2019 VCE Mathematical Methods 1 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have been included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a.

$y = 2e^{x} - e^{-x}$ so $\frac{dy}{dx} = 2e^{x} + e^{-x}$

Or $\frac{dy}{dx} = \frac{4e^{2x}e^{x} - (2e^{2x} - 1)e^{x}}{(e^{x})^{2}} = \frac{2e^{3x} + e^{x}}{e^{2x}}$ (quotient rule)

Some students used a combination of product and chain rules.

Question 1b.

$$f'(x) = 2x\cos(3x) - 3x^{2}\sin(3x)$$

$$f'(\tfrac{\pi}{3}) = -\frac{2\pi}{3}$$

Question 2

$$f(x) = \frac{2}{3}x^{3} - \frac{3}{4}x^{\frac{1}{3}} + c$$

where $c = f(1) - \frac{2}{3} + \frac{3}{4} = -\frac{7}{4} - \frac{2}{3} + \frac{3}{4} = -\frac{5}{3}$

So $f(x) = \frac{2}{3}x^{3} - \frac{3}{4}x^{\frac{1}{3}} - \frac{5}{3}$

Question 3a.

$$\int_{2}^{7} \frac{1}{x-\sqrt{3}}dx = \left[\log_{e}(x-\sqrt{3})\right]_{2}^{7} = \log_{e}\left(\frac{7-\sqrt{3}}{2-\sqrt{3}}\right)$$

$$\int_{2}^{7} \frac{1}{x+\sqrt{3}}dx = \left[\log_{e}(x+\sqrt{3})\right]_{2}^{7} = \log_{e}\left(\frac{7+\sqrt{3}}{2+\sqrt{3}}\right)$$

Question 3b.

$$\frac{1}{2}\left(\frac{1}{x-\sqrt{3}}+\frac{1}{x+\sqrt{3}}\right)$$

$$=\frac{1}{2}\left(\frac{\left(x+\sqrt{3}\right)+\left(x-\sqrt{3}\right)}{x^2-3}\right)$$

$$=\frac{x}{x^2-3}$$

Question 3c.

$$\int_2^7 \frac{x}{x^2-3}dx$$

$$=\frac{1}{2}\int_2^7 \frac{1}{x-\sqrt{3}}+\frac{1}{x+\sqrt{3}}\,dx$$

$$=\frac{1}{2}\log_e\left(\frac{(7-\sqrt{3})(7+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}\right)$$

$$=\frac{1}{2}\log_e(46)$$

Question 4a.

$g(x) = \log_e(x-3) + 2$

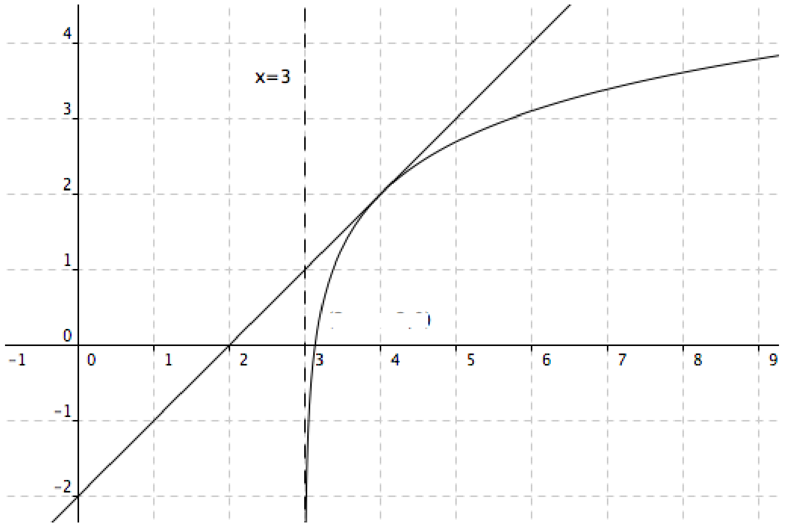
Domain: $x > 3$ or $(3,\infty)$
Range: R

Question 4bi.

$$g'(x)=\frac{1}{x-3}$$

Using $g(4)=2$ and $g'(4)=1$ the tangent is $y=x-2$

Question 4bii.



Question 5a.

$\left[(h(x))^2\right]=1$ so $h(x)=1$ *or* -1

$$\sqrt{2x+3}-2=1,-1$$
$$\sqrt{2x+3}=3,1$$
$$2x+3=9,1$$
$$x=-1,3$$

Both values are in the domain of h.

Question 5b.

Let $y=h^{-1}(x)$

$$x=\sqrt{2y+3}-2$$
$$x+2=\sqrt{2y+3}$$
$$(x+2)^2=2y+3$$
$$y=\frac{1}{2}(x+2)^2-\frac{3}{2}$$

Hence $h^{-1}(x)=\frac{1}{2}(x+2)^2-\frac{3}{2}$

Range: $[-2,\infty)$

Question 6a.

Since first two tosses are heads, required probability is

Pr(2 heads out of next 3)+Pr(3 heads out of next 3)

$$=\binom{3}{2}\left(\frac{1}{2}\right)^3+\binom{3}{3}\left(\frac{1}{2}\right)^3=\frac{3+1}{8}=\frac{1}{2}$$

Use of a tree diagram or any other appropriate method was accepted.

Question 6b.

$$\left(\frac{2}{3}-\frac{33}{20}\sqrt{\frac{2}{3}\times\frac{1}{3}\times\frac{1}{18}},\frac{2}{3}+\frac{33}{20}\sqrt{\frac{2}{3}\times\frac{1}{3}\times\frac{1}{18}}\right)$$

$$=\left(\frac{2}{3}-\frac{33}{20}\sqrt{\frac{1}{9^2}},\frac{2}{3}+\frac{33}{20}\sqrt{\frac{1}{9^2}}\right)$$

$$=\left(\frac{29}{60},\frac{51}{60}\right)$$

Question 7a.

$$A(a)=\int_0^a\left(\sin\left(\pi x\right)-\sin\left(\pi a\right)\right)dx$$

$$=\left[-\frac{\cos\left(\pi x\right)}{\pi}-x\sin\left(\pi a\right)\right]_0^a$$

$$=\frac{-\cos\left(\pi a\right)+1}{\pi}-a\sin\left(\pi a\right)$$

$$=\frac{1}{\pi}-\frac{1}{\pi}\cos\left(a\pi\right)-a\sin\left(a\pi\right)$$

Question 7b.

$$A(1)=\frac{2}{\pi},\quad A\left(\frac{3}{2}\right)=\frac{1}{\pi}+\frac{3}{2}$$

Range: $\left[\frac{2}{\pi},\frac{2+3\pi}{2\pi}\right]$

Question 7ci.

$$\text{Area}=\int_0^{\frac{4}{3}}(2\sin(\pi x)+\sqrt{3})dx$$

$$=2\int_0^{\frac{4}{3}}(\sin(\pi x)+\frac{\sqrt{3}}{2})dx$$

$$=2\int_0^{\frac{4}{3}}(\sin(\pi x)-\sin(\frac{4\pi}{3}))dx$$

$$=2A(a)\text{ with }a=\frac{4}{3}$$

Or observe that required area is a dilation by factor 2 of the original area, width $a=\frac{4}{3}$

Question 7cii.

$$A\left(\frac{4}{3}\right) = \frac{4}{\sqrt{3}} + \frac{3}{\pi}$$
$$= \frac{4\pi\sqrt{3}+9}{3\pi}$$

Question 8a.

$$\Pr\left(W = k\right) = \binom{50}{k}\left(\frac{1}{6}\right)^{k}\left(\frac{5}{6}\right)^{50-k}$$

Question 8b.

$$\frac{\Pr(W = k+1)}{\Pr(W = k)} = \binom{50}{k+1}\frac{1}{6}^{k+1} \times \frac{5}{6}^{49-k} / \left(\binom{50}{k}\frac{1}{6}^{k} \times \frac{5}{6}^{50-k}\right)$$
$$= \frac{(k)!\times(50-k)!}{(k+1)!\times\left(49-k\right)!} \times \frac{1}{6} \times \frac{6}{5}$$
$$= \frac{(50-k)}{5\left(k+1\right)}$$

Question 8c.

$$\Pr(W = k+1) < \Pr(W = k)$$
$$(50-k) > 5(k+1)$$
$$k > \frac{45}{6} \quad \left(\frac{15}{2}\right)$$

Hence

$$\Pr(W = 7) < \Pr\left(W = 8\right)$$
$$\Pr(W = 9) < \Pr(W = 8)$$

So greatest for $k = 8$

Or by argument from features of binomial distribution.