VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

MATHEMATICAL METHODS
Written examination 1

Friday 31 May 2019

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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DO NOT WRITE IN THIS AREA

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \dfrac{2e^{2x} - 1}{e^x}$.

Find $\dfrac{dy}{dx}$. 2 marks

b. Let $f(x) = x^2 \cos(3x)$.

Find $f'\left(\dfrac{\pi}{3}\right)$. 2 marks

Question 2 (2 marks)

Find $f(x)$ given that $f(1) = -\frac{7}{4}$ and $f'(x) = 2x^2 - \frac{1}{4}x^{-\frac{2}{3}}$.

Question 3 (4 marks)

a. Evaluate $\int_2^7 \frac{1}{x+\sqrt{3}}\,dx$ and $\int_2^7 \frac{1}{x-\sqrt{3}}\,dx$. 2 marks

b. Show that $\frac{1}{2}\left(\frac{1}{x-\sqrt{3}}+\frac{1}{x+\sqrt{3}}\right)=\frac{x}{x^2-3}$. 1 mark

c. Use your answers to **part a.** and **part b.** to evaluate $\int_2^7 \frac{x}{x^2-3}\,dx$ in the form $\frac{1}{a}\log_e(b)$, where a and b are positive integers. 1 mark

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Question 4 (8 marks)

A function g has rule $g(x) = \log_e(x - 3) + 2$.

a. State the maximal domain of g and the range of g over its maximal domain. 2 marks

b. i. Find the equation of the tangent to the graph of g at (4, 2). 2 marks

ii. On the axes on page 7, sketch the graph of the function g, labelling any asymptote with its equation. Also draw the tangent to the graph of g at (4, 2). 4 marks

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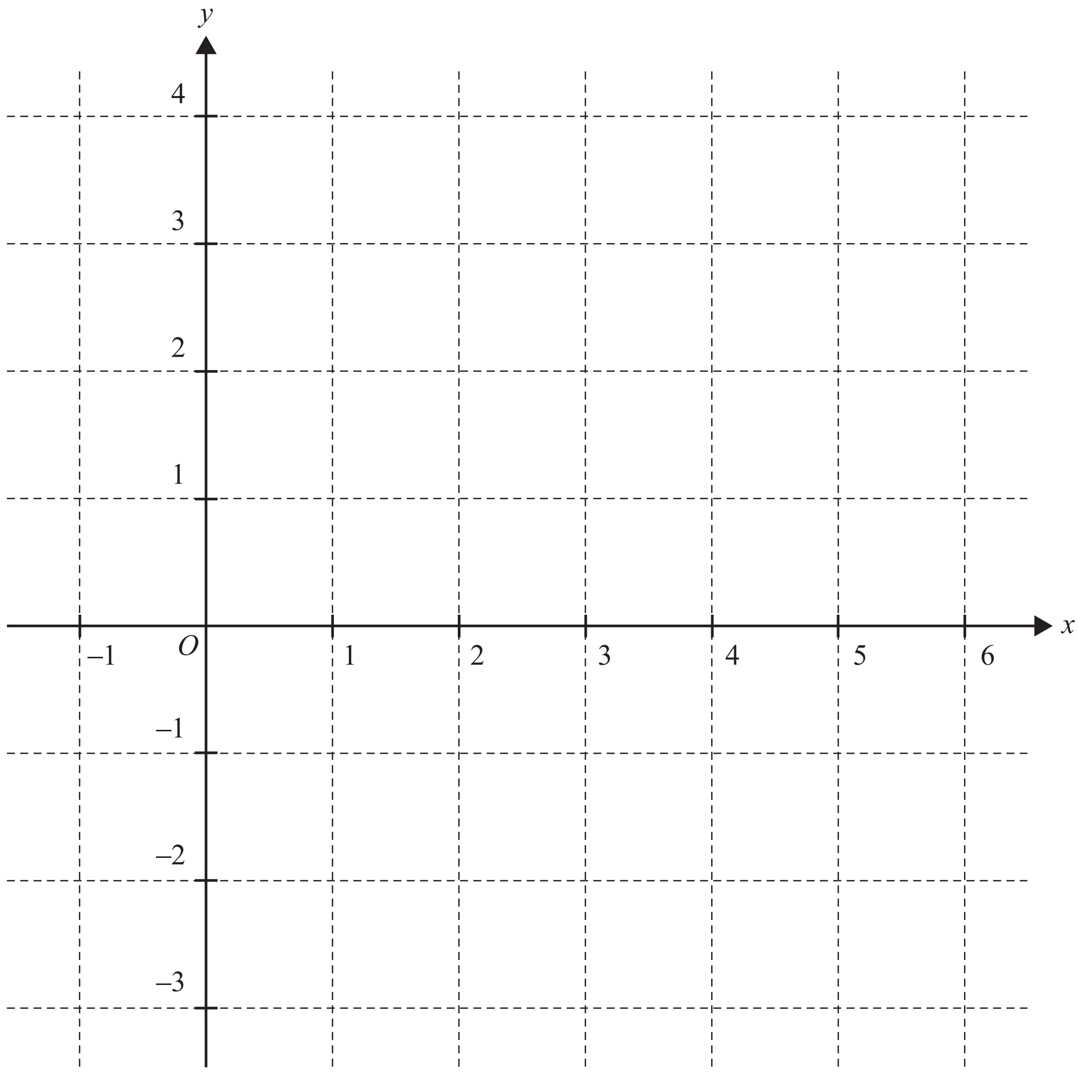

Question 5 (5 marks)

Let $h:\left[-\frac{3}{2}, \infty\right) \rightarrow R,\ h(x) = \sqrt{2x+3} - 2$.

a. Find the value(s) of x such that $[h(x)]^2 = 1$. 2 marks

b. Find the domain and the rule of the inverse function h^{-1}. 3 marks

Question 6 (4 marks)

Jacinta tosses a coin five times.

a. Assuming that the coin is fair and given that Jacinta observes a head on the first two tosses, find the probability that she observes a total of either four or five heads. 2 marks

b. Albin suspects that the coin Jacinta tossed is not actually a fair coin and he tosses it 18 times. Albin observes a total of 12 heads from the 18 tosses.

Based on this sample, find the approximate 90% confidence interval for the probability of observing a head when this coin is tossed. Use the z value $\frac{33}{20}$. 2 marks

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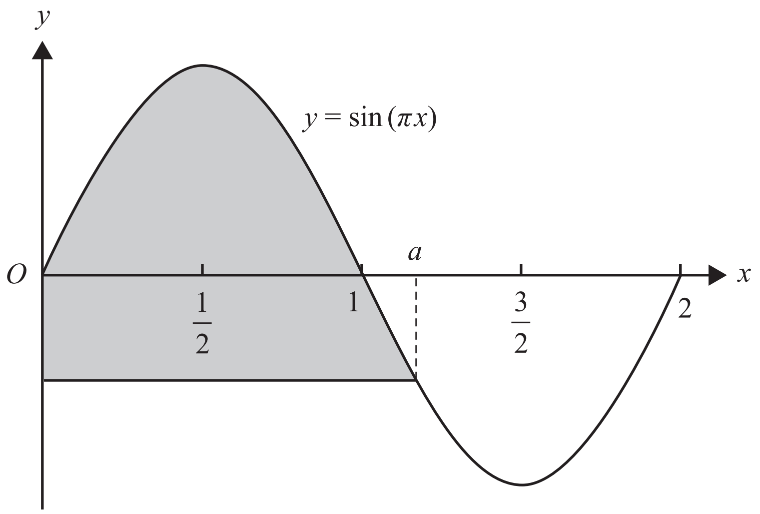
Question 7 (8 marks)

The shaded region in the diagram below is bounded by the vertical axis, the graph of the function with rule $f(x) = \sin(\pi x)$ and the horizontal line segment that meets the graph at $x = a$, where $1 \le a \le \frac{3}{2}$.



Let $A(a)$ be the area of the shaded region.

a. Show that $A(a) = \frac{1}{\pi} - \frac{1}{\pi}\cos(a\pi) - a\sin(a\pi)$. 3 marks

b. Determine the range of values of $A(a)$. 2 marks

c. **i.** Express in terms of $A(a)$, for a specific value of a, the area bounded by the vertical axis, the graph of $y = 2\left(\sin(\pi x) + \frac{\sqrt{3}}{2}\right)$ and the horizontal axis. 2 marks

ii. Hence, or otherwise, find the area described in **part c.i.** 1 mark

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Question 8 (5 marks)

A fair standard die is rolled 50 times. Let W be a random variable with binomial distribution that represents the number of times the face with a six on it appears uppermost.

a. Write down the expression for $\Pr(W = k)$, where $k \in \{0, 1, 2, \ldots, 50\}$. 1 mark

b. Show that $\dfrac{\Pr(W = k+1)}{\Pr(W = k)} = \dfrac{(50-k)}{5(k+1)}$. 2 marks

c. Hence, or otherwise, find the value of k for which $\Pr(W = k)$ is the greatest. 2 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET