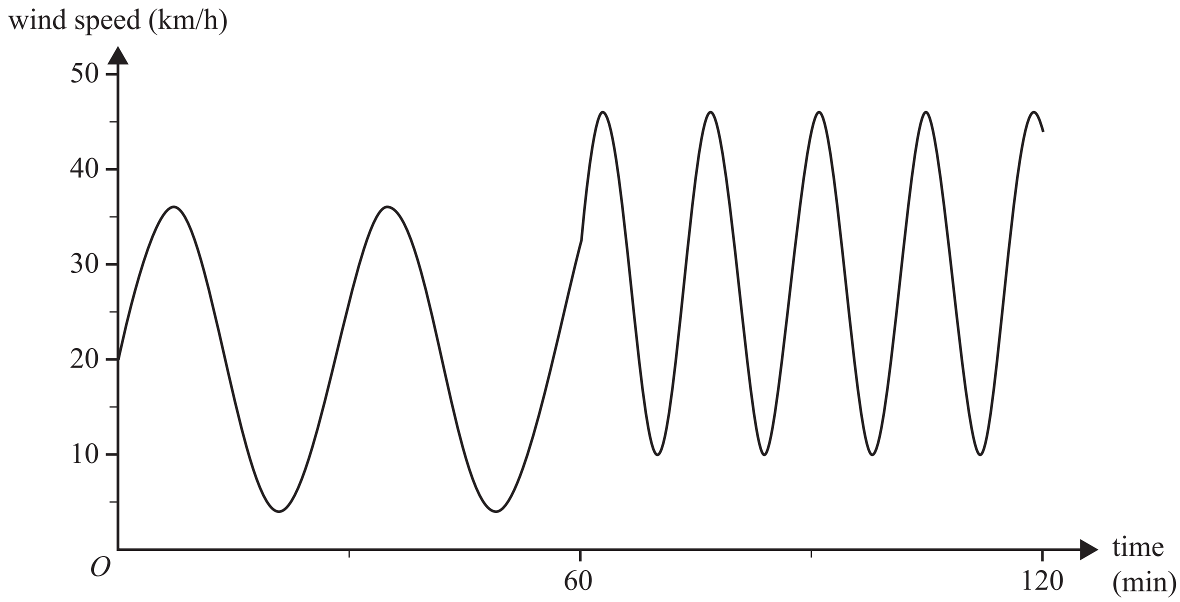
c. Find $v(60)$, correct to four decimal places. 1 mark

d. Find the average value of $v(t)$ for the first 60 minutes, correct to two decimal places. 2 marks

A sudden wind change occurs at 10 am. From that point in time, the wind speed varies according to the new function

$$v_1(t) = 28 + 18\sin\left(\frac{\pi(t-k)}{7}\right)$$

where v_1 is the speed of the wind, in kilometres per hour, t is the time, in minutes, after 9 am and $k \in R^+$. The wind speed after 9 am is shown in the diagram below.



e. Find the smallest value of k, correct to four decimal places, such that $v(t)$ and $v_1(t)$ are equal and are both increasing at 10 am. 2 marks

f. Another possible value of k was found to be 31.4358

Using this value of k, the weather monitoring station sends a signal when the wind speed is greater than 38 km/h.

i. Find the value of t at which a signal is first sent, correct to two decimal places. 1 mark

ii. Find the proportion of one cycle, to the nearest whole per cent, for which $v_1 > 38$. 2 marks

g. Let $f(x) = 20 + 16\sin\left(\frac{\pi x}{14}\right)$ and $g(x) = 28 + 18\sin\left(\frac{\pi(x-k)}{7}\right)$.

The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of f onto the graph of g.

State the values of a, b, c and d, in terms of k where appropriate. 3 marks

Question 3 (14 marks)

Concerts at the Mathsland Concert Hall begin L minutes after the scheduled starting time. L is a random variable that is normally distributed with a mean of 10 minutes and a standard deviation of four minutes.

a. What proportion of concerts begin before the scheduled starting time, correct to four decimal places? 1 mark

b. Find the probability that a concert begins more than 15 minutes after the scheduled starting time, correct to four decimal places. 1 mark

If a concert begins more than 15 minutes after the scheduled starting time, the cleaner is given an extra payment of $200. If a concert begins up to 15 minutes after the scheduled starting time, the cleaner is given an extra payment of $100. If a concert begins at or before the scheduled starting time, there is no extra payment for the cleaner.

Let C be the random variable that represents the extra payment for the cleaner, in dollars.

c. **i.** Using your responses from **part a.** and **part b.**, complete the following table, correct to three decimal places. 1 mark

c	0	100	200
$\Pr(C = c)$			

ii. Calculate the expected value of the extra payment for the cleaner, to the nearest dollar. 1 mark

iii. Calculate the standard deviation of C, correct to the nearest dollar. 1 mark

The owners of the Mathsland Concert Hall decide to review their operation. They study information from 1000 concerts at other similar venues, collected as a simple random sample. The sample value for the number of concerts that start more than 15 minutes after the scheduled starting time is 43.

d. **i.** Find the 95% confidence interval for the proportion of concerts that begin more than 15 minutes after the scheduled starting time. Give values correct to three decimal places. 1 mark

ii. Explain why this confidence interval suggests that the proportion of concerts that begin more than 15 minutes after the scheduled starting time at the Mathsland Concert Hall is different from the proportion at the venues in the sample. 1 mark

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The owners of the Mathsland Concert Hall decide that concerts must not begin before the scheduled starting time. They also make changes to reduce the number of concerts that begin after the scheduled starting time. Following these changes, M is the random variable that represents the number of minutes after the scheduled starting time that concerts begin. The probability density function for M is

$$f(x) = \begin{cases} \dfrac{8}{(x+2)^3} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

where x is the time, in minutes, after the scheduled starting time.

e. Calculate the expected value of M. 2 marks

f. **i.** Find the probability that a concert now begins more than 15 minutes after the scheduled starting time. 1 mark

ii. Find the probability that each of the next nine concerts begins no more than 15 minutes after the scheduled starting time and the 10th concert begins more than 15 minutes after the scheduled starting time. Give your answer correct to four decimal places. 2 marks

iii. Find the probability that a concert begins up to 20 minutes after the scheduled starting time, given that it begins more than 15 minutes after the scheduled starting time. Give your answer correct to three decimal places. 2 marks

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Question 4 (13 marks)

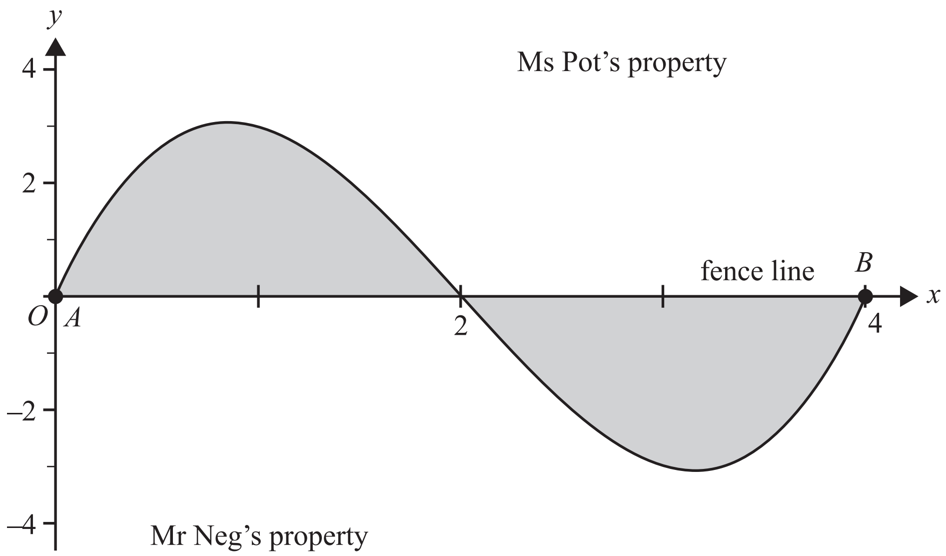
A mining company has found deposits of gold between two points, A and B, that are located on a straight fence line that separates Ms Pot's property and Mr Neg's property. The distance between A and B is 4 units.

The mining company believes that the gold could be found on both Ms Pot's property and Mr Neg's property.

The mining company initially models the boundary of its proposed mining area using the fence line and the graph of

$$f : [0, 4] \to R,\ f(x) = x(x-2)(x-4)$$

where x is the number of units from point A in the direction of point B and y is the number of units perpendicular to the fence line, with the positive direction towards Ms Pot's property. The mining company will only mine from the boundary curve to the fence line, as indicated by the shaded area below.



a. Determine the total number of square units that will be mined according to this model. 2 marks

The mining company offers to pay Mr Neg \$100 000 per square unit of his land mined and Ms Pot \$120 000 per square unit of her land mined.

b. Determine the total amount of money that the mining company offers to pay. 1 mark

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The mining company reviews its model to use the fence line and the graph of

$$p:[0,4] \to R,\ p(x) = x\left(x - 4 + \frac{4}{1+a}\right)(x-4)$$

where $a > 0$.

c. Find the value of a for which $p(x) = f(x)$ for all x. 1 mark

d. Solve $p'(x) = 0$ for x in terms of a. 2 marks

Mr Neg does not want his property to be mined further than 4 units measured perpendicular from the fence line.

e. Find the smallest value of a, correct to three decimal places, for this condition to be met. 2 marks

f. Find the value of a for which the total area of land mined is a minimum. 3 marks

g. The mining company offers to pay Ms Pot \$120 000 per square unit of her land mined and Mr Neg \$100 000 per square unit of his land mined.

Determine the value of a that will minimise the total cost of the land purchase for the mining company. Give your answer correct to three decimal places. 2 marks

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Question 5 (10 marks)

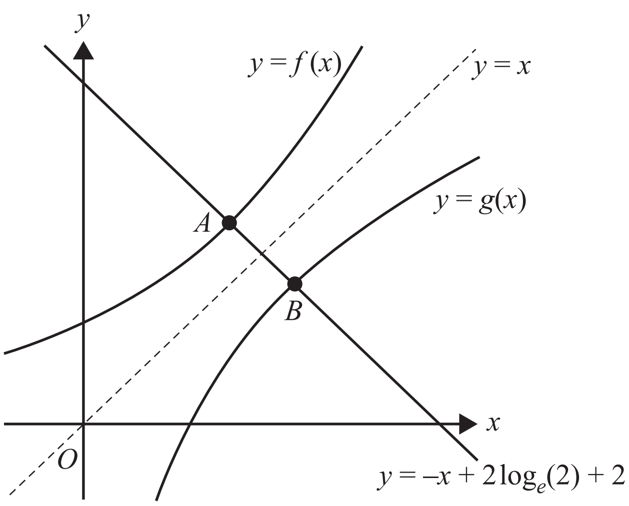
Let $f: R \to R, f(x) = e^{\left(\frac{x}{2}\right)}$ and $g: R^+ \to R, g(x) = 2\log_e(x)$.

a. Find $g^{-1}(x)$. 1 mark

b. Find the coordinates of point A, where the tangent to the graph of f at A is parallel to the graph of $y = x$. 2 marks

c. Show that the equation of the line that is perpendicular to the graph of $y = x$ and goes through point A is $y = -x + 2\log_e(2) + 2$. 1 mark

Let B be the point of intersection of the graphs of g and $y = -x + 2\log_e(2) + 2$, as shown in the diagram below.



d. Determine the coordinates of point B. 1 mark

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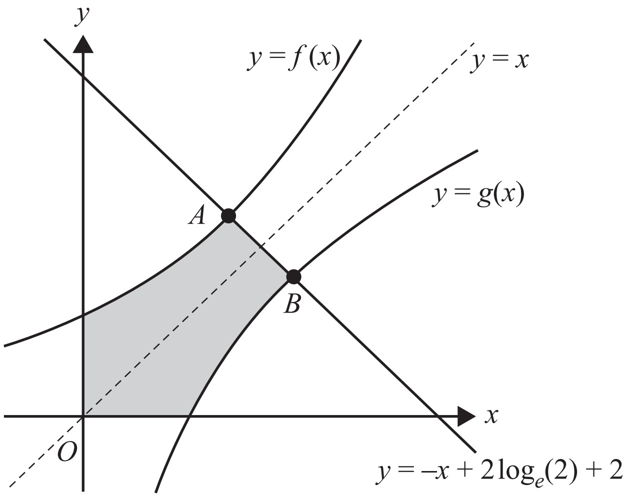
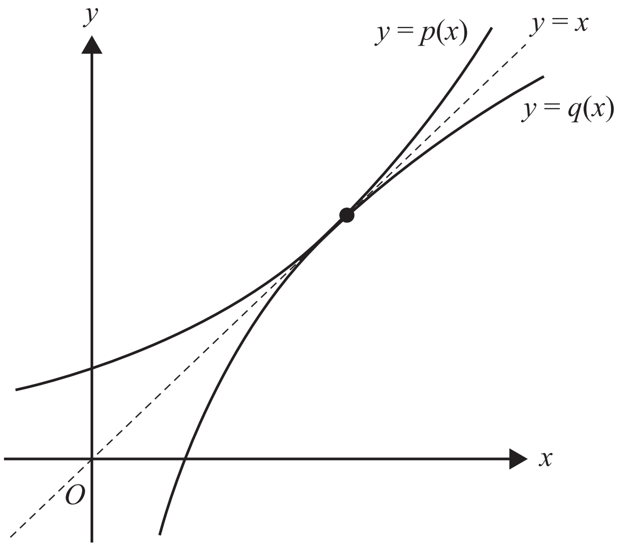
e. The shaded region below is enclosed by the axes, the graphs of f and g, and the line $y = -x + 2\log_e(2) + 2$.



Find the area of the shaded region. 2 marks

Let $p: R \to R,\ p(x) = e^{kx}$ and $q: R^+ \to R,\ q(x) = \frac{1}{k}\log_e(x)$.

f. The graphs of p, q and $y = x$ are shown in the diagram below. The graphs of p and q touch but do not cross.



Find the value of k. 2 marks

g. Find the value of k, $k > 0$, for which the tangent to the graph of p at its y-intercept and the tangent to the graph of q at its x-intercept are parallel. 1 mark

END OF QUESTION AND ANSWER BOOK

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VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2019

MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x}\,dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET