Name: ANSWERS

MM12 Applications of Calculus and Antidiffrentiation Test 2019

Section A: Short Answer **Time allowed: 20 minutes**

Total 19 marks

1. Find the equation of the tangent to the curve $y = 3x^2 - 4x + 1$ at the point (2, 5)

$\frac{dy}{dx} = 6x - 4$ 1

$x = 2 \quad \frac{dy}{dx} = 8$

$M_T = 8$ 1

$y - y_1 = m(x - x_1)$

$y - 5 = 8(x - 2)$

$y = 8x - 11$ 1

(3 marks)

2. For the graph of $f:[-1,2] \to R, f(x) = x^2(1-x)$

a. Find the exact coordinates of the turning points.

$f(x) = x^2 - x^3$

$f'(x) = 2x - 3x^2$ 1

Let $2x - 3x^2 = 0$

$x(2 - 3x) = 0$

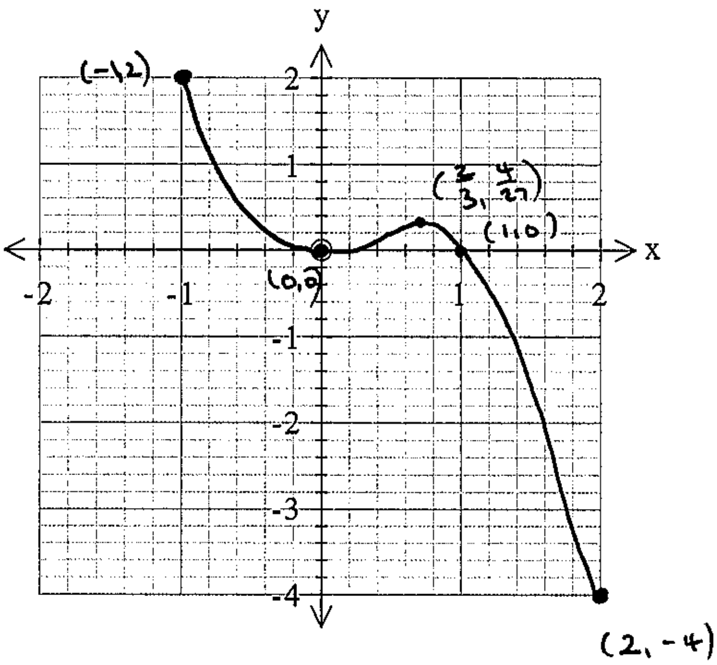
$x = 0 \quad x = 2/3$ 1

Turning points

$(0,0) \quad \left(\frac{2}{3}, \frac{4}{27}\right)$ 1

(3 marks)

b. Sketch the graph of $y = f(x)$, clearly labelling coordinates of intercepts, turning points and endpoints.

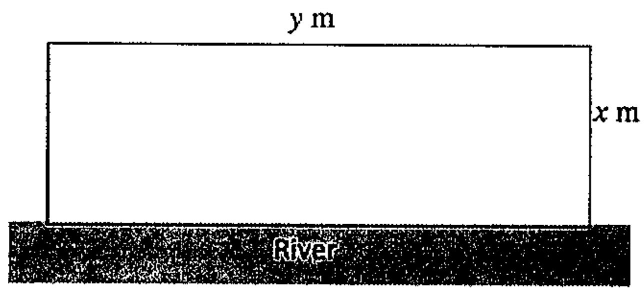


1 for shape

$\frac{1}{2}$ for each point (round down).

(3 marks)

3. A farmer wishes to build a fence around a rectangular field. He has 100m of fencing wire and will use a river running through his land as a boundary.



a. Find an expression for y in terms of x.

$2x + y = 100$

$y = 100 - 2x$ 1

(1 mark)

b. Show that the area, A of the field is given by the function $A = 100x - 2x^2$.

$A = x \times y$

$= x(100 - 2x)$ 1

$= 100x - 2x^2$

(1 mark)

c. Find $\frac{dA}{dx}$

$\frac{dA}{dx} = 100 - 4x$ 1

(1 mark)

d. Hence, find the maximum area of the field.

Let $100 - 4x = 0$ 1

$x = 25$ 1

Area $= 25 \times 50$

$= 1250 \text{ m}^2$ 1

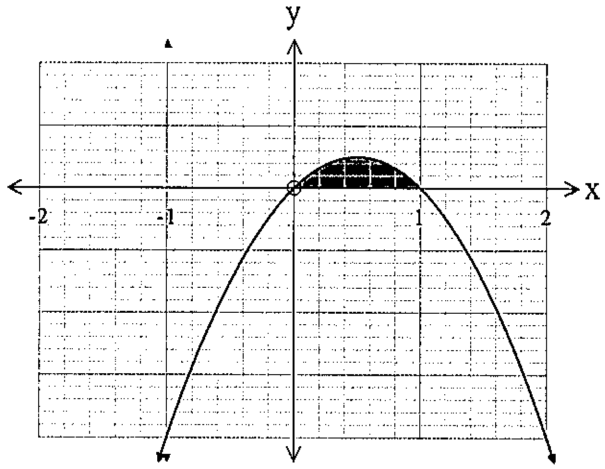
(3 marks)

4 a. Find $\int (x - x^2)dx$

$$= \underbrace{\frac{x^2}{2} - \frac{x^3}{3}}_{1} + \underbrace{c}_{1}$$

(2 marks)

b. **Hence** evaluate the area of the shaded region of the graph of $y = x(1-x)$



$$\text{Area} = \int_0^1 (x - x^2)\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \quad 1$$

$$= \frac{1}{2} - \frac{1}{3}$$

(2 marks)

$$= \frac{1}{6} \text{ sq units.} \quad 1$$

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Section B: Multiple Choice Calculators are allowed **Time allowed: 25 minutes**

7+19 = 26 marks

Circle Correct Response

1. If $f(x) = x^2(x-2)$ then $f'(2)$ equals

A. -1
B. 0
C. 4
D. -4
E. 10

2. The equation of the tangent to the curve $f(x) = 2x^2 - 3x + 1$ at the y-intercept is

A. $y = -3x + 1$
B. $y = 3x + 1$
C. $y = -3x - 1$
D. $y = -3x$
E. $y = 3$

$x = 0$

3. A car is driving in a straight line. Its position, x (in metres), from the origin, is given by the following equation.
$x(t) = 2t^2 + 5t + 10$ for $t \geq 0$, whre t is the time in seconds.

The velocity of the car is 25 m/s when the time is

A. 10 seconds
B. -10 seconds
C. -5 seconds
D. 5 seconds
E. 0 seconds

$v(t) = 4t + 5$

$4t + 5 = 25$

$t = 5$

4. If the curve with the equation $y = ax^2 + x + 1$ has a stationary point at $x = -2$, then a equals

A. $\frac{-1}{4}$
B. $\frac{1}{4}$
C. $\frac{1}{2}$
D. $\frac{-1}{2}$
E. -1

$\frac{dy}{dx} = 2ax + 1$

$2a(-2) + 1 = 0$

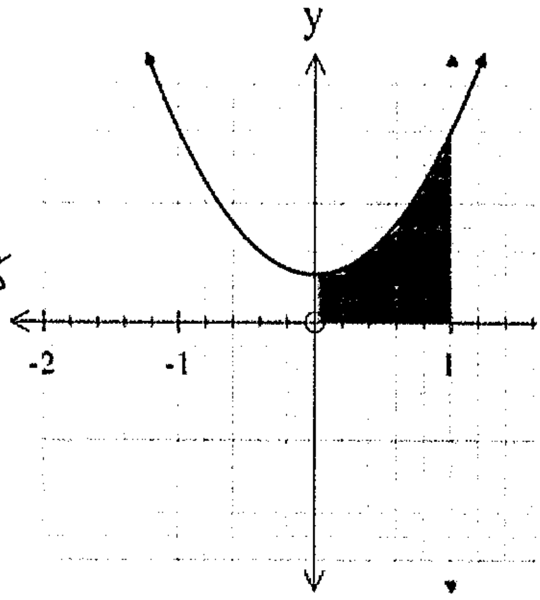
$-4a + 1 = 0$

$a = \frac{1}{4}$

5. The graph with equation $y = bx^2 + 4$ is shown. The area shaded is 8 square units. The value of b is;

A. -12
B. -15
C. 12
D. 15
E. 30

$\int_0^1 bx^2 + 4 \, dx = 8$



6. If $\frac{dy}{dx} = 2x + 4$ and (0, 1) is a point on the curve $y = f(x)$, then an expression for y is

A. $y = x^2 + 4x$
B. $y = x^2 + 4x - 1$
C. $y = x + 4$
D. $y = x^2 + 4x+1$
E. $y = 2x^2 + 4x - 1$

$y = \int 2x + 4 \, dx$

$y = x^2 + 4x + c$ $\quad x = 0$

$y = 1$

$y = x^2 + 4x + 1$ $\quad c = 1$

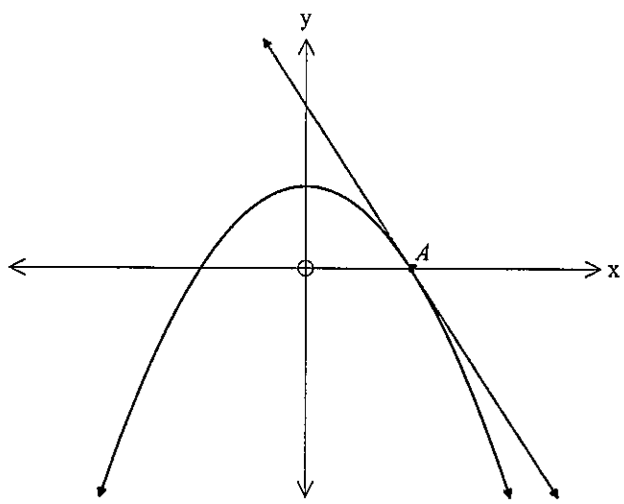
7. The value of $\int_1^2 (6x^2 - 5)dx$ is

A. 0
B. 3
C. 9
D. 20
E. 37

SECTION C: ANALYSIS **Calculators allowed**
Unless otherwise stated, exact answers should be given

Answer all questions in the space allocated.

Question 1 (7 marks)
The graph of $y = 4 - x^2$ is shown below. A tangent to the graph is drawn at point A, one of the X-intercepts of the graph.



a. Find the coordinates of the point A. (1 mark)

$(2, 0)$

b. Show the equation of the tangent is $y = -4x + 8$. (2 marks)

$\frac{dy}{dx} = -2x$

$x = 2 \quad \frac{dy}{dx} = -4$

$m_T = -4$

$y - y_1 = m(x - x_1)$

$y = -4(x - 2)$

$y = -4x + 8$

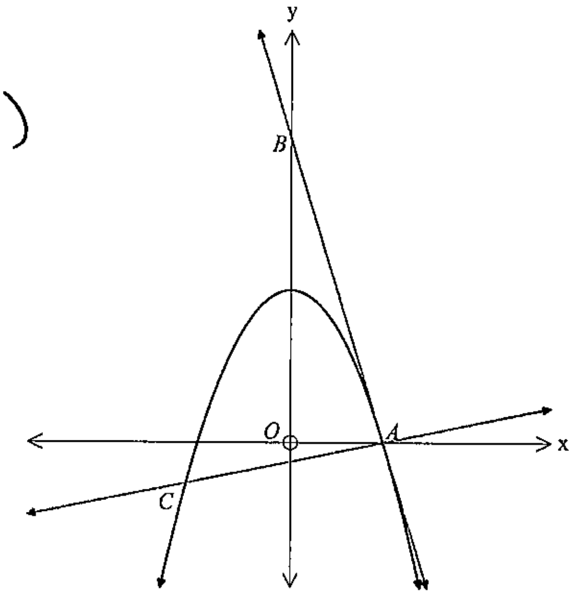
c. Find the equation of the normal to the curve at point A.

(1 mark)

$M_N = \frac{1}{4}$

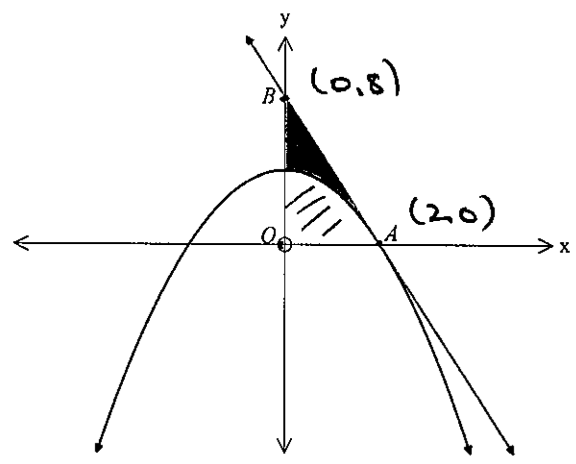
$y - 0 = \frac{1}{4}(x-2)$

$y = \frac{x}{4} - \frac{1}{2}$



d. Find the area of triangle OAB and hence find the exact value of the shaded area. Showing all relevant working.

(3 marks)



Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 2 \times 8$

$= 8$ sq units.

Shaded /// area $= \int_0^2 4 - x^2 \, dx$

$= \frac{16}{3}$

Required area $= 8 - \frac{16}{3}$

$= 8/3$ sq. units

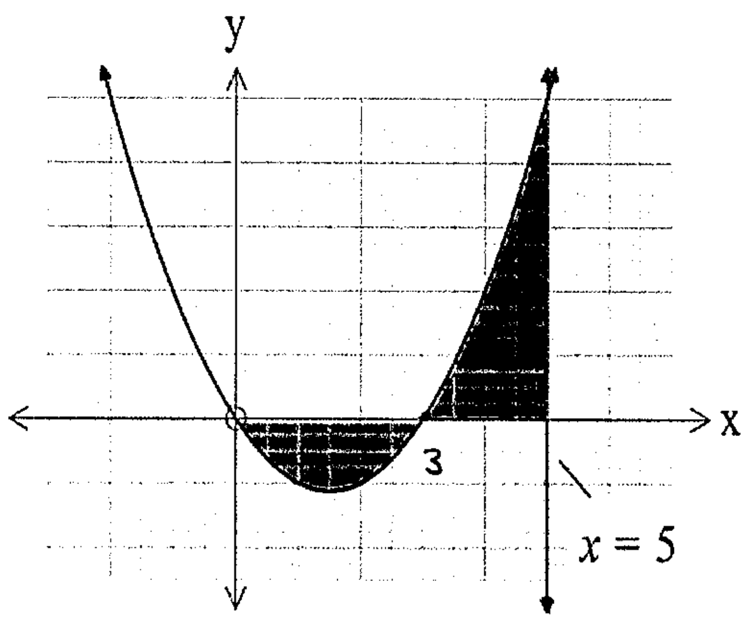
Question 2 (5 marks)

a. Find $\int(x^2 - 3x)dx$

(2 marks)

$$= \frac{x^3}{3} - \frac{3x^2}{2} + C$$

Part of the graph of $y = x^2 - 3x$ is shown.



b. **Write** the integral expression that could be used to find the shaded area.

(2 marks)

$$A = -\int_0^3 (x^2-3x)\,dx + \int_3^5 (x^2-3x)\,dx$$

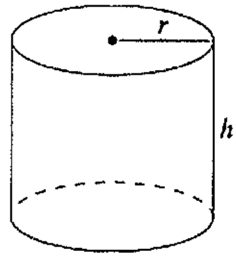
c. Hence, find the area of the shaded region.

(1 mark)

$$A = \frac{79}{6} \text{ sq. units.}$$

Question 3 (7 marks)

A closed cylindrical tank of height h metres and radius r metres has a volume of 64π cubic metres.



a. Given that $V = \pi r^2 h$, find and expression for h in terms of r.

(1 mark)

$$64\pi = \pi r^2 h$$

$$h = \frac{64}{r^2}$$

b. Given that surface area, A, is given by $A = 2\pi r^2 + 2\pi rh$, write an expression for A in terms of r.

(1 mark)

$$A = 2\pi r^2 + 2\pi r \times \frac{64}{r^2}$$

$$= 2\pi r^2 + \frac{128\pi}{r}$$

c. Find $\frac{dA}{dr}$

(1 mark)

$$\frac{dA}{dr} = 4\pi r - \frac{128\pi}{r^2}$$

d. Hence find the values of r and h, (correct to 2 decimal places) that would give a minimum surface area. Justify that your values give a minimum.

(4 marks)

let $4\pi r - \frac{128\pi}{r^2} = 0$

$r = 3.17$ 1

$h = 6.35$ 1

r	3	3.17	4
$\frac{dA}{dr}$	-6.98	0	25.13

1

local minimum. 1