Presbyterian Ladies' College

MELBOURNE

VCE
Year 11 + 10 Accelerated

2019 Semester 2
Mathematical Methods Unit 2 Examination

PAPER 1

Class/Teacher: **Lang** **Lewis** **Mendan** **Smith** **Taylor**

10MATHAC/Lewis

(Please circle)

Name: ANSWERS. **Form:** ________

READING TIME: (15 minutes in total for Papers 1 + 2)
WRITING TIME: (40 minutes)

Section	Number of Questions	Number of Questions to be answered	Number of Marks
A	9	9	40

No. of Pages: 9

Instructions

1. No calculators are permitted.
2. No notes are permitted.
3. You can refer to the formula sheet provided.

SECTION A: Short Answer: All questions should be answered in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A: **Short Answer: No notes and no calculators allowed.**
Exact answers should be given unless instructed otherwise.

Question 1 (3 marks)

At a country show, Billy plays a game of chance that involves trying to knock a pineapple off a stand by throwing a ball. He has three throws and on each throw, the probability of success is $\frac{1}{5}$.

a. What is the probability that Bill knocks the pineapple off twice?

2 marks

$^3C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)$

$= {}^{12}/_{125}$

b. What is the probability that he knocks the pineapple off with his last throw only?

1 mark

${}^4/_5 \times {}^4/_5 \times {}^1/_5$

$= {}^{16}/_{125}$

Question 2 (3 marks)

A workplace has 6 female and 5 male employees.

a. In how many ways can two employees fill the positions of manager and assistant manager?

1 mark

$11 \times 10 = 110$

b. In how many ways can an advisory committee of 5 be chosen so that there are 3 women and 2 men?

2 marks

$^6C_3 \times {}^5C_2$

$= 20 \times 10$

$= 200$

Question 3 (6 marks)

a. Let $g(x) = \frac{5}{x^2} + 2\sqrt{x} + 1, \quad x > 0.$

Differentiate g with respect to x.

2 marks

$g(x) = 5x^{-2} + 2x^{1/2} + 1$

$g'(x) = -10x^{-3} + x^{-1/2}$

b. Let $h: R \rightarrow R, \quad h(x) = 2x^2(x^2 - 1)$

Evaluate $h'(-1)$

2 marks

$h(x) = 2x^4 - 2x^2$

$h'(x) = 8x^3 - 4x$

$h'(-1) = -4$

c. If $y = (2x - x^3)^5$, find $\frac{dy}{dx}$

2 marks

$\frac{dy}{dx} = 5(2-3x^2)(2x-x^3)^4$

Question 4 (4 marks)

a. If $f'(x) = 3x^2 - 2$ and $f(1) = 2$, find $f(x)$

2 marks

$f(x) = \int (3x^2 - 2)\,dx$

$f(x) = x^3 - 2x + c$

sub $(1,2)$ $\quad 2 = 1 - 2 + c$

$f(x) = x^3 - 2x + 3$

b. Evaluate $\int_1^2 \frac{x^3 - 3x}{x}\,dx$

2 marks

$\int_1^2 x^2 - 3\,dx$

$= \left[x^3/3 - 3x \right]_1^2$

$= (8/3 - 6) - (1/3 - 3)$

$= -2/3$

Question 5 (3marks)

Solve the equation $2\log_5(a) - \log_5 24 + \log_5 6 = 0$ for a.

$\log_5 a^2 - \log_5 24 + \log_5 6 = 0$

$\log_5 a^2/4 = 0$

$a^2/4 = 1$

$a^2 = 4$

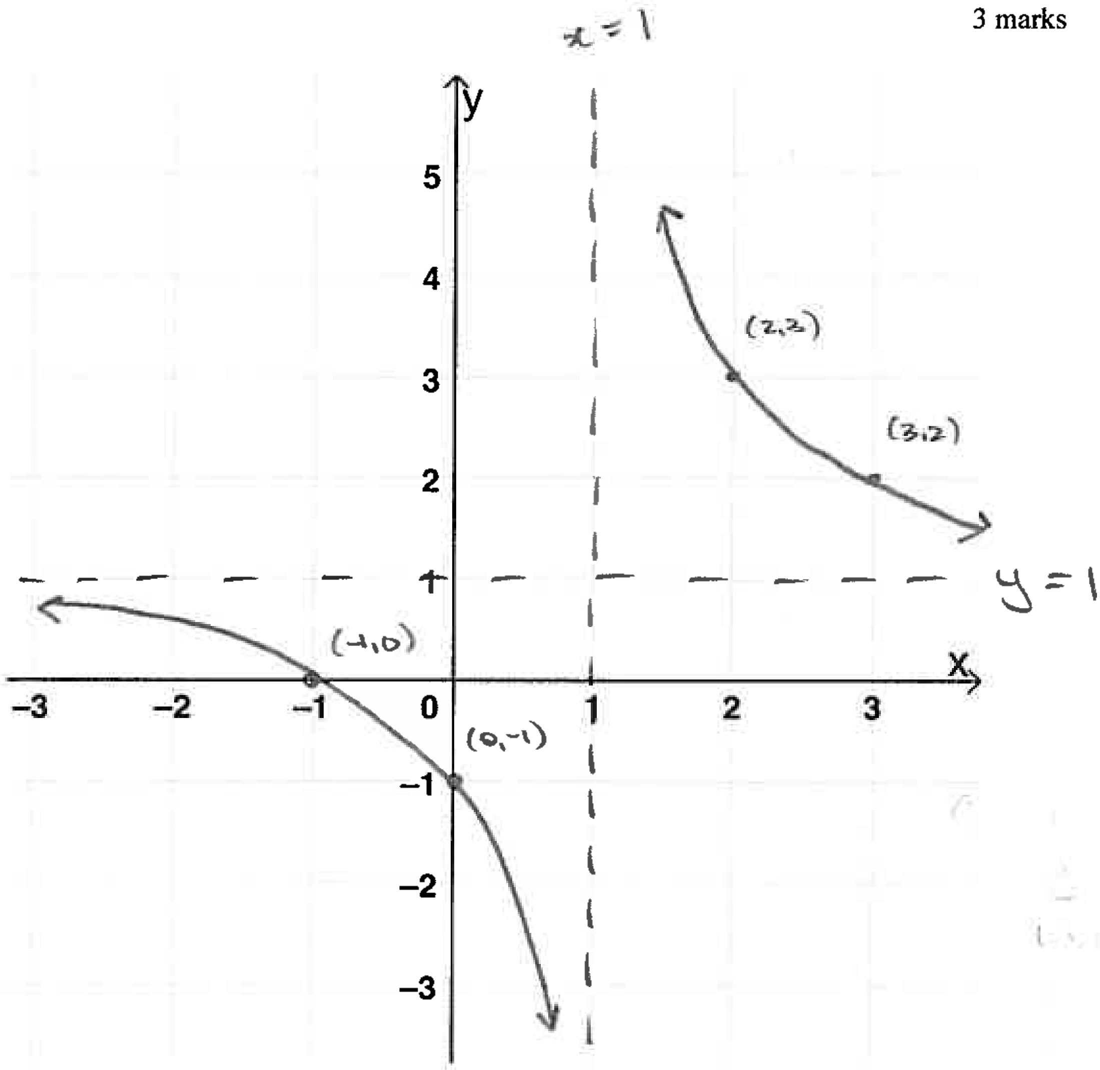
$a = \pm 2$

$a > 0$ so $a = 2$

Question 6 (5 marks)

a. Sketch the graph of $y = \frac{2}{x-1} + 1$ on the axes below. Label the asymptotes. The coordinates of the axial intercepts should also be labelled.

3 marks



b. State the domain and range of the graph.

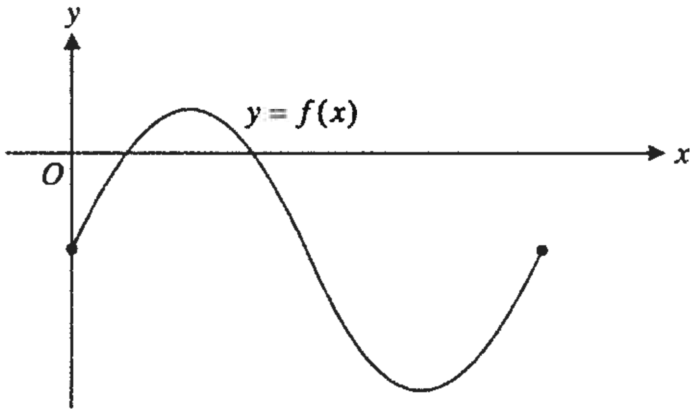
2 marks

dom $x \in R \setminus \{1\}$

ran $y \in R \setminus \{1\}$

Question 7 (6 marks)

Let $f: [0, a] \rightarrow R, \ f(x) = \sqrt{2}\sin(4x) - 1$
The graph of f is shown below.



a. Find a, given that $f(0) = f(a)$

1 mark

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$a = \frac{\pi}{2}$

b. Solve $f(x) = 0$, for x.

2 marks

$\sqrt{2}\sin(4x) - 1 = 0$

$\sin(4x) = \frac{1}{\sqrt{2}}$

$4x = \frac{\pi}{4}, \frac{3\pi}{4}$

$x = \frac{\pi}{16}, \frac{3\pi}{16}$

c. Find the gradient of the line segment connecting the points where $x = 0$ and $x = \frac{\pi}{12}$

Express your answer in the form $\frac{m\sqrt{m}}{n}$ where m and n are constants.

3 marks

$x = 0 \quad y = -1$

$x = \frac{\pi}{12} \quad y = \frac{\sqrt{6}}{2} - 1$

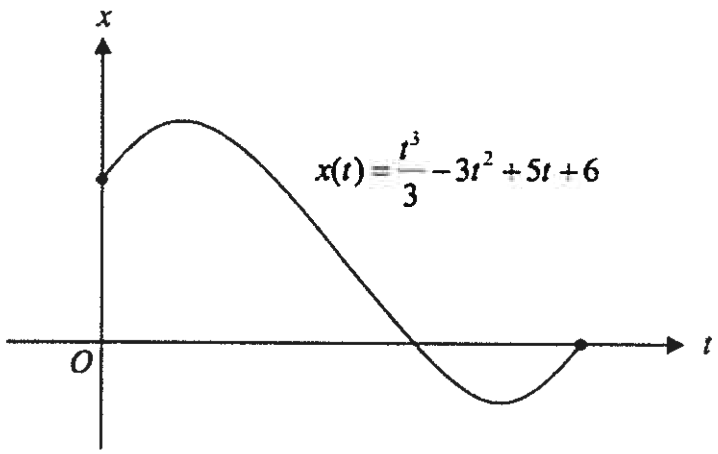
$m = \frac{\frac{\sqrt{6}}{2} - 1 + 1}{\frac{\pi}{2} - 0}$

$m = \frac{6\sqrt{6}}{\pi}$

Question 8 (4 marks)

A particle moves in a straight line such that its position, x centimetres from a fixed origin at O, at time t seconds, is given by $x(t) = \frac{t^3}{3} - 3t^2 + 5t + 6,\ t \in [0,6]$.

A graph of this function is shown below.



a. At what times did the particle change direction?

2 marks

$v(t) = t^2 - 6t + 5$

$v(t) = 0 \qquad t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 5$ or $t = 1$ changed direction

b. What is the total distance travelled by the particle when it **first** reaches the origin?

2 marks

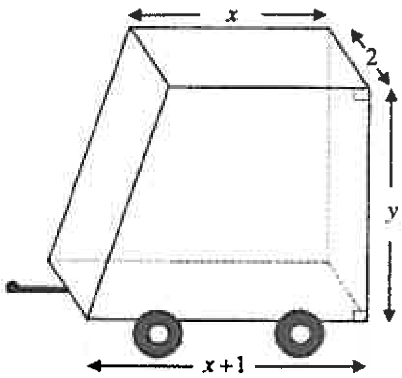
$x(0) = 6$

$x(1) = 8\frac{1}{3}$

Distance $= 2\frac{1}{3} + 8\frac{1}{3}$

$= 10\frac{2}{3}$ cm

Question 9 (6 marks)
An enclosed trailer in the shape of a trapezoidal prism is shown below.



The length of the prism along the top of the trailer is x metres and along the base it is $x + 1$ metres. The height of the trailer is y metres and the width is two metres.
The length of the top of the trailer together with the height, must add to equal 6 metres.

a. Show that the volume of the trailer, V, in cubic metres, is given by $V = -2x^2 + 11x + 6$

3 marks

$x + y = 6 \Rightarrow y = 6 - x$

$V = \frac{1}{2}(x + x + 1)y \times 2$

$= (2x+1)(6-x)$

$= -2x^2 + 12x + 6 - x$

$= -2x^2 + 11x + 6$

b. Find the value of x for which the volume is a maximum.

2 marks

$\frac{dV}{dx} = -4x + 11$

Let $-4x + 11 = 0$

$x = 11/4$ cm

c. Find the maximum volume of the trailer.

1 mark

$V = -2\left(\frac{11}{4}\right)^2 + 11\left(\frac{11}{4}\right) + 6$

$= 169/8$

End of Section A

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Name

ANSWERS

Teacher

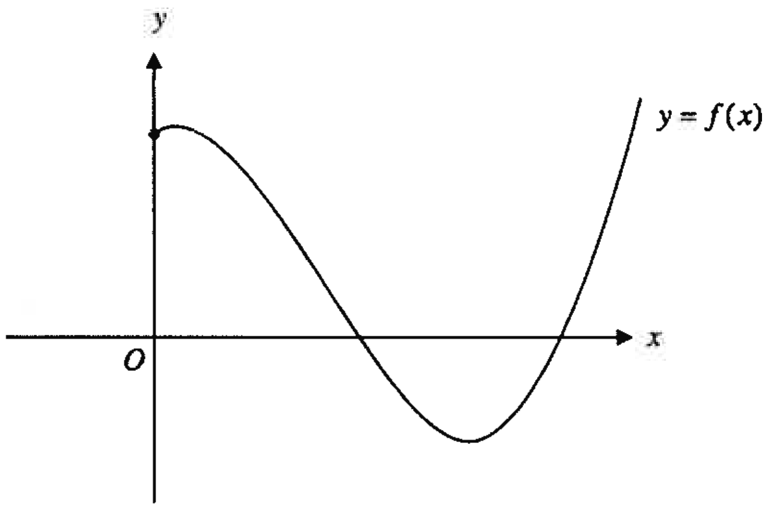
1	A	●	C	D	E
2	●	B	C	D	E
3	A	B	C	●	E
4	A	B	C	●	E
5	A	B	C	D	●
6	A	B	●	D	E
7	●	B	C	D	E
8	A	B	●	D	E
9	A	B	C	●	E
10	A	B	C	●	E
11	A	B	●	D	E
12	A	B	●	D	E
13	●	B	C	D	E
14	A	B	●	D	E
15	A	●	C	D	E
16	●	B	C	D	E
17	A	●	C	D	E
18	A	●	C	D	E
19	A	B	C	●	E
20	A	B	C	D	●

2019 MM EXAM 2 (0940)

Section C: Analysis: Notes and calculators allowed.
Exact answers should be given unless instructed otherwise.

Question 1 (15 marks)

Let $f: [0, \infty) \to R, f(x) = x^3 - 5x^2 + 2x + 8$.
Part of the graph of f is shown below.



a. Find the **coordinates** of the y-intercept of the graph.

1 mark

$(0, 8)$

b. Find the **coordinates** of the x-intercepts of the graph.

2 marks

$(2, 0)$ $(4, 0)$

c. Find the derivative of f and hence find the exact value of the **x-coordinates** of the turning points

3 marks

$f'(x) = 3x^2 - 10x + 2$

Let $f'(x) = 0$ $3x^2 - 10x + 2 = 0$

$x = \frac{5 \pm \sqrt{19}}{3}$

d. Find the range of f, expressing any endpoints of the interval in **exact** form.

2 marks

range $\left[\frac{56-38\sqrt{19}}{27}, \infty\right)$

e. Find the gradient of the tangent to the graph of f, at the point where $x = 1$.

1 mark

-5

f. Find the equation of the tangent to f at the point where $x = 1$.

1 mark

$y = 11 - 5x$

g. The tangent to the curve at $x = 1$ crosses the curve of f at one other point. Find the coordinates of this point.

2 marks

$11 - 5x = f(x)$

$(3, -4)$

h. Evaluate the integral $\int_0^2 f(x)dx$.

1 mark

$32/3$

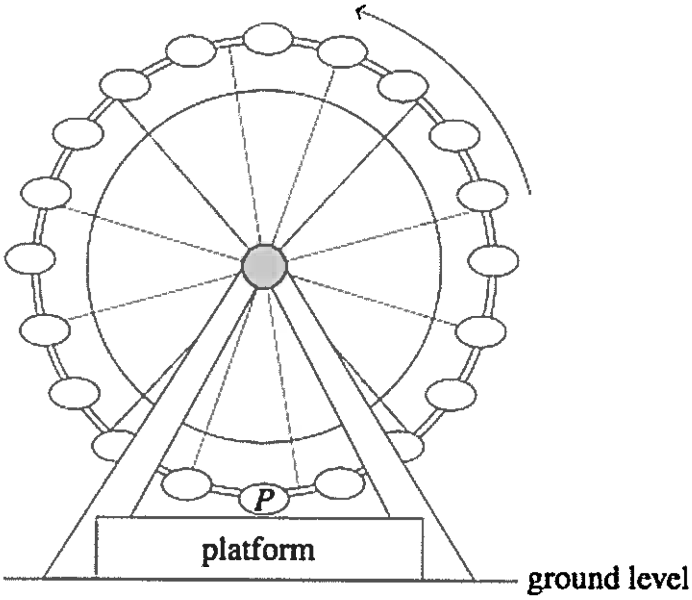
i. Find the exact area bound by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 5$

2 marks

$A = \int_0^2 f(x)\,dx - \int_2^4 f(x)\,dx + \int_4^5 f(x)\,dx$

$= \frac{283}{12}$ units^2

Question 2 (14 marks)
A large Ferris wheel is a tourist attraction that lets people experience views of the city. This Ferris wheel has 20 pods that are evenly spaced around the wheel and rotates at a constant rate in an anticlockwise direction. To ride the wheel, people enter the pods from a platform that is above ground level. The Ferris wheel does not stop at any time but moves slowly enough for people to enter and exit the pods safely.



A camera is attached to a point P on the side of a pod as shown in the diagram above. This happens when the point P is at its lowest point, 4 metres above the ground at 2.00 pm.

The height, h metres, of the point P above ground level at any time t hours after 2.00 pm is given by

$$h(t) = 64 - 60cos\left(\frac{5\pi t}{2}\right)$$

a. What is the amplitude of $h(t)$?

1 mark

60

b. What is the maximum height reached by point P?

1 mark

124 m

c. Find the period of the graph of $h(t) = 64 - 60cos\left(\frac{5\pi t}{2}\right)$ and explain, in a simple sentence, what is meant by the period in this context.

2 marks

period = 4/5

1 revolution takes 4/5 hour.

d. At what time after 2.00 pm does the point P first return to its lowest point?

2 marks

$t = 0.8$

2.48 pm

e. What is the radius of the Ferris wheel?

1 mark

60 m

f. Find the distance travelled by the point P in one rotation of the Ferris wheel correct to the nearest metre.

2 marks

$d = 2\pi \times 60$

$= 120\pi = 377m$

g. Find the height of the point P above the ground at 2.36 pm

2 marks

$t = 0.6$

$h(0.6) = 64m$

h. The camera starts filming when the point P reaches 94 metres above the ground and continues filming as long as the point P remains 94 metres or more above the ground. When the point P is less than 94 metres the camera stops filming. For what fraction of one rotation is the camera filming?

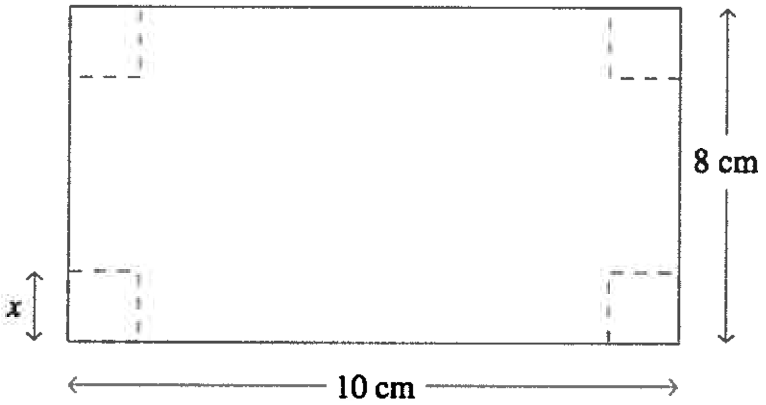
3 marks

$h(t) = 94$

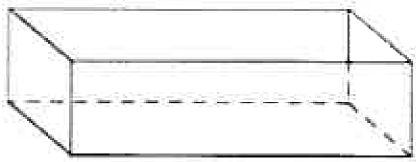
$t = \frac{4}{15}, \frac{8}{15}$

above 94m $\quad \frac{4/15}{4/5} = \frac{1}{3}$ of rotation

Question 3 (8 marks)
A rectangular sheet of cardboard is 10 cm long and 8 cm wide. A square of side length x cm is cut from each corner as shown in the diagram below



The remaining cardboard is then folded to make an open box as shown in the diagram below.



a. Show that the volume of the box, V cm^3, is given by $V(x) = 4x(5-x)(4-x)$

2 marks

$V = x(8-2x)(10-2x)$

$= x \times 2(4-x) \times 2(5-x)$

b. The domain of V is $a < x < b$
State the values of a and b.

2 marks

$a = 0$ $\quad$ $b = 4$

c. Find the value(s) of x for which the box has a volume of 24 cm^3 correct to one decimal place where appropriate.

2 marks

$V(x) = 24$

$x = 0.4 \; , \; 3.0$

d. Find the maximum possible volume for this box and the value of x for which this occurs. Give your answers correct to one decimal place.

2 marks

$x = 1.5$

$V = 52.5$

Question 4 (13 marks)
At a family gathering there are 8 cousins. Five are boys and three are girls. They are asked to stand in a line so that they can have their photo taken.

a. In how many ways can this be done if

i. There are no restrictions

1 mark

$8! = 40320$

ii. The tallest boy is first and the tallest girl is second in the line.

1 mark

$1 \times 1 \times 6! = 720$

b. (i) Three of the cousins are randomly chosen to serve afternoon tea. In how many ways could two boys be chosen?

2 marks

${}^5C_2 \times {}^3C_1$

$= 30$

(ii) What is the **probability** that two boys are chosen?

2 marks

$\frac{30}{{}^8C_3}$

$= 30/56$

$= 15/28$

At the family gathering, the cousins play a board game that involves rolling a die. When a player rolls a 6 they win a small prize.
If a player has 5 rolls of the die, find the probability that they win (3 dp).

c. Exactly two prizes

1 mark

$\frac{652}{3888}$ (~ 0.161)

d. At least one prize.

1 mark

$\frac{4651}{7776}$ (~ 0.598)

e. Three or four prizes, given that they win at least one prize.

2 marks

$\frac{Pr(3 \leq x \leq 4)}{Pr(x \geq 1)} = \frac{0.035365}{0.598122}$

$= 0.059$

A new game is introduced that involves 6 trials. The probability of success is p.
Let $X \sim Bi(6, p)$

f. **Show that** $\Pr(3 \leq X \leq 4) = 5p^3(1-p)^2(4-p)$

3 marks

$Pr(3 \leq x \leq 4) = {}^6C_3\, p^3(1-p)^3 + {}^6C_4\, p^4(1-p)^2$

$= 20p^3(1-p)^3 + 15p^4(1-p)^2$

$= 5p^3(1-p)^2[4(1-p) + 3p]$

$= 5p^3(1-p)^2(4-p)$

End of Section C

End of Examination