TSSM
Creating VCE Success

2019 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

									Letter

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference book, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A
Answer **all** questions. Choose the response that is correct for the question. A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers. No marks will be given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f: R \to R, f(x) = -2\cos\left(\frac{x-\pi}{4}\right) - 1$

The period and range, respectively, for this function are

A. 8π and $[-3, -1]$

B. $\frac{\pi}{2}$ and $[-3, 1]$

C. 8π and $[-3, 1]$

D. $\frac{\pi}{2}$ and $[-3, -1]$

E. π and $[1, 3]$

Question 2

Part of the graph of $y = f(x)$ is shown at right.
The equation of $f(x)$ could be

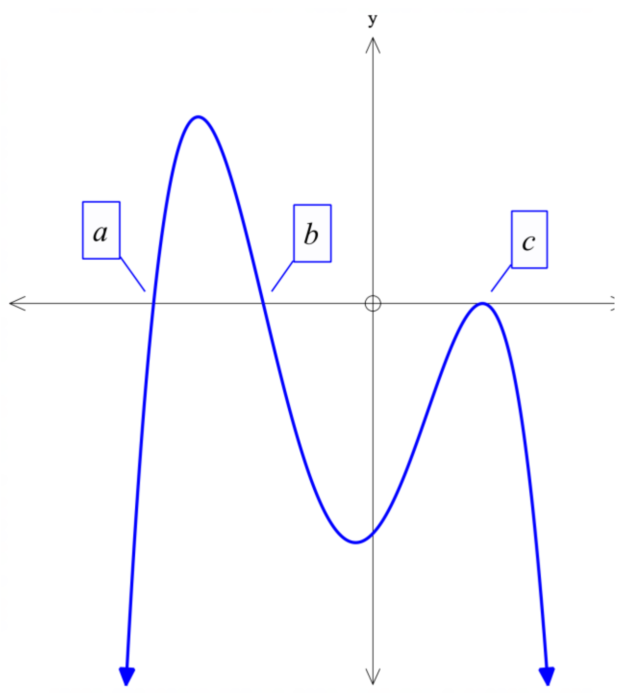
A. $f(x) = -(x+a)(x+b)(x-c)^2$

B. $f(x) = (a-x)(x-b)(x-c)^2$

C. $f(x) = -(x-a)(x-b)(x-c)$

D. $f(x) = -(x+a)(x+b)(x+c)^2$

E. $f(x) = (x-a)(x-b)(x-c)^2$



SECTION A – continued

Question 3
If $y = \ln(1 - 3x) + 1$, then x is equal to

A. $\frac{(e^y - e)e^y}{3}$
B. $\frac{1}{3} - \frac{e^y}{3e}$
C. $\frac{1 - e^{y-1}}{3}$
D. $\frac{1}{3} - \frac{e}{3e^{-y}}$
E. $\frac{1}{3} - \frac{1}{3e \times e^y}$

Question 4
The inverse, g^{-1}, of the function defined by $g: (-\infty, 3) \to R, g(x) = \frac{1}{(x-3)^2}$, is

A. $g^{-1}: (0, \infty) \to R, g^{-1}(x) = \frac{1}{3-x}$
B. $g^{-1}: (-\infty, 3) \to R, g^{-1}(x) = 3 - \frac{1}{\sqrt{x}}$
C. $g^{-1}: (0, \infty) \to R, g^{-1}(x) = 3 + \frac{1}{\sqrt{x}}$
D. $g^{-1}: (-\infty, 3) \to R, g^{-1}(x) = x - 3$
E. $g^{-1}: (0, \infty) \to R, g^{-1}(x) = 3 - \frac{1}{\sqrt{x}}$

Question 5
Let $f: [0,10] \to R, f(x) = -x + 10$ and $g: R \to R, g(x) = e^{-x}$
The range of the function $g(f(x))$

A. $[e^{-10}, 1]$
B. R
C. $[0, 10]$
D. $(0, \infty)$
E. $[9, 10 - e^{-10}]$

SECTION A – continued
TURN OVER

Question 6
The simultaneous equations given below,

$$mx + 5y = 1$$
$$x + (m+4)y = m$$

where m is a real constant, have infinitely many solutions if

A. $m = -5$
B. $m \in \{-5, 1\}$
C. $m \in R$
D. $m = 1$
E. $m \in R\backslash\{-5, 1\}$

Question 7
The sum of the solutions of $-\sqrt{3}\sin(2x) = \cos(2x)$for $x \in [0,2\pi]$ is

A. $\frac{5\pi}{12}$
B. $\frac{14\pi}{3}$
C. $\frac{11\pi}{4}$
D. $\frac{10\pi}{3}$
E. $\frac{7\pi}{4}$

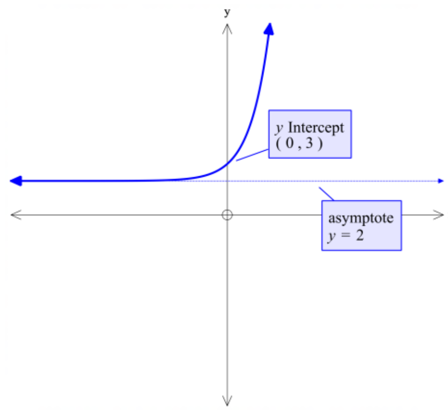
Question 8
If events A and B are independent and $\Pr(A) = 0.3$ and $\Pr(B) = 0.45$, then $\Pr(A \cup B)$ equals

A. 0.135
B. 0.865
C. 0.75
D. 0.615
E. 0.25

SECTION A – continued

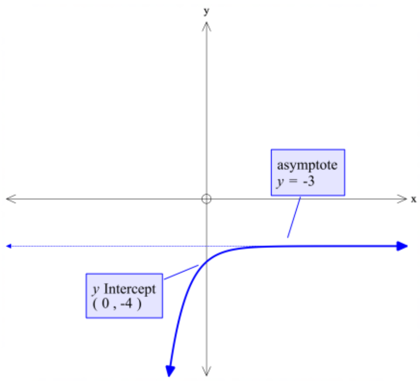
Question 9

The graph of $f(x)$ is shown to the right

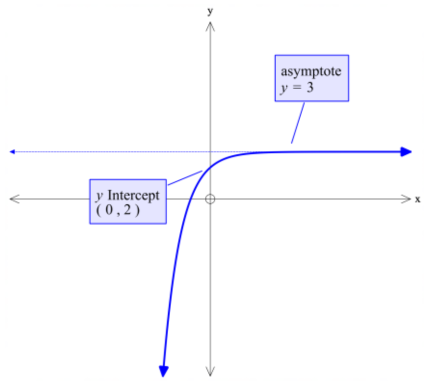


Which of the following shows $-f(-x)+1$?

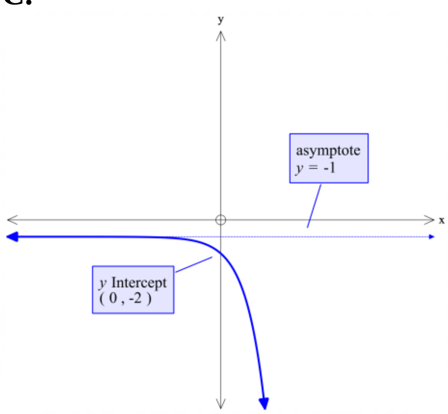
A.



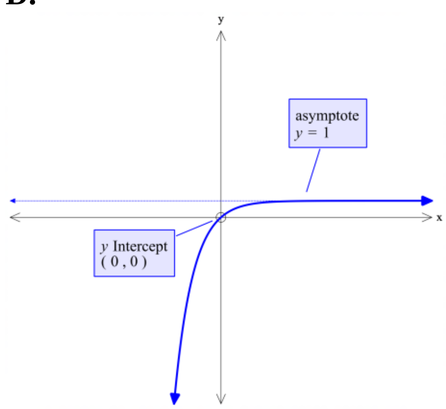
B.



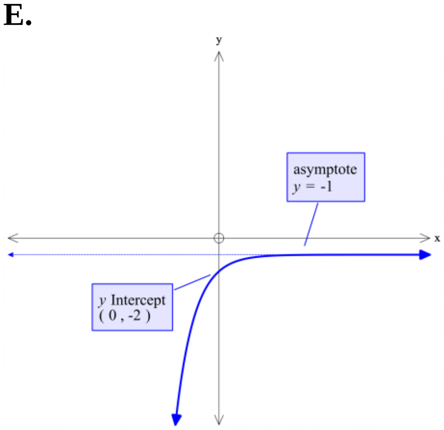
C.



D.



E.



Question 10
The equation perpendicular to a tangent to the curve of $y = (2x+1)^3$ at the point of $(-1,-1)$ can be expressed as

A. $y = 6x + 7$
B. $y = -\frac{x}{6} + 7$
C. $y = -\frac{x}{6} - \frac{7}{6}$
D. $y = 6x - 1$
E. $y = -\frac{x}{6} - 1$

Question 11
Which of the following functions makes $f(x+y) = f(x)f(y)$ true?

A. $f(x) = 2x + 3$
B. $f(x) = 2x^3$
C. $f(x) = \sin(2x)$
D. $f(x) = \frac{2}{3x}$
E. $f(x) = 3^{2x}$

Question 12
If a fair die is rolled 10 times, the probability that no more than four 5's are obtained, correct to 3 decimal places, is closest to

A. 0.985
B. 0.930
C. 0.984
D. 0.054
E. 0.983

Question 13
The transformation matrix $T: R^2 \to R^2$ which maps the curve with the equation $y = \frac{1}{x^2}$ to the curve with the equation $y = \frac{-2}{(x+3)^2} + 4$, could be

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

SECTION A – continued

Question 14
Consider the following discrete probability distribution for the random variable X

x	1	2	3	4	5
$\Pr(X = x)$	p	$2p$	p	$\frac{p}{2}$	$\frac{3p}{2}$

The mean of this distribution is

A. $\frac{35}{12}$
B. $\frac{1}{6}$
C. $\frac{35}{6}$
D. 3
E. $\frac{1}{2}$

Question 15
A cubic function h has a local maximum at $(m, 4)$ and a local minimum at $(n, 1)$.
For $h(x) - p = 0$ to have exactly three solutions, then the values p can be
A. $p > 1$
B. $p > 4$ or $p < 1$
C. $-4 \leq p \leq -1$
D. $1 \leq p \leq 4$
E. $1 < p < 4$

Question 16
If $f(x) = \log_e(x^2 + 2)$ and $f'(x) = \frac{2x}{x^2+2}$, then the antiderivative of $g(x) = \frac{3x}{x^2+2}$ is equal to

A. $3 \times \int \frac{2x}{x^2+2}\, dx + c$
B. $6 \log_e(x^2 + 2) + c$
C. $\log_e(x^2 + 2) + c$
D. $\frac{3}{2} \log_e(x^2 + 2) + c$
E. $\frac{1}{2} \log_e(x^2 + 2) + c$

Question 17
A random sample of 50 students were sampled and asked about their preferred flavour of milk. Eight students said they preferred strawberry over chocolate. A 95% confidence interval for the proportion of students in the population who prefer chocolate flavoured milk is given by

A. $(0.738, 0.942)$
B. $(0.058, 0.262)$
C. $(0.16, 0.84)$
D. $(0.058, 0.261)$
E. $(0.738, 0.941)$

SECTION A – continued
TURN OVER

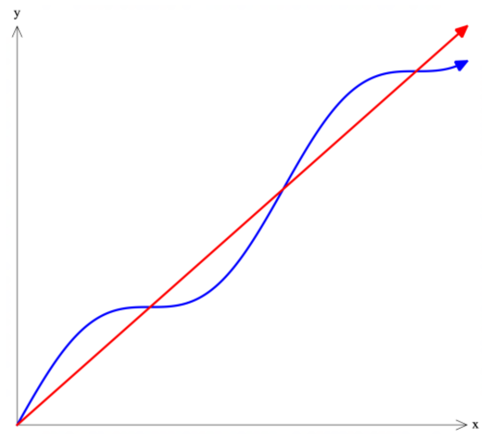
Question 18
The average value of $h(x) = \sin^2(x)$ from $[-a, a]$ equals 0.4
The value of a is approximately

A. 0.4246
B. 1.2979
C. 1.1636
D. 1.2989
E. 2.0523

Question 19
With normal use, a certain brand of pen runs out of ink X weeks after production, where X is normally distributed random variable. The variable X has a mean of 14 weeks and a standard deviation of σ weeks. From a sample of 400 pens, there are 10 which are expected to run out in the first 6 weeks. The value of σ is closest to

A. 1 week
B. 3 weeks
C. 4 weeks
D. 6 weeks
E. 7 weeks

Question 20
The graphs of $f: R \to R, f(x) = x$ and $g: R \to R, g(x) = x + \sin(x)$ are shown in the diagram below.



For the region of $x \in [0, 3\pi]$, an expression for the area between the two graphs is

A. $\int_0^{3\pi} (g(x) - f(x))\, dx$
B. $2\int_0^{\pi} (g(x) - f(x))\, dx + \int_{2\pi}^{3\pi} \big(f(x) - g(x)\big)\, dx$
C. $3\int_{2\pi}^{3\pi} \big(f(x) - g(x)\big)\, dx$
D. $\int_0^{\pi} \big(g(x) - f(x)\big)\, dx + \int_{\pi}^{2\pi} \big(g(x) - f(x)\big)\, dx + \int_{2\pi}^{3\pi} \big(g(x) - f(x)\big)\, dx$
E. $3\int_0^{\pi} \big(g(x) - f(x)\big)\, dx$

END OF SECTION A

SECTION B – Extended response questions

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (9 marks)

The number of bacteria $B(t)$ in a particular petri dish in a laboratory experiment can be modelled by the rule $B(t) = Ae^{kt}$, where t is the time in minutes after the experiment started. It was found that after 23 minutes there was double the amount of bacteria compared to at the start of the experiment.

a. Show that $k = 0.03$. 3 marks

b. If the population of bacteria was 15000 after one hour, calculate the initial number of bacteria correct to the nearest whole number. 2 marks

c. Find $B(90)$, correct to the nearest whole number. 1 mark

SECTION B – Question 1 – continued
TURN OVER

Another petri dish with the same bacteria was also included in the experiment, but this second lot of bacteria was found to follow the rule $P(t) = 3900e^{0.02t}$.

d. State the initial amount of bacteria found in this second petri dish $P(t)$. 1 mark

e. Calculate the time, correct to the nearest second, at which the two petri dishes contain the same amount of bacteria. 2 marks

Total 9 marks

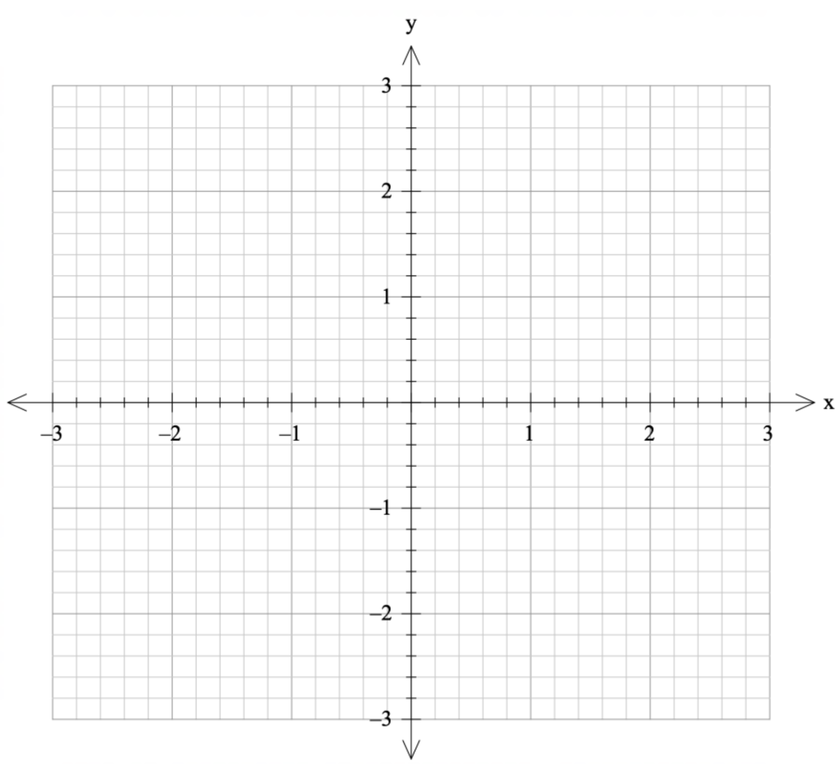
SECTION B – continued

Question 2 (15 marks)
Consider the quartic function $f: R \rightarrow R, f(x) = 2x^4 - x^3 - 3x^2$.

a. State the coordinates of the x-intercepts of $f(x)$. 1 mark

b. Find the coordinates of the stationary points of $f(x)$, and state their nature.
Give your answer correct to two decimal places 4 marks

c. Sketch the graph of $f(x)$ on the axes provide, label all key features. 3 marks



SECTION B – Question 2 – continued
TURN OVER

d. Determine the value(s) of c, for which $f(x) + c$ has no x-intercepts. 1 mark

A tangent to the function $f(x)$ touches the graph at $x = 1$.

e. Determine the equation of the tangent. 2 marks

The tangent intersects the function $f(x)$ at two other points.

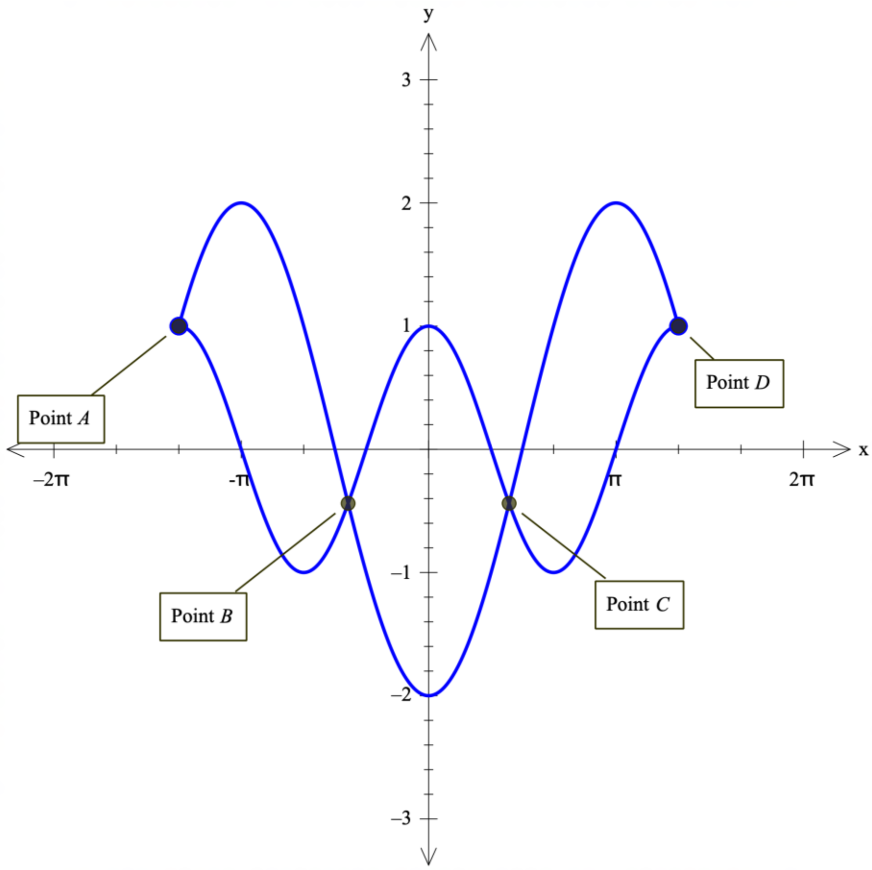
f. State the coordinates of these points of intersection between the function $f(x)$ and the tangent. 2 marks

g. Calculate the area bound between the tangent and the graph of $f(x)$. 2 marks

Total 15 marks

SECTION B – continued

Question 3 (12 marks)
Jim is designing a feature garden bed in his backyard which is based on the graphs of the functions $f(x) = \cos\left(\frac{3}{2}x\right)$ and $g(x) = -2\cos(x)$, within the domain $-2\pi \leq x \leq 2\pi$.
The sketch he made is shown in the diagram below, in which the measurements in the x and y directions are in metres.



Jim needs to finalise his plans for the garden bed before he can purchase the soil and plants he needs.

a. State the period and amplitude of

i. $f(x)$.

___

___

ii. $g(x)$.

___

___

2 + 2 = 4 marks

SECTION B – Question 3 – continued
TURN OVER

The functions $f(x)$ and $g(x)$ intersect at points a, b, c and d within the domain $-2\pi \leq x \leq 2\pi$.

b. Find the coordinates of points a, b, c and d correct to 3 decimal places. 3 marks

c. State the domain of both functions. 1 mark

d. State the range of

i. $f(x)$.

ii. $g(x)$.

1 + 1 = 2 marks

The area where the garden bed needs to be cleared of grass before new top soil can be added. This area is bounded by the two graphs between the points a and d.

e. Calculate the area, to the nearest square metre, that needs to be cleared. 2 marks

Total 12 marks

SECTION B – continued

Question 4 (15 marks)
Jim is planting broccoli seeds in his vegetable garden during early Autumn for a Winter harvest. Broccoli seeds can be purchased in packets of 50, 500 and 1000. The probability that the particular brand of broccoli seed Jim uses fails to sprout is $\frac{1}{8}$.

a. Calculate the expected number of seeds to sprout in the packet of 1000 seeds 2 marks

Jim purchases a packet of 500 broccoli seeds and plants 400 of them according to the directions on the packet.

b. Determine the probability that no more than 355 seeds sprout, correct to 3 decimal places.
2 marks

Jim notices that more than $\frac{1}{4}$ of the seeds he planted have already sprouted in the first week after planting.

c. Given this, calculate the probability that less than 350 of the seeds will spout, correct to 3 decimal places.
2 marks

SECTION B – Question 4 – continued
TURN OVER

In April, Jim would like to water his garden twice each day, once in the morning before the sun rises and once in the afternoon. However, Jim knows it is best to water his garden when it is not too hot so that the water has time to soak into the soil and if it is too hot the water will just evaporate.

He checks the MyWeatherApp on his phone which says that during the month of April there is a 35% chance that each day the temperature will be at the right temperature for watering his garden, and this is independent of whether it has rained the day before.

d. If each days temperature in independent of the day before it, determine the probability that for at least 10 days during the month of April the conditions are right for watering in the afternoon. Give your answer correct to three decimal places. 2 marks

The optimum temperature for Jim to water in the afternoon is less than $20°C$. The temperature of each day during the month is normally distributed. It is found that 9% of the days have a temperature of less than $16.57°C$, while 20% of the days have a temperature of more than $25.08°C$.

e. Determine the mean and standard deviation for the temperature of the month of April, correct to two decimal places. 3 marks

SECTION B – Question 4 - continued

At time of harvest, Jim has noted that his broccoli plants are all different sizes. According to the suppliers website the height of the plants are normally distributed with a mean of 47.2cm and a standard deviation of 3.6cm.

Jim contacts his friends who have also planted the same broccoli seeds and gathers a sample of 150 plants. For samples of 150 from the population with mean 47.2cm and standard deviation 3.6cm, $\hat{P}$ is the random variable of the distribution of sample proportions of broccoli plants that are smaller than 42cm.

f. Calculate the probability, correct to four decimal places, that $\Pr(\hat{P} \geq 0.05 | \hat{P} > 0.02)$. Do not use normal approximation. 3 marks

From the sample of 150, it was found that 10 broccoli plants were less than 42cm tall.

g. Calculate the 95% confidence interval for Jim's estimate of the population proportion, correct to four decimal places 1 mark

Total 15 marks

SECTION B – continued
TURN OVER

Question 5 (9 marks)
Consider the function $f: R \to R, f(x) = -x^2(x-n)(x+n)$, where n is a positive real number

a. Find the coordinates of the stationary points of f in terms of n. 1 mark

The area bounded between the curve of $f(x)$ and the x-axis, between the x-intercepts of f, is equal to $\frac{4}{15}$.

b. Find the value of n, and hence state the x-intercepts of $f(x)$. 2 marks

Consider the graph of the function $g: R \to R, g(x) = -mx^2 + m$, where m is a positive real number,

c. Show that $g(x)$ has two of the same x-intercepts as the function $f(x)$. 1 mark

d. Find the area enclosed by the graphs of $f(x)$ and $g(x)$ in terms of m. 2 marks

SECTION B – Question 5 – continued

e. The graph of $f(x)$ has a tangent at the point $x = b$.

i. Evaluate $f'(b)$.

ii. Find the equation of the tangent to the graph at $x = b$.

1 + 2 = 3 marks

Total 9 marks

END OF QUESTION AND ANSWER BOOK